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# Output Consensus Control of Nonlinear Non-minimum Phase Multi-agent Systems Using Output Redefinition Method

F. Shamsi, H. A. Talebi*, F. Abdollahi

Department of Electrical Engineering, Amirkabir University of Technology, Tehran, Iran

**ABSTRACT:** This paper concerns the problem of output consensus in nonlinear non-minimum phase systems. The main contribution of the paper is to guarantee achieving consensus in the presence of unstable zero dynamics. To achieve this goal, an output redefinition method is proposed. The new outputs of agents are functions of original outputs and internal states and defined such that the dynamics of agents are minimum phase. However, since the main objective is to achieve consensus on original outputs of agents, the consensus invariant set in the new coordinate of the agents dynamics should be defined such that if the new states of the agents converge to this invariant set, the output consensus in original system is achieved. On the other words, achieving consensus in minimum phase system with redefined output is equivalent to output consensus in original system. After defining the proper invariant set, a consensus protocol is designed to guarantee that the redefined outputs and the internal states to this set. Theoretical results are mathematically proved based on Lyapunov criterion. Numerical examples are employed to show the effectiveness of the proposed approach.

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## 1- Introduction

Cooperative control of multi-agent systems has received lots of attentions in the past few years. Some examples of multi-agent systems applications are robotics [1,2], unmanned aerial vehicles [3], autonomous underwater vehicles [4], and space systems [5].

Control problems such as sensor fusion [6,7], formation control in multi-robot systems [8-13], rendezvous for unmanned vehicles [14], and synchronization [15-19] are some of the problems that have arisen in multi-agent systems research area. One of the challenging problems in this area of research is the consensus which means agreement of agents on a certain quantity of interest [20-23].

In the past few years consensus control of multi-agent systems has been studied over various classes of agents dynamics. For the first time, consensus control of agents with single-integrator models was studied in [20,21]. Since then, several methods for achieving consensus among agents with more complex models such as double-integrators [22-25], and general linear dynamics [26,27], have been proposed. In many real applications, the dynamics of agents are nonlinear, and linearization of agents dynamics will affect the performance and the region of convergence. Therefore, some studies have been devoted to the control of multi-agent systems with different types of nonlinear models.

The consensus problem over various classes of nonlinear dynamics have been investigated in the literature. For instance, consensus control of multi-agent systems with first-order nonlinear models were studied in [28-31]. The consensus problem in second-order nonlinear multi-agent systems were investigated in [32-37]. In [38], a control scheme for achieving consensus in a multi-agent system in which the dynamics of the agents were nonlinear with relative degree two and stable internal dynamics was proposed. A technique for output synchronization has been studied in [39] for a group of agents with nonlinear dynamics by using the feedback linearization approach and based on the asymptotic stability assumption of the agents zero dynamics. Finally, in [40-42], consensus control of multi-agent systems with Lagrangian dynamics of agents were studied.

All of the above mentioned methods for solving consensus problems are generally based on dynamics inversion. In other words, they have considered nonlinear systems without internal dynamics or systems with stable internal dynamics. However, there are many systems which are non-minimum phase, i.e., poses unstable zero dynamics, (e.g. flexible manipulators, satellites with flexible panels, etc). In such systems, if the instability of internal dynamics is not considered in controller design, the internal states of the systems would grow unboundedly. Therefore, since the inverse dynamics of a non-minimum phase system is unstable, dynamics inversion based methods cannot be applied to multi-agent systems with non-minimum phase dynamics of agents. To the best of the authors' knowledge, the problem of consensus control for a leaderless multi-agent systems with nonlinear non-minimum phase dynamics is still open to study.

Control of non-minimum phase systems is a challenging problem which has been studied in literature extensively [43-48]. In such systems, perfect reference tracking cannot be achieved due to the limitations which are caused by unstable zero dynamics. Recently, the performance limitations of non-minimum phase systems have been studied in [48], and new classifications for non-minimum phase systems based on value of minimum norm of the regulation error is proposed.

To deal with non-minimum phase systems problem, several methods are proposed such as output regulation [49,50], sliding mode [51]-[53], output redefinition [54,55] etc.

Corresponding author, E-mail: alit@aut.ac.ir

Output Redefinition is one of the most common methods that is presented to solve the tracking problem in non-minimum phase system [54]. The main ideas of this method are:

- define a new output such that the zero dynamics with respect to this redefined output are stable (e.g. the system with the modified output is minimum phase,
- define a modified desired trajectory such that if the new output tracks the new desired trajectory, the original output tracks the original desired trajectory, as well.

In [55], a new method for output redefinition is proposed. In this method, the possible relative degrees for systems with new output is determined. Some conditions are considered to guarantee the minimum phase property of the system with redefined outputs.

In this paper, based on the idea of the output redefinition method, a consensus strategy is proposed to achieve output consensus in nonlinear non-minimum phase multi-agent systems. In this method, a structure for redefining a new output is presented such that the zero dynamics of the agents are exponentially stable with respect to this output. Since, there is no desired trajectory to be modified, based on this definition, a new consensus invariant set is proposed for the states of the agents to converge such that the output consensus for the original outputs of the agents is achieved, which is the main objective of control design.

The following notations are considered throughout the paper. $R$ is the set of real numbers. $R_+$ denotes the set of positive real numbers $1$ is a vector with all elements equal to $1$, $I_n$ denotes an $n{\times}n$ identity matrix, $\otimes$ stands for the Kronecker product, and $\|.\|$ indicates the 2-norm. Moreover, $\lambda_{min}(M)$ and $\lambda_{max}(M)$ denote the minimum and maximum eigenvalues of the matrix $M$, respectively.

The rest of the paper is organized as follows. Some preliminaries are presented in Section 2. In Sections 3 and 4, agents model definition and the main result for output consensus problem for nonlinear non-minimum phase multi-agent systems are investigated, respectively. A numerical example is presented in Section 5, and Section 6 concludes the paper.

## 2- Preliminaries

### 2- 1- GRAPH THERORY

The interconnection among agents is modelled using an undirected graph $G$ described by a node (agent) set V={1,2,...,N} and an edge set $\varepsilon \subseteq v{\times}v$ which $(i,j)\in\varepsilon$ if there exists an edge between the $ith$ and $jth$ nodes. The neighbouring set of the node can be defined as $N_i \subset V$ which $j\in N_i$ if $(i,j)\in\varepsilon$. In the graph, a path between two nodes is a sequence of edges connecting these two nodes. Moreover, a connected graph is a graph in which there exists a path between each two distinct nodes. The Laplacian matrix associated with the graph can be defined as $L=[l_{ij}]_{N{\times}N}$ where $l_{ij}= l_{ji}=-1, i\neq j$ if $(i,j)\in\varepsilon$ while $l_{ij}=0$ otherwise, and $l_{ii}=-\sum_{i=1}^{N} l_{ij}$. It is known that for an undirected graph, the Laplacian matrix is symmetric. It can be said that for a connected graph, the Laplacian matrix is positive semi-definite with a simple zero eigenvalue.

### 2- 2- Non-Minimum Phase Systems

Let us consider the following -order nonlinear systems [45]:

$$y^{(r)} = a(\mu,z) + b(\mu,z)u, \qquad \dot{z} = w(\mu,z), \tag{1}$$

in which $\mu = \begin{bmatrix} y & \dot{y} & \dots & y^{(r-1)} \end{bmatrix}^\top$ where $y\in R$ is the output, $z\in R^{(n-r)}$ denotes the internal states vector, and $\dot{z} = w(\mu,z)$ stands for the internal dynamics.

*Definition 1:* In (1), let $u$ is selected such that $y$ and all of its derivatives stay identically at zero. In this condition, the internal dynamics of the system are called its zero dynamics described as follows [45]:

$$\dot{z} = w(0,z). \tag{2}$$

*Definition 2:* If the zero dynamics of a nonlinear system are unstable, the nonlinear system is called non-minimum phase [45].

## 3- Problem Formulation

Consider a group of agents with identical nonlinear non-minimum phase dynamics as follows:

$$\dot{y}_i = f(y_i,z_i) + u_i, \qquad \dot{z}_i = g(y_i,z_i), i\in\{1,2,\cdots,N\}, \tag{3}$$

where $y_i$, $z_i$, and $u_i\in R$ are the output, internal state, and the input of the $ith$ agent, respectively. Moreover, $f$ and $g$ are continuous nonlinear functions. It is assumed that the dynamics of the agents are non-minimum phase. This means that the zero dynamics of agents defined by the following equation, is unstable.

$$\dot{z}_i = g(0,z_i). \tag{4}$$

The objective is to find $u_i$ such that the consensus among outputs of the agents, $y_i, i\in\{1,2,...,N\}$, is achieved, i.e. $\lim_{t\to\infty}(y_i - y_j) = 0, i,j\in\{1,2,\dots,N\}$ Therefore, the consensus invariant set can be defined by the following set:

$$M_1 = \{(y_1,y_2,\dots,y_N) \mid y_1 = y_2 = \dots = y_N\} \tag{5}$$

It can be seen that if the outputs of the agents converge to $M_1$ output consensus is obtained.

Before presenting the main results, the following assumptions are considered:

*Assumption 1:* Nonlinear functions $f$ and $g$ are Lipschitz with respect to their arguments:

$$|f(y_i,z_i) - f(y_j,z_j)| \le \gamma_1|y_i - y_j| + \gamma_2|z_i - z_j|, \qquad |g(y_i,z_i) - g(y_j,z_j)| \le \gamma_3|y_i - y_j| + \gamma_4|z_i - z_j|, \tag{6}$$

where $\gamma_1,\gamma_2,\gamma_3,\gamma_4 \in \mathbb{R}_+\cup\{0\}$ are Lipschitz constants for every $(y_i,z_i)$ and $(y_j,z_j)\in\mathbb{R}^2$.

*Assumption 2:* There exists a nonlinear Lipschitz function $\varphi(z_i)$ (with Lipschitz constant $\eta_1$) such that $\varphi(0)=0$ and the nonlinear system

$$\dot{z}_i = g(\varphi(z_i),z_i) \tag{7}$$

is exponentially stable. We assume that

$$g(y_i + \varphi(z_i),z_i) = -\alpha z_i + g_1(y_i,z_i). \tag{8}$$

It should be noted that since nonlinear functions $g(.,.)$ and $\varphi(.)$ are Lipschitz, and considering (8), it can be concluded that $g_1$ is also a Lipschitz function as follows:

$$|g_1(y_i,z_i) - g_1(y_j,z_j)| \le \gamma_5|y_i - y_j| + \gamma_6|z_i - z_j|. \tag{9}$$

It is also assumed that $\alpha > \gamma_6$ .

More details on defining new output can be found in [54] and [55].

*Assumption 3:* The states of the agents, i.e. both output and internal state, are available for transmission among the neighbouring agents.
*Remark 1:* It should be noted that by considering local Lipschitz conditions, our analysis will be the same as it for global Lipschitz conditions; however, in this condition, the results are valid locally.

## 4- Control Design

In this section, a control protocol based on the output redefinition is proposed. In this method, we first redefine a new output for agents (a new set of coordinates for describing the agents dynamics) such that the system with respect to this redefined output is minimum phase and then we define a new consensus invariant set such that if the modified outputs converge to the new invariant set, the original outputs would also converge to the original consensus invariant set, $M_1$ , as well.
First, Let us define the modified outputs of the agents as follows:

$$y_{inew} = y_i - \varphi(z_i), \tag{10}$$

where $\varphi(z_i)$ is a nonlinear Lipschitz function which satisfies Assumption 2. It can be easily seen that $(y_{inew}, z_i)$ qualifies as a new set of coordinates for the dynamics of the agents. Then, in the new coordinates, the dynamics of the agents can be rewritten as:

$$\begin{aligned} \dot{y}_{inew} &= f(y_{inew} + \varphi(z_i), z_i) \\ &\quad - \frac{\partial \varphi(z_i)}{\partial z_i} g(y_{inew} + \varphi(z_i), z_i) + u_i, \\ \dot{z}_i &= g(y_{inew} + \varphi(z_i), z_i). \end{aligned} \tag{11}$$

We claim that the new system (11) with the redefined output $y_{inew}$ is minimum phase. To prove this, the stability of the zero dynamics of the new system should be investigated. The zero dynamics of the system (11) can be obtained as follows:

$$\dot{z}_i = g(\varphi(z_i), z_i). \tag{12}$$

Using Assumption 2, it can be concluded that the zero dynamics of the new system are exponentially stable. Therefore, the new system with the redefined output is minimum phase.
The main objective of this paper is to achieve consensus among the original outputs of the agents, $y_i$,i=1,...,N , i.e. the original outputs of the agents converge to the consensus invariant set $M_1$. Therefore, we should redefine the consensus invariant set in the new coordinates such that if the states of the new system converge to the redefined set, the original outputs converge to $M_1$, as well. In this condition, it can be concluded that the proposed approach guarantees achieving output consensus in the multi-agent system with non-minimum phase dynamics of the agents.
Consider the following invariant set in the new coordinates:

$$\begin{aligned} M_2 = \{(y_{1new}, z_1, y_{2new}, z_2, \ldots, y_{Nnew}, z_N) \mid \\ y_{1new} + \varphi(z_1) = \ldots = y_{Nnew} + \varphi(z_N)\} \end{aligned} \tag{13}$$

Using the definition of $M_1$ and $y_{inew}$, it can be seen that $M_1$=$M_2$.
Let us define $M_3$ as follows:

$$\begin{aligned} M_3 = \{(y_{1new}, z_1, y_{2new}, z_2, \ldots, y_{Nnew}, z_N) \mid \\ y_{1new} = \ldots = y_{Nnew}, z_1 = \ldots = z_N\} \end{aligned} \tag{14}$$

It can be easily verified that $M_3 \subset M_2$ and since $M_1 \equiv M_2$, it can be concluded that $M_3 \subset M_1$. Therefore, if the states of the new system (11) converge to $M_3$, the original outputs of the agents will also converge to $M_1$, and hence the output consensus will be achieved.
For simplicity, we consider the control signal as follows:

$$u_i = \frac{\partial \varphi(z_i)}{\partial z_i} g(y_{inew} + \varphi(z_i), z_i) + v_i \tag{15}$$

Considering (11) and (15), the model of the agents in the new coordinates can be obtained as

$$\begin{aligned} \dot{y}_{inew} &= f_1(y_{inew} + \varphi(z_i), z_i) + v_i, \\ \dot{z}_i &= g(y_{inew} + \varphi(z_i), z_i). \end{aligned} \tag{16}$$

The control signal is designed as follows:

$$v_i = k \sum_{j \in \mathcal{N}_i} (y_{jnew} - y_{inew}), \tag{17}$$

where $k$ is a positive constant. The main results of this paper are summarized in Theorem 1.
*Theorem 1:* Consider a group of $N$ agents with uniform and nonlinear non-minimum phase dynamics (3). Suppose that the communication graph among the agents is undirected and connected, and Assumptions 1, 2 and 3 hold. Then, the control signal proposed in (15) and (17) asymptotically solves the output consensus problem of the agents if the following condition hold:

$$k > \frac{\gamma_1}{\lambda_2(\mathcal{L})} + \frac{(\gamma_2' + \gamma_5)^2}{4(\alpha - \gamma_6)\lambda_2(\mathcal{L})} \tag{18}$$

where $\gamma_2' = \gamma_1 \eta_1 + \gamma_2$
*Proof:*
According to the definition of Laplacian matrix, we have

$$\sum_{j \in \mathcal{N}_i} (y_{jnew} - y_{inew}) = -\sum_{j=1}^{N} l_{ij} y_{jnew}. \tag{19}$$

where $l_{ij}$ is the (i,j)th element of the Laplacian matrix. The closed loop dynamics of the agents can be obtained by substituting (17) into (16) and considering (19) as follows:

$$\begin{aligned} \dot{y}_{inew} &= f(y_{inew} + \varphi(z_i), z_i) - k \sum_{j=1}^{N} l_{ij} y_{jnew}, \\ \dot{z}_i &= g(y_{inew} + \varphi(z_i), z_i). \end{aligned} \tag{20}$$

Considering (8), the internal dynamics of the system (20) can be rewritten as

$$\dot{z}_i = -\alpha z_i + g_1(y_{inew}, z_i). \tag{21}$$

The consensus error for the redefined output and the error of internal state for the *ith* agent are defined as follows:

$$\begin{aligned} e_i &= y_{inew} - s, \\ \varepsilon_i &= z_i - q, \end{aligned} \tag{22}$$

where

$$s = \frac{1}{N} \sum_{j=1}^{N} y_{jnew}, q = \frac{1}{N} \sum_{j=1}^{N} z_j. \tag{23}$$

Considering this definitions, it can be concluded that if $\lim_{t\to\infty} e_i = 0$ and $\lim_{t\to\infty} \varepsilon_i = 0, i \in \{1,2,\cdots,N\}$ then the states of the system (11) converge to . Now, by using (20), (21), and (22) and considering $\sum_{j=1}^{N} l_{ij} s = 0$ , the error dynamics can be obtained as follows:

$$\begin{aligned} \dot{e}_i &= f(y_{inew} + \varphi(z_i), z_i)) \\ &\quad - \frac{1}{N}\sum_{j=1}^{N} f(y_{jnew} + \varphi(z_j), z_j) - k\sum_{j=1}^{N} l_{ij} e_j, \\ \dot{\varepsilon}_i &= -\alpha\varepsilon_i + g_1(y_{inew}, z_i) - \frac{1}{N}\sum_{j=1}^{N} g_1(y_{jnew}, z_j). \end{aligned} \tag{24}$$

To study the asymptotic stability of (24), the following Lyapunov candidate function is used:

$$V = \frac{1}{2}\sum_{i=1}^{N} e_i^2 + \varepsilon_i^2.$$

Now, taking the time derivative of $V$ along the error trajectories of (24) yields in

$$\begin{aligned} \dot{V} &= \sum_{i=1}^{N}\Big(e_i (f(y_{inew} + \varphi(z_i), z_i) \\ &\quad - \frac{1}{N}\sum_{j=1}^{N} f(y_{jnew} + \varphi(z_j), z_j) - k\sum_{j=1}^{N} l_{ij} e_j) \\ &\quad + \varepsilon_i (-\alpha\varepsilon_i + g_1(y_{inew}, z_i) - \frac{1}{N}\sum_{j=1}^{N} g_1(y_{jnew}, z_j))\Big) \end{aligned} \tag{25}$$

Now, by adding $f(s+\varphi(q),q)$ and $g_1(s,q)$ to and subtracting them from the right-hand-side of (25), and after some manipulation, one can get

$$\begin{aligned} \dot{V} &= \sum_{i=1}^{N}\Big(e_i (f(y_{inew} + \varphi(z_i), z_i) - f(s + \varphi(q), q)) \\ &\quad + e_i (f(s + \varphi(q), q) - \frac{1}{N}\sum_{j=1}^{N} f(y_{jnew} + \varphi(z_j), z_j)) \\ &\quad - \alpha\varepsilon_i^2 + \varepsilon_i (g_1(y_{inew}, z_i) - g_1(s, q)) + \varepsilon_i (g_1(s, q) \\ &\quad - \frac{1}{N}\sum_{j=1}^{N} g_1(y_{jnew}, z_j))\Big) - k\sum_{i=1}^{N} e_i \sum_{j=1}^{N} l_{ij} e_j. \end{aligned} \tag{26}$$

where $s$ and $q$ are defined in (23). Using the definition of $e_i$ and $\varepsilon_i$ in (22), it can be verified that $\sum_{i=1}^{N} e_i = 0$ and $\sum_{i=1}^{N} \varepsilon_i = 0$ . Therefore, it can be concluded that

$$\begin{aligned} &\sum_{i=1}^{N} e_i (f(s + \varphi(q), q) - \\ &\qquad \frac{1}{N}\sum_{j=1}^{N} f(y_{jnew} + \varphi(z_j), z_j)) = 0 \\ &\sum_{i=1}^{N} \varepsilon_i (g_1(s, q) - \frac{1}{N}\sum_{j=1}^{N} g_1(y_{jnew}, z_j)) = 0, \end{aligned} \tag{27}$$

Since two terms $(f(s + \varphi(q), q) - \frac{1}{N}\sum_{j=1}^{N} f(y_{jnew} + \varphi(z_j), z_j))$ and $(g_1(s,q) - \frac{1}{N}\sum_{j=1}^{N} g_1(y_{jnew}, z_j))$ are independent of the summation variable $i$. Considering (26) and (27), one can obtain

$$\begin{aligned} \dot{V} &= \sum_{i=1}^{N}\Big(e_i (f(y_{inew} + \varphi(z_i), z_i) - f(s + \varphi(q), q)) \\ &\quad - \alpha\varepsilon_i^2 + \varepsilon_i (g_1(y_{inew}, z_i) - g_1(s, q))\Big) \\ &\quad - k\sum_{i=1}^{N} e_i \sum_{j=1}^{N} l_{ij} e_j. \end{aligned} \tag{28}$$

Assumption 1 and the Lipschitz conditions (6) and (9) yield in

$$\begin{aligned} &|f(y_{inew} + \varphi(z_i), z_i) - f(s + \varphi(q), q)| \\ &\le \gamma_1 |e_i| + \gamma_2' |\varepsilon_i|, \\ &|(g_1(y_{inew}, z_i) - g_1(s, q))| \le \gamma_5 |e_i| + \gamma_6 |\varepsilon_i|. \end{aligned} \tag{29}$$

where $\gamma_2'$ is defined in Theorem 1. Therefore, by using (28) and (29), it can be concluded that:

$$\begin{aligned} \dot{V} &\le \sum_{i=1}^{N}\Big(\gamma_1 |e_i|^2 + (\gamma_2' + \gamma_5)|e_i||\varepsilon_i| \\ &\quad + (\gamma_6 - \alpha)|\varepsilon_i|^2\Big) - k\sum_{i=1}^{N} e_i \sum_{j=1}^{N} l_{ij} e_j. \end{aligned} \tag{30}$$

Considering the definition of the Laplacian matrix, one can get

$$-k\sum_{i=1}^{N} e_i \sum_{j=1}^{N} l_{ij} e_j = -k e^\top \mathcal{L} e \tag{31}$$

where $L$ is the Laplacian matrix, and $e = \begin{bmatrix} e_1 & e_2 & \cdots & e_N \end{bmatrix}^\top$ In this condition, substituting (31) into (30) results in

$$\begin{aligned} \dot{V} &\le \sum_{i=1}^{N}\Big(\gamma_1 |e_i|^2 + (\gamma_2' + \gamma_5)|e_i||\varepsilon_i| \\ &\quad + (\gamma_6 - \alpha)|\varepsilon_i|^2\Big) - k e^\top \mathcal{L} e. \end{aligned} \tag{32}$$

It is assumed that the communication graph of the network is connected. Therefore, the Laplacian matrix $L$ has exactly one zero eigenvalue and the other eigenvalues are positive real numbers denoted by $\lambda_2 \le \ldots \le \lambda_N$. Moreover, the right and left eigenvectors corresponding to the zero eigenvalue are $\frac{1}{\sqrt{N}}\mathbf{1}$ and $\frac{1}{\sqrt{N}}\mathbf{1}^\top$, respectively [46]. There exists a unitary matrix $\Phi$ such that $\Phi^\top \mathcal{L} \Phi = \Lambda = \mathrm{diag}\{0, \lambda_2, \cdots, \lambda_N\}$, where the first column of $\Phi$ is $\frac{1}{\sqrt{N}}\mathbf{1}$. Now, consider the following definition:

$$\omega \triangleq \begin{bmatrix} \omega_1 & \omega_2 & \cdots & \omega_N \end{bmatrix}^\top = \Phi^\top e, \tag{33}$$

Since the left eigenvector associated with the zero eigenvalue of the Laplacian matrix is $\frac{1}{\sqrt{N}}\mathbf{1}^\top$, it can be verified that

$$\omega_1 = \frac{\mathbf{1}^\top}{\sqrt{N}} e = 0 \tag{34}$$

Therefore, by defining $\zeta^\top = \begin{bmatrix} \omega_2 & \cdots & \omega_N \end{bmatrix}^\top$, one can conclude that $\omega = \begin{bmatrix} 0 & \zeta^\top \end{bmatrix}^\top$, where $\zeta \in \mathbb{R}^{N-1}$. By substituting $\mathcal{L} = \Phi\Lambda\Phi^\top$ into (32), one can obtain

$$\begin{aligned} \dot{V} &\le \sum_{i=1}^{N}\Big(\gamma_1 |e_i|^2 + (\gamma_2' + \gamma_5)|e_i||\varepsilon_i| \\ &\quad + (\gamma_6 - \alpha)|\varepsilon_i|^2\Big) - k e^\top \Phi\Lambda\Phi^\top e. \end{aligned} \tag{35}$$

By using the definition (33), (35) can be restated as follows:

$$\dot{V} \leq \sum_{i=1}^{N}\Big(\gamma_1 |e_i|^2 + (\gamma_2' + \gamma_5)|e_i||\varepsilon_i| + (\gamma_6 - \alpha)|\varepsilon_i|^2\Big) - k\,\omega^{\top}\Lambda\omega. \tag{36}$$

Considering that $\Lambda$ is a diagonal matrix containing the eigenvalues of the Laplacian matrix and by using (34), the following inequality can be concluded from (36):

$$\dot{V} \leq \sum_{i=1}^{N}\Big(\gamma_1 |e_i|^2 + (\gamma_2' + \gamma_5)|e_i||\varepsilon_i| + (\gamma_6 - \alpha)|\varepsilon_i|^2\Big) - k\,\zeta^{\top}\Gamma\zeta. \tag{37}$$

where $\Gamma = \mathrm{diag}\{\lambda_2, \cdots, \lambda_N\}$. Since $\Gamma$ is a diagonal positive definite matrix, it can be concluded that

$$-k\,\zeta^{\top}\Gamma\zeta \leq -k\,\lambda_2(\mathcal{L})\|\zeta\|^2 . \tag{38}$$

Now, substituting (38) into (37) results in

$$\dot{V} \leq \sum_{i=1}^{N}\Big(\gamma_1 |e_i|^2 + (\gamma_2' + \gamma_5)|e_i||\varepsilon_i| + (\gamma_6 - \alpha)|\varepsilon_i|^2\Big) - k\,\lambda_2(\mathcal{L})\|\zeta\|^2 . \tag{39}$$

Since $\Phi$ is a unitary matrix, it can be verified that $\|\omega\|=\|\Phi e\|=\|e\|$. Moreover, based on the definitions of $\omega$, one can say that $\|\zeta\|=\|\omega\|$, implying that $\|\zeta\|^2=\|e\|^2$, and therefore

$$\|\zeta\|^2 = \sum_{i=1}^{N}|e_i|^2 . \tag{40}$$

In this condition, substituting (40) into (39) yields in

$$\dot{V} \leq \sum_{i=1}^{N}\Big((\gamma_1 - k\,\lambda_2(\mathcal{L}))|e_i|^2 + (\gamma_2' + \gamma_5)|e_i||\varepsilon_i| + (\gamma_6 - \alpha)|\varepsilon_i|^2\Big) \tag{41}$$

One can restate (41) as the following form:

$$\dot{V} \leq -\delta_i^{\top} Q\,\delta_i , \tag{42}$$

where

$$\delta_i = \big[|e_i| \quad |\varepsilon_i|\big]^{\top}, \tag{43}$$

$$Q = \begin{bmatrix} (k\,\lambda_2(\mathcal{L}) - \gamma_1) & -\dfrac{(\gamma_2' + \gamma_5)}{2} \\ -\dfrac{(\gamma_2' + \gamma_5)}{2} & (\alpha - \gamma_6) \end{bmatrix}. \tag{44}$$

Now, let us investigate the sign of $\dot{V}$. Considering (42), (43), and (44), it can be concluded that if we select control gain $k$ such that $Q$ is positive definite, $\dot{V}$ would be negative definite, and therefore the consensus errors of the redefined output and internal state converge to zero asymptotically. Hence, the states of redefined system (11) converge to $M_3$. It can be verified that condition (18) guarantees that $Q$ is a positive definite matrix. On the other hand, since $M_3 \subset M_2$ and $M_1 \equiv M_2$, one can conclude that the original outputs of the agents converge to $M_1$ and therefore, output consensus is achieved. This completes the proof.

## 5- Simulation

In this section, two examples are considered to show the effectiveness of the proposed approach.

*Example 1:* Consider a group of four agents described by the following nonlinear non-minimum phase dynamics:

$$\dot{y}_i = 2\sin(y_i) + 0.5 z_i + u_i ,$$
$$\dot{z}_i = \tanh(z_i) + y_i .$$

It is assumed that the communication graph is as shown in Fig. 1. Therefore, the minimum eigenvalue of the Laplacian matrix is 2. Moreover, the Lipschitz constants can be calculated as $\gamma_1=2$, $\gamma_2=0.5$, $\gamma_3=1$ and $\gamma_4=1$. To redefine the output, $\varphi(z_i)=-2z_i$ is selected. It can be verified that using $y_{inew}=y_i+2z_i$, the redefined system is minimum phase. In this condition, to satisfy (18), the controller gain is set to $k$=5. Now, let us define the agent coupling error with respect to neighbouring agents as follows:

$$e_{ci} = y_i - \frac{1}{n_i}\sum_{j\in\mathcal{N}_i} y_j ,$$

where $n_i$ denotes the number of the neighbours of *ith* agent. The original outputs and the coupling errors of the agents are depicted in Fig. 2 and Fig. 3, respectively and as shown in these figures, the outputs of the agents converge to a common value and also the coupling errors converge to zero, confirming that output consensus is achieved. The internal states of the agents are shown in Fig. 4. It can be seen that, the internal states of the agents are bounded. Furthermore, the control signal of each agent is depicted in Fig. 5, which are also bounded.

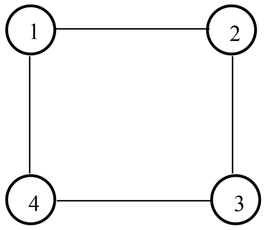


**Fig. 1. The communication topology associated with the MAS network.**

*Example 2:* In this example, we consider a network of four disturbed forced Van der Pol oscillators which have a wide range of applications in various areas such as physics, biological systems, social networks, engineering, etc. [47]. The dynamical model of each oscillator is

$$\dot{y}_i = -0.1 z_i + 0.3\left(1 - z_i^2\right) y_i + \sin(t) + u_i ,$$
$$\dot{z}_i = y_i .$$

Based on Definition 2, the zero dynamics of the agents can be obtained as $\dot{z}_i = 0$ which are not asymptotically stable. Thus, they are non-minimum phase. The communication topology of the multi-agent is as the previous example. We consider $\varphi(z_i)=-1.5z_i$ for output redefinition. Based on the results of Theorem 1, the controller gain is set to $k$=2. It should be noted that the nonlinear terms in the dynamics of the agents are locally Lipschitz. Therefore, the results of Theorem 4.2 are valid locally. The main outputs of the agents

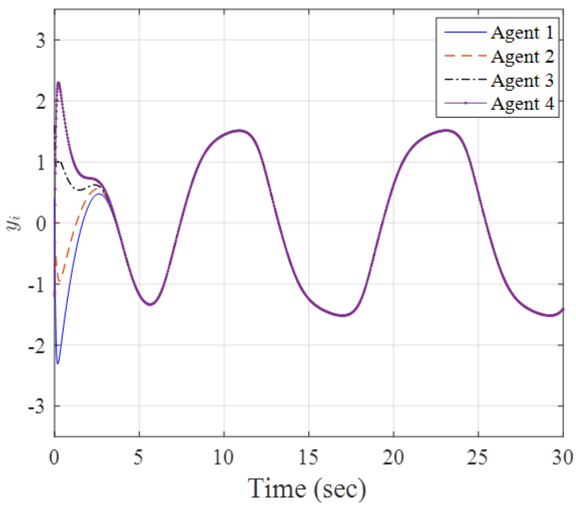


Fig. 2. The outputs of the agents in Example 1.

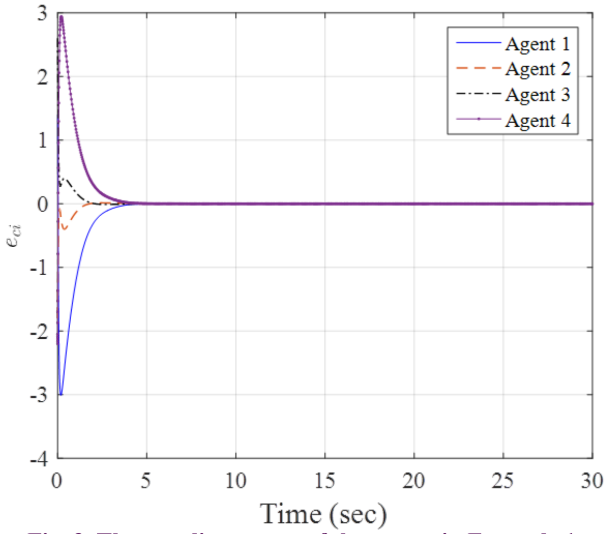


Fig. 3. The coupling errors of the agents in Example 1.

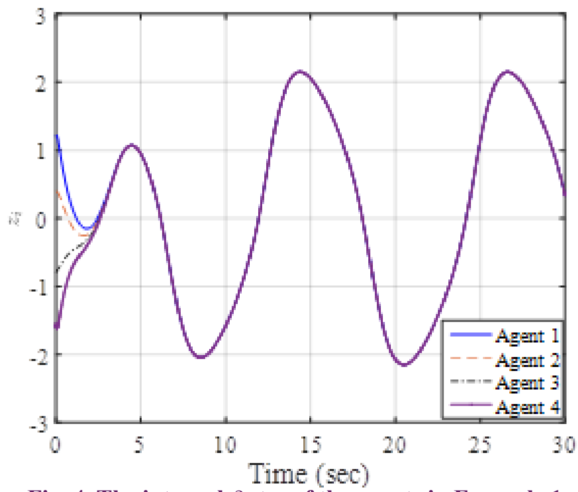


Fig. 4. The internal states of the agents in Example 1.

and the consensus coupling errors are depicted in Fig. 6 and Fig. 7, respectively. It can be seen that the outputs consensus is achieved, and as shown in Fig. 8, the internal states of the agents are bounded. Furthermore, the control signal of each agent is given in Fig. 9.

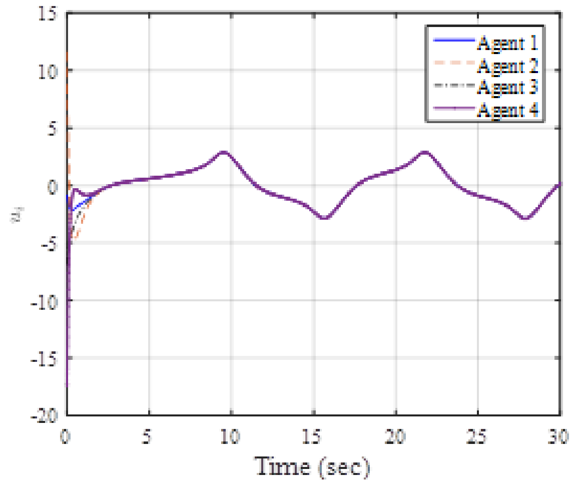


Fig. 5. The inputs of the agents in Example 1.

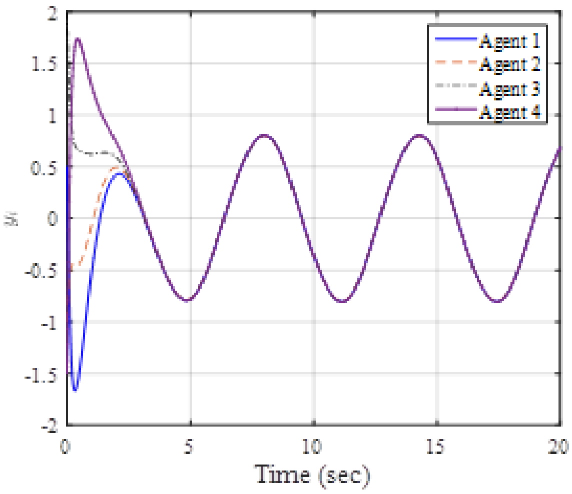


Fig. 5. The inputs of the agents in Example 1.

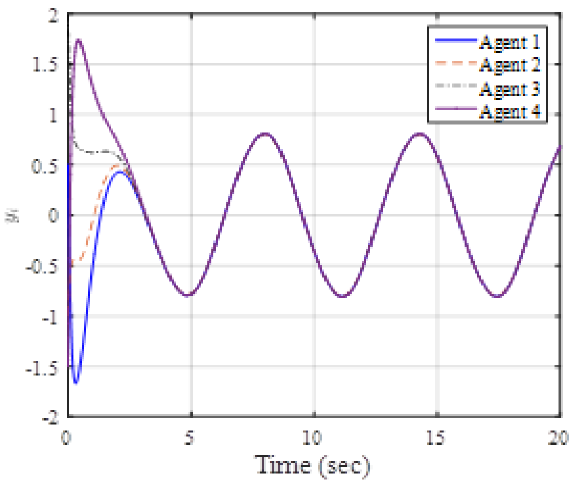


Fig 6. The outputs of the agents in Example 2.

## 6- Conclusion

A new consensus strategy for a network of agents with nonlinear and non-minimum phase dynamics based on output redefinition was proposed in this paper. In this method, an output redefinition was done such that the new system is minimum phase with respect to the redefined output. Then,

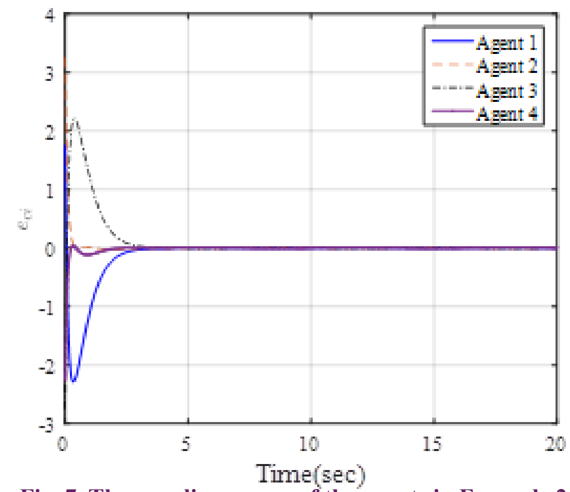


**Fig. 7. The coupling errors of the agents in Example 2.**

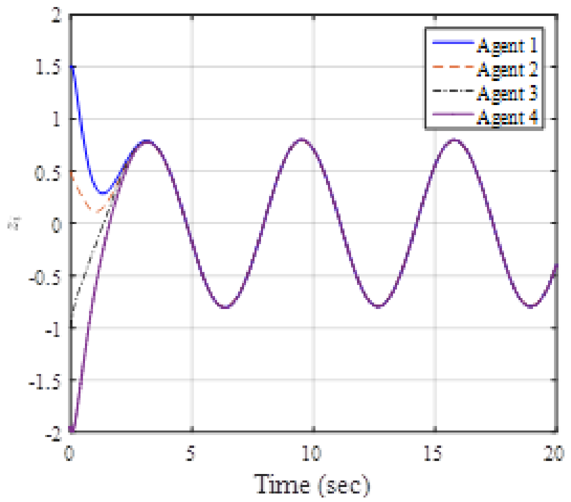


**Fig. 8. The internal states of the agents in Example 2.**

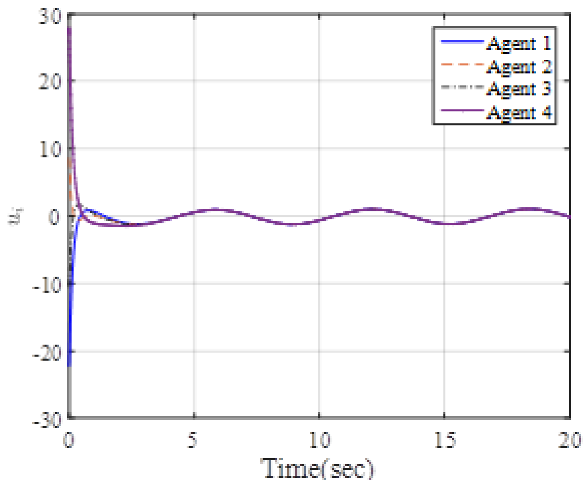


**Fig. 9. The inputs of the agents in Example 2.**

we defined a new consensus invariant set such that if the states of the redefined system converge to this set, the original outputs converge to original consensus invariant set, as well. Therefore, the proposed strategy guaranteed achieving output consensus in the network of agents with nonlinear non-minimum phase dynamics.

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