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# Conjugate problem of combined radiation and laminar forced convection separated flow

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**ABSTRACT:** This paper presents a numerical investigation for laminar forced convection flow of a radiating gas in a rectangular duct with a solid element that makes a backward facing step. The fluid is treated as a gray, absorbing, emitting and scattering medium. The governing differential equations consisting the continuity, momentum and energy are solved numerically by the computational fluid dynamics techniques. Since the present problem is a conjugate one and both gas and solid elements are considered in the computational domain, simultaneously, the numerical solution of Laplace equation is obtained in the solid element for temperature calculation in this area. Discretized forms of these equations are obtained using the finite volume method and solved by the SIMPLE algorithm. The radiative transfer equation (RTE) is also solved numerically by the discrete ordinate method (DOM) for computation of the radiative term in the gas energy equation.The streamline and isotherm plots in the gas flow and the distributions of convective, radiative and total Nusselt numbers along the solid-gas interface are presented. Besides, the effects of radiation conduction parameter and also solid to gas conduction ratio as two important parameters on thermo hydrodynamic characteristics of such thermal system are explored. It is revealed that the radiative Nusselt number on the interface surface is much affected by RC parameter but the radiation conduction parameter has not considerable effect on the convective Nusselt number. Comparison between the present numerical results with those obtained by other investigators for the case of non-conjugate problems shows good consistency.

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## 1- Introduction

Two-dimensional forced convection flow over a backward or forward facing step in ducts is widely encountered in a great number of engineering applications. In this type of convection flow, separating and reattaching regions exist because of the sudden contraction and expansion in flow geometry. Regarding to this subject, one can mention to convection flow for cooling in electronic systems, power generating equipment, gas turbine blades, heat exchangers, combustion chamber and ducts flows in industrial applications. In some of the mentioned advices, specially, for high temperature thermal systems and when soot particles exist in the combustion product, the radiation effect may become considerable. Hence, in order to have credible results in the analysis of these types of convective flow, working gas must be considered as a radiating medium that can absorb, emit and scatter thermal radiation. The fluid flow over backward facing step (BFS) or forward facing step (FFS) has the most features of convective separated flows. Although the geometry of BFS or FFS flow is very simple, the heat transfer and fluid flow in ducts with sudden expansion and contraction (because of the step) contain most of complexities. Conjugate heat transfer happens at the same time that heat transfer in fluid region is coupled with heat transfer in solid zone. While the governing energy equations of solid and fluid regions need to be solved, simultaneously.

Several investigators expressed many theoretical studies about laminar convection flow over BFS in a 2-D duct [1-6] in a forced convection problem, when the flowing gas acts as a participating medium, its complex absorption, emission and scattering caused a considerable difficulty in the simulation of these flows. There aren't many numbers of literatures about the radiative transfer problems in convection flows with complex 2-D and 3-D geometries. Bouali and Mezrhab [7] studied heat transfer by laminar forced convection with considering surface radiation in a divided vertical channel with isotherm side walls. They found that the surface radiation has considerable effect on the Nusselt number in convective flow with high Reynolds numbers

Two-dimensional forced convection laminar flow of radiating gas over a backward facing step in a duct was analyzed by Ansari and Gandjalikhan Nassab [8]. Effects of wall emissivity, Reynolds number and its interaction with the conduction-radiation parameter on heat transfer behavior of the system were searched.

Recently many investigators pay more attention to conjugate heat transfer study. Vynnycky et al. [9] presented closed form relations for interface temperature, local Nusselt number and average Nusselt number for laminar convection flow over a flat plate as conjugate case. A study by Chiu et al. [10-11] deals to conjugate heat transfer of horizontal channel both experimentally and numerically. It was determined that the conjugate heat transfer significantly affects the temperature and heat transfer rates at the surface of the heated region.

Laminar mixed convection with surface radiation from a vertical plate with a heat source as conjugate case was investigated by Rao et al. [12]. The governing equations using stream function–vorticity formulation is solved. Steady and unsteady conjugate heat transfer from a cylinder in laminar convection flow, are presented by Juncu. [13-14]. the numerical investigation for Reynolds number ranging from 2 to 20 was carried out and also the alternating direction

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implicit (ADI) method for solving the governing equations, numerically was followed by him.

About conjugate heat transfer in laminar convection flow, there are limited numbers of research work, such that all of them are related to forced and mixed convection without considering radiation effects. Related to this subject, a conjugate turbulent mixed convection flow from a vertical channel with four heat sources was analyzed numerically by Mathews et al. [15]. The series of governing equations was solved by the CFD method whereas the standard turbulent model was employed in calculation of turbulent fluctuations. The effect of several important parameters, such as Reynolds and Richardson numbers on thermal characteristics of the system were investigated.

Recently, Kanna and Das [16] presented closed form solution for laminar wall jet flow as conjugate case. They have explored closed form solution for local and average Nusselt numbers, and interface temperature and validated the same by numerical simulations.

Despite the conjugate combined convection and radiation heat transfer in laminar channel flow is relatively important in engineering application, and the problem is far more involved, it is still not studied by any investigator. Toward this end, the present work deals to numerical solution of the governing equations for convection laminar flow of participating gas including continuity, momentum and energy equations. For computation of radiative heat flux inside the participating working gas, the RTE is solved using the DOM. As the problem is a conjugate one, the conduction equation in the solid element is also solved simultaneously. By this numerical strategy, the pressure, velocity and temperature fields for the fluid flow and also the temperature distribution in the solid element are obtained. Also, an attempt is made to investigate the effects of several important factors including radiating parameters on thermal characteristics of the laminar convection flow.

## 2- Theory

Two-dimensional combined convection, conduction and radiation heat transfer in fluid flow and conduction heat transfer in solid region in horizontal heated rectangular duct with a BFS step are numerically simulated. Schematic of the computational domain is shown in Fig.1.upstream and downstream heights of the duct are h and H, respectively, while the expansion ratio (ER=S/H) is considered 1/2. The upstream and downstream length of the duct is considered to be L1=5H and L2=30H. This is made to ensure that the flow at the exit section becomes fully developed.

### 2- 1- Basic Equations

The governing equations for incompressible, steady and two-dimensional laminar flow are the conservations of mass, momentum and energy that can be expressed as follows:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\frac{\mu}{\rho}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right) \tag{2}$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\frac{\mu}{\rho}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right) \tag{3}$$

$$\begin{aligned}&\frac{\partial}{\partial x}\left(\rho u c_p T\right)+\frac{\partial}{\partial y}\left(\rho u c_p T\right)\\&=k_f\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)-\nabla . q_r\end{aligned} \tag{4}$$

Also, the Laplace equation is used to find the temperature distribution inside the solid element.

$$\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)=0 \tag{5}$$

### 2- 2- Gas Radiation Modeling

The energy equation in flow includes the convective and conductive terms, the radiative term as the divergence of the radiative heat flux, i.e$\nabla . q_r$ also presented. This radiative term can be calculated as follow [17]:

$$\nabla . \vec{q}_r=\sigma_a\left(4\pi I_b(\vec{r})-\int_{4\pi} I(\vec{r},\vec{s})d\Omega\right) \tag{6}$$

In the equation (6), $\sigma_a$ is the scattering coefficient, $I(\vec{r},\vec{s})$ is the radiation intensity at the situation $\vec{r}$ and in the direction $\vec{s}$ and $I_b(\vec{r})=\sigma\left(T(\vec{r})\right)^4/\pi$ is the black body radiation intensity for calculation of $\nabla . q_r$ , the radiation intensity field is already needed. To obtain this term, it is necessary to solve the radiative transfer equation. This equation for an absorbing, emitting and scattering gray medium can be expressed as [17]:

$$\begin{aligned}&(\vec{s}.\nabla)I(\vec{r},\vec{s})=-\beta I(\vec{r},\vec{s})+\sigma_a I_b(\vec{r})\\&+\frac{\sigma_s}{4\pi}\int_{4\pi} I(\vec{r},\vec{s})\varphi(\vec{s},\vec{s}')d\Omega'\end{aligned} \tag{7}$$

Where, $\sigma_s$ is the scattering coefficient, $\beta=\sigma_a+\sigma_s$ the extinction coefficient and is $\varphi(\vec{s},\vec{s}')$ the scattering phase function for the radiation from incoming direction s' and confined within the solid angle dΩ' to scattered direction $\vec{s}$ confined within the solid angle. In this study, the phase function is equal to unity because the assumption of isotropic scattering medium. The boundary condition for a diffusely emitting and reflecting gray wall is:

$$I(\vec{r}_w,\vec{s})=\varepsilon_w I_b(\vec{r}_w)+\frac{(1-\varepsilon_w)}{\pi}\int_{\vec{n}_w.\vec{s}'<0} I(\vec{r}_w,\vec{s})\left|\vec{n}_w.\vec{s}'\right|d\Omega' \qquad \vec{n}_w.\vec{s}>0 \tag{8}$$

Thus $\varepsilon_w$ is the wall emissivity, $I_b(\vec{r}_w)$ is the black body radiation intensity at the temperature of the boundary surface and $\vec{n}_w$ is the outward unit vector normal to the surface. Since, the RTE depends on the temperature fields through the emission term, it must be solved simultaneously with overall energy equation. RTE is an integro-differential equation which can be solved with discrete ordinates method.

In the DOM, equation (6) is solved for a set of n different directions, $\vec{s}_i$ , i=1, 2, 3, n and integrals over solid angle are replaced by the numerical quadrature, that is shown as follow,

$$\int_{4\pi} f(\vec{s})d\Omega \cong \sum_{i=1}^{n} w_i f(\vec{s}_i) \tag{9}$$

Where $W_i$ is the quadrature weights associated with the directions $\vec{s}_i$. Thus, according to this method, equation (6) is approximated by a set of n equations, as given:

$$(\vec{s}_i.\nabla)I(\vec{r},\vec{s}_i) = -\beta I(\vec{r},\vec{s}_i) + \sigma_a I_b(\vec{r}) + \frac{\sigma_s}{4\pi}\sum_{j=1}^{n} I(\vec{r},\vec{s}_j)\varphi(\vec{s}_j,\vec{s}_i)w_j \qquad i = 1,2,3,...,n \tag{10}$$

Subjected to the boundary conditions, as follow:

$$I(\vec{r}_w,\vec{s}_i) = \frac{(1-\varepsilon_w)}{\pi}\sum_{\vec{n}_w.\vec{s}_j<0} I(\vec{r}_w,\vec{s}_j)\left|\vec{n}_w.\vec{s}_j\right| w_j + \varepsilon_w I_b(\vec{r}_w) \quad \vec{n}_w.\vec{s}_i > 0 \tag{11}$$

Also $\nabla.\vec{q}_r$ is expressed as:

$$\nabla.\vec{q}_r = \sigma_a\left(4\pi I_b(\vec{r}) - \sum_{i=1}^{n} I(\vec{r},\vec{s}_i)w_i\right) \tag{12}$$

At any arbitrary surface, heat flux may also be determined from the surface energy balance as follow:

$$\vec{q}.n(r_w) = \varepsilon_w\left(\pi I_b(r_w) - \sum_{\vec{n}_w.\vec{s}_j<0} I(\vec{r}_w)\left|\vec{n}_w.\vec{s}_j\right| w_i\right) \tag{13}$$

The details of the numerical solution of RTE by DOM were also described in the previous work by the second author, in which the thermal characteristics of porous radiant burners were investigated [18].

The walls in contact with the fluid are assumed to emit and reflect diffusely with constant wall emissivity $\varepsilon_w$=0.8, for the radiative boundary conditions. As well as, the inlet and outlet sections are considered for radiative transfer as black walls at the fluid temperatures respectively, in inlet and outlet sections.

## 2- 3- Non-dimensional forms of the Governing Equations

In numerical solution of this study, the set of governing equation including the continuity, momentum and energy, the following dimensionless parameters are used to calculate the non-dimensional form of these equations:

$$(X,Y) = \left(\frac{x}{H},\frac{y}{H}\right),(U,V) = \left(\frac{u}{U_0},\frac{v}{v_0}\right) \qquad P = \frac{p}{\rho U_\circ^2}$$

$$\Theta = \frac{T - T_{in}}{T_w - T_{in}} \quad \theta_1 = \frac{T_{in}}{T_w - T_{in}} \quad \theta_2 = \frac{T_w}{T_{in}} \quad I^* = \frac{I}{\sigma T_w^4} \quad \tau = \beta.H$$

$$K = \frac{k_s}{k_f} \quad 1-\omega) = \frac{\sigma_a}{\beta} \quad \Pr = \frac{\nu}{\alpha} \quad \mathrm{Re} = \frac{\rho U_0 H}{\mu} \quad Pe = \mathrm{Re}.\Pr$$

$$RC = \frac{\sigma T_w^3 H}{k_f} \quad q_r^* = \frac{q_r}{\sigma T_w^4} \tag{14}$$

By using the upper dimensionless equations (14), the non-dimensional forms of the governing equations become as follows:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial X} = 0 \tag{15}$$

$$\frac{\partial}{\partial X}\left(U^2 - \frac{1}{Re}\frac{\partial U}{\partial X}\right) + \frac{\partial}{\partial Y}\left(UV - \frac{1}{Re}\frac{\partial U}{\partial Y}\right) = -\frac{\partial P}{\partial X} \tag{16}$$

$$\frac{\partial}{\partial X}\left(UV - \frac{1}{Re}\frac{\partial V}{\partial X}\right) + \frac{\partial}{\partial Y}\left(V^2 - \frac{1}{Re}\frac{\partial V}{\partial Y}\right) = -\frac{\partial P}{\partial Y} \tag{17}$$

$$\frac{\partial}{\partial X}\left(U\Theta - \frac{1}{Pe}\frac{\partial \Theta}{\partial X}\right) + \frac{\partial}{\partial Y}\left(V\Theta - \frac{1}{Pe}\frac{\partial \Theta}{\partial Y}\right) + \frac{\tau(1-\omega)RC\,\theta_1\theta_2}{Pe}\left[\frac{4}{\theta_2^4}\left(\frac{\Theta}{\theta_1}+1\right)^4 - \sum_{I=1}^{n} I_i^* w_i\right] = 0 \tag{18}$$

## 2- 4- Boundary Condition

1. The flow at the inlet is assumed to be fully developed, where a dimensionless velocity distribution and is shown as [19]

$$U(Y) = 12\left(Y - Y^2\right) \tag{19}$$

The dimensionless temperature distribution at the inlet is shown as

$$\Theta_{in} = 0 \tag{20}$$

2. The temperature is considered a constant value ($\Theta = 1$) on the step vertical wall in contact with the flow, at the top flat wall, and the bottom solid region boundary. The left and right solid region boundary is isolated. A no-slip velocity boundary condition is applied for the walls in contact with the fluid.

3.On the interface surface between the gas flow and solid element, in addition to condition of having the same local temperature for both solid and gas phases, the following criteria is imposed based on the continuity of heat flux at this boundary:

$$-K\frac{\partial\Theta}{\partial n}\Big|_{n=0} = -\frac{\partial\Theta}{\partial n}\Big|_{n=0} + RC\,\theta_1\theta_2\varepsilon_w\left[\frac{1}{\theta_2^4}\left(\frac{\Theta}{\theta_1}+1\right)^4 - \sum_{n.s_w}\overrightarrow{I^*}(\vec{r}_w)\left|r_W.s_w\right| w_i\right] \tag{21}$$

$$\Theta_s = \Theta_f \tag{22}$$

Where n is the normal vector to the wall, $\Theta_f$ is temperature for fluid region and $\Theta_s$ is solid temperature at interface.

4. At the duct outlet section, a zero gradient in stream-wise direction is considered for all existent dependent variables in this study.

## 2- 5- The Main Physical Quantities

The main physical values of interest in heat transfer study are the mean bulk temperature and Nusselt number.

The mean bulk temperature along the channel was computed using the following equation:

$$\Theta_b = \frac{\int_0^1 \Theta U dY}{\int_0^1 U dY} \tag{23}$$

The energy transport from the duct wall to the gas flow, in the combined convection-radiation heat transfer, depends on two related factors:

1.Fluid temperature gradient on the wall

2.Rate of radiative heat exchange on boundary surface

So, total heat flux on the wall is calculated by this equation:

$$q_t = q_c + q_r = -k_f(\frac{\partial T}{\partial y}) + q_r \tag{24}$$

Thus, the function of total Nusselt number ( $Nu_t = \frac{q_t H}{k_f (T_W - T_M)}$ ) is the sum of local convective Nusselt number ($Nu_c$), and local radiative Nusselt number ($Nu_r$).

Total Nusselt numbers is given as follows [20]:

$$Nu_t = Nu_c + Nu_r = \frac{-1}{\Theta_w - \Theta_b} \frac{\partial \Theta}{\partial Y}\bigg|_{Y=0} + \frac{RC.\theta_1.\theta_2}{\Theta_w - \Theta_b} q_r^* \tag{25}$$

Equation (25) includes two parts. The first term on the right hand side illustrates the convective Nusselt number, whilst the second term is the radiative Nusselt number. It should be attended that for pure convective hat transfer, total Nusselt number is equal to the convective one.

## 3- Numerical Procedure

Integration of continuity, momentum and energy equations over an elemental cell volume with staggered control volumes for the x-, and y- velocity components gives the finite difference forms of these equations (1) to (5). Other variables of interest were calculated at the grid nodes. Discrete procedure utilizes the line-by-line method connected to the finite volumes technique with the Hybrid technique [21] for the calculations of convective terms at the faces of control volumes and central differencing is used to discretize the diffusion terms. These computations are coded into a computer program in FORTRAN. The discretized forms of the governing equations were numerically solved by the SIMPLE algorithm of Patankar and Spalding [21]. Numerical calculations were carried out by writing a computer program in FORTRAN. To simulate the solid element in the computational domain, the blocked off method is used in this study. In addition, for computating the divergence of radiative heat flux, which is needed for the numerical solution of the energy equation, $S_4$ approximation has been used. Numerical solutions are obtained iteratively such that iterations are ended when sum of the absolute residuals become less than $10^{-5}$ for momentum and energy equations. But in the numerical solution of RTE, the maximum difference between the radiative intensities calculated during two consecutive iteration levels did not exceed $10^{-6}$ at each nodal point for the converged solution. This numerical strategy can obtain the velocity, temperature and radiation intensity distributions inside the flow domain and the temperature distribution in the solid element.

The grid-independence study was achieved by a grid size of 350×80 with clustering near to the duct walls, while using denser mesh of 400×90 resulted in less than 3.1% difference in the value of Nusselt numbers on the interface surface. Therefore, the grid size of 350×80 is sufficient for this simulation. The flow chart of present numerical computations in solving the set of governing equations is drawn in Fig. 2.

The computational procedure is validated against the numerical results of Abu Nada [22] for a laminar convection flow in a duct with sudden expansion, while the working gas is non-participating medium and also the bottom wall is kept at constant temperature. It should be mentioned that the present computations can simulate such convective flow where the solid-gas conduction ratio is considered to be high. The distributions of convective Nusselt number along the solid-gas interface for three different values of K are shown in Fig. 3. This figure predicts zero value for convection Nusselt number at the corner x=0 where the gas is at rest. Then convection coefficient increases at the recirculated zone up to the reattachment point where the maximum value of $Nu_c$ takes place, and finally the convection Nusselt number approaches to a constant value far from the inlet section. A good consistency is shown in Fig. 3 between the present result and theoretical finding in Ref. [22] for the case of K=100 with a maximum discrepancy of about 5% in computation of maximum Nusselt number. In another test case for laminar convection flow of radiating gas in a recess, the distribution of convective Nusselt number along the bottom wall is shown in Fig. 4 and compared with numerical results by Atashafrooz et al. [23]. This figure also reveals very good consistency between these findings. It is worth mentioning that again in this case, the present computations are done for a large value of solid-gas conduction ratio for simulation of isotherm bottom wall.

## 4- Results and Discussion

First to show the flow pattern of convection duct flow with BFS, the streamline distributions at Re=400 are plotted in Fig. 5. As seen, a large recirculated region exist on the bottom wall after the step location and the reattachment point takes place at X=4. Besides, numerical results depicts a small recirculated bubble on the duct upper wall that occurs above the reattachment point. The variations of convective and radiative Nusselt numbers along the bottom wall are shown in Fig.6 for different values of the RC parameter. This figure depicts zero value of the convection coefficient at the step corner where the fluid is at rest, after which $Nu_c$ increases in the recirculated domain because of flow vortices and maximum value of this parameter occurs just at the reattachment point and then convective Nusselt number decreases and approaches to a constant value far from the step. Besides, Fig.6 shows small effect of radiation conduction parameter on the convection coefficient. Fig.6 also shows the variation of radiative Nusselt number along the step wall. As seen, after the step location $Nu_r$ decreases along the heated surface because of decreasing in outgoing radiative heat flux from the bottom wall due to the presence of step vertical surface and its radiosity toward the bottom wall. Then radiative Nusselt number increases and reaches to its maximum value after which $Nu_r$ decreases and approaches to a constant value far from the step. As it is expected, Fig. 6 shows increasing trend for radiative Nusselt number by increasing in the value of RC parameters. Distributions of temperature at the interface surface between gas flow and solid element are drawn in Fig. 7 for different values of the conduction-radiation parameter. This figure shows temperature increase at the vicinity of the step location especially at low values of RC. This is due to sharp radiative Nusselt number decrease in this region, as shown in Fig. 6. After that, interface temperature decreases in the recirculated zone, such that the minimum temperature occurs at the reattachment point, and then θ increases and approaches to a constant value far from the step. Also, Fig. 7 shows that high interface temperature takes place under the condition of having high values for RC, because of more radiation heat transfer form high temperature upper wall via the radiation dominance participating gas flow into the interface surface. For showing more clear figures to presenting the thermal behavior of the duct flow, the temperature fields in both gas and solid phases are drawn in Fig.8. This figure shows how heat transfer takes place from the solid element and also from the upper wall into the gas flow. The minimum temperature of the solid element takes place in the vicinity of duct inlet section because of high convection coefficient in this region. Also, there is a low temperature region inside the solid element near to the reattachment point, which is also due to high convection Nusselt number on the interface surface. The effect of RC parameter on thermal characteristics of the proposed system can also be studied in Fig. 8 by comparing temperature distributions at different values of RC with each other. This figure reveals that more heat transfer takes place from the heated surfaces into the convection flow at high values of RC that causes rapid rate of thermal developing in the convective flow.

One of the main parameter that has high effect on thermal behavior of the conjugate problems is the solid to gas conduction ratio. For study the effect of this parameter, distributions of total heat flux along the interface surface are drawn in Fig. 9 for a broad range of parameter K. As seen, this figure depicts high effect of conduction ratio on the rate of heat transfer from the solid element into the gas flow via the interface surface. For all values of K, maximum rate of heat transfer occurs at the reattachment point, where high convection coefficient exists. Besides, an increasing trend is seen for the rate of heat transfer with conduction ratio, especially for the range of $5 \le K \le 100$.

## Conclusions

In this research work, a conjugate problem of laminar convection flow of radiating gas inside a duct with sudden expansion is analyzed by numerical scheme. Toward this end, the Navier-Stokes equations along with gas energy equations are solved by CFD method for computations of the velocity and temperature fields in the gas flow. Since, the present problem is a conjugate one, the Laplace equation for conduction process inside the solid element must be solved simultaneously with the energy equation for gas flow. Also, for computation of radiative term in gas energy equation, RTE is solve numerically by employing the DOM. It is worth mentioning that the analysis of conjugate problem of separated forced convection flow of radiating gas in a channel in which the set of governing equations in both solid element and gas flow are solved simultaneously is done here for the first time. Numerical results reveal that thermal behavior of such system is much affected by the conduction ratio. Such that under the condition of having high values for K, more heat transfer takes place from the solid element toward the gas flow. Numerical results show that increasing in the value of RC parameter causes considerable increase in radiative Nusselt number along the gas-solid interface, but the convective Nusselt number is not much affected by the radiation conduction parameter. In the present analysis, it is seen that high rate of heat transfer takes place between the gas flow and solid element under high values of solid to gas conduction ratio, such that the parameter K has a great effect on temperature pattern inside the convective flow. Also it is found that minimum temperature on the solid to gas interface takes place just at the reattachment point, such that this minimum temperature increases by increasing in RC parameter.

## Figures

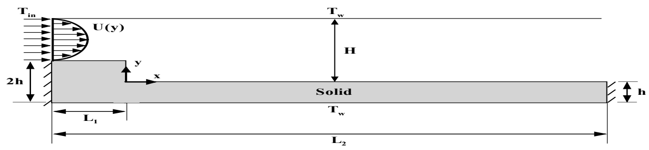


Fig. 1. Schematic of the problem

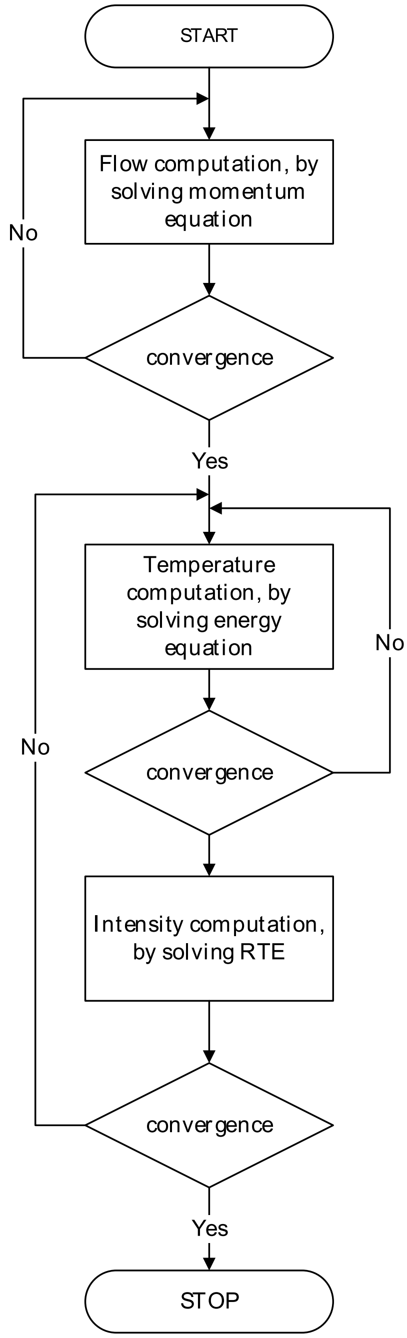


Fig. 2: Flow chart of numerical computations

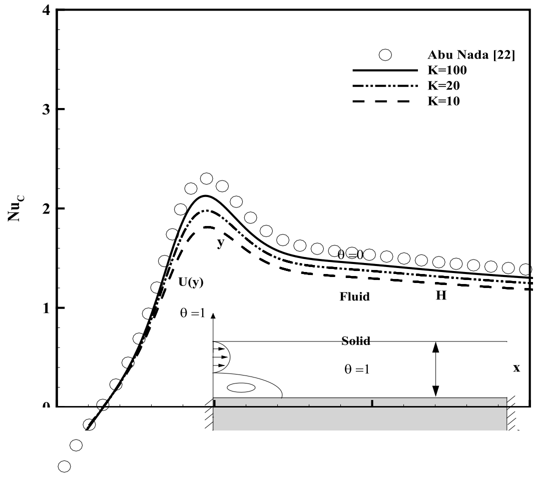


**Fig. 3. Convective Nusselt number distribution on the solid-liquid interface, comparison to theoretical findings in Ref. [22]**

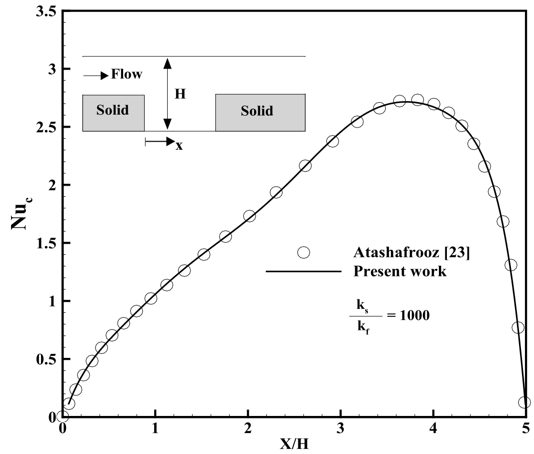


**Fig. 4. Distribution of convective Nusselt number along the bottom wall, comparison to theoretical findings in Ref. [23]**

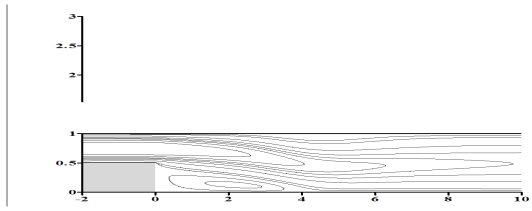

**Fig. 5. Streamline distribution in separated duct flow**

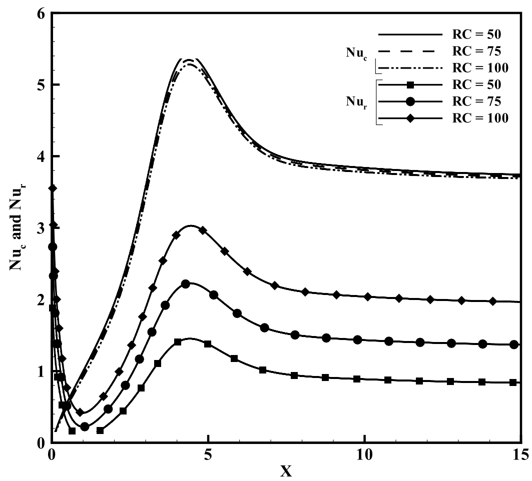


**Fig. 6. Distribution of radiative and convective nusselt number along on the solid-liquid interface [K=5, Pr=0.7]**

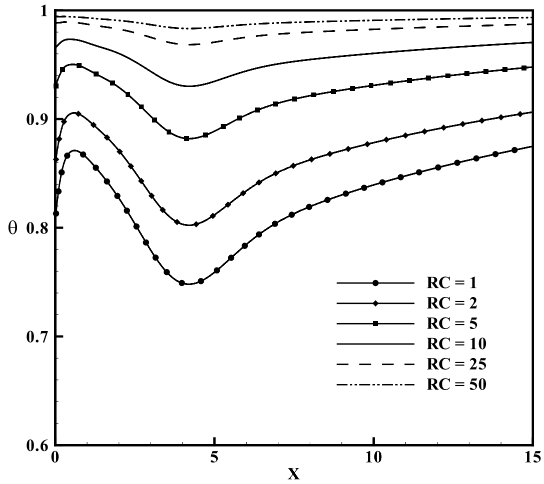


**Fig. 7. Variation of non-dimension temperature along on the solid-liquid interface[K=5, Pr=0.7]**

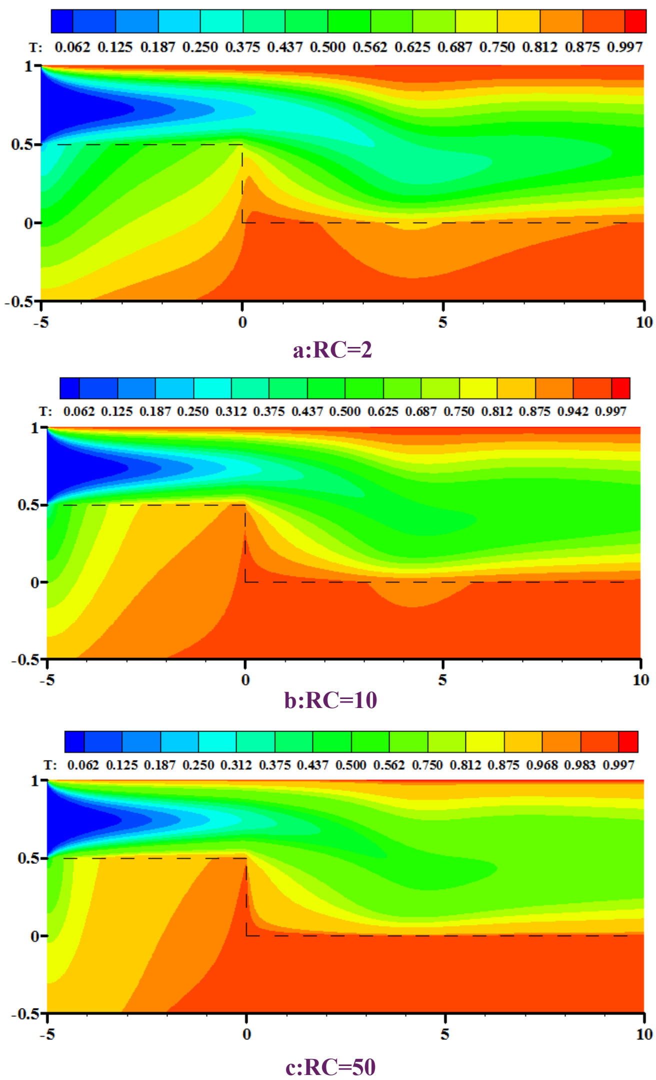


**Fig. 8. isotherm line temperature [K=5, Pr=0.7]**

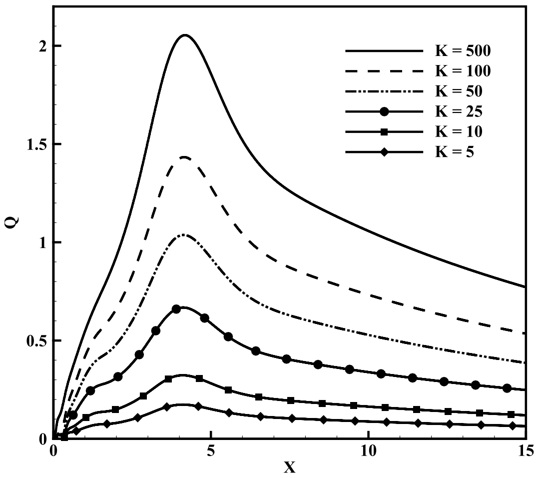


**Fig. 9. Distribution of non-dimension heat transfer along on the solid-gas interface**

## References

[1] Vradis, George C., and Lara Vannostrand. "Laminar coupled flow downstream of an asymmetric sudden expansion." Journal of thermophysics and heat transfer 6.2 (1992): 288-295.

[2] Kondoh, T., Y. Nagano, and T. Tsuji. "Computational study of laminar heat transfer downstream of a backward-facing step." International Journal of Heat and Mass Transfer 36.3 (1993): 577-591.

[3] Erturk, Ercan. "Numerical solutions of 2-D steady incompressible flow over a backward-facing step, Part I: High Reynolds number solutions." Computers & Fluids 37.6 (2008): 633-655.

[4] Abu-Nada, Eiyad. "Numerical prediction of entropy generation in separated flows." Entropy 7.4 (2005): 234-252.

[5] Abu-Nada, Eiyad. "Entropy generation due to heat and fluid flow in backward facing step flow with various expansion ratios." International Journal of Exergy 3.4 (2006): 419-435.

[6] B. F. Armaly, A. Li and J. H. Nie, Measurements in threedimensional laminar separated flow, International Journal of Heat and Mass Transfer, 46 (2003) 3573-3582.

[7] Armaly, Bassem F., An Li, and J. H. Nie. "Measurements in three-dimensional laminar separated flow." International Journal of Heat and Mass Transfer 46.19 (2003): 3573-3582.

[8] Ansari, A. B., and SA Gandjalikhan Nassab. "Thermal characteristics of convective flows encountered in a 2-D backward facing step under the effect of radiative heat transfer." International Review of Mechanical Engineering 4.6 (2010): 711-718.

[9] Vynnycky, M., et al. "Forced convection heat transfer from a flat plate: the conjugate problem." International Journal of Heat and Mass Transfer 41.1 (1998): 45-59.

[10] Chiu, Han-Chieh, Jer-Huan Jang, and Wei-Mon Yan. "Mixed convection heat transfer in horizontal rectangular ducts with radiation effects." International Journal of Heat and Mass Transfer 50.15 (2007): 2874-2882.

[11] Chiu, Han-Chieh, and Wei-Mon Yan. "Mixed convection heat transfer in inclined rectangular ducts with radiation effects." International Journal of Heat and Mass Transfer 51.5 (2008): 1085-1094.

[12] Rao, C. Gururaja, C. Balaji, and S. P. Venkateshan. "Effect of surface radiation on conjugate mixed convection in a vertical channel with a discrete heat source in each wall." International journal of heat and mass transfer 45.16 (2002): 3331-3347.

[13] Juncu, Gh. "Conjugate heat/mass transfer from a circular cylinder with an internal heat/mass source in laminar cross flow at low Reynolds numbers." International journal of heat and mass transfer 48.2 (2005): 419-424.

[14] Juncu, G. (2008). Unsteady heat transfer from an elliptic cylinder. International Journal of Heat and Mass Transfer, 51(3), 920-928.

[15] Mathews, Roy N., C. Balaji, and T. Sundararajan. "Computation of conjugate heat transfer in the turbulent mixed convection regime in a vertical channel with multiple heat sources." Heat and mass transfer 43.10 (2007): 1063-1074.

[16] Kanna, P. Rajesh, and Manab Kumar Das. "Conjugate heat transfer study of backward-facing step flow–a benchmark problem." International journal of heat and mass transfer 49.21 (2006): 3929-3941.

[17] Modest, Michael F. Radiative heat transfer. Academic press, 2013.

[18] Keshtkar, M. M., and SA Gandjalikhan Nassab. "Theoretical analysis of porous radiant burners under 2-D radiation field using discrete ordinates method." Journal of Quantitative Spectroscopy and Radiative Transfer 110.17 (2009): 1894-1907.

[19] Bejan, Adrian. Convection heat transfer. John wiley & sons, 2013.

[20] Ansari, A. B., and SA Gandjalikhan Nassab. "Study of laminar forced convection of radiating gas over an inclined backward facing step under bleeding condition using the blocked-off method." ASME Journal of heat transfer 133.7 (2011): 072702.

[21] Patankar, Suhas V., and D. Brian Spalding. "A calculation procedure for heat, mass and momentum transfer in three-dimensional parabolic flows." International Journal of Heat and Mass Transfer 15.10 (1972): 1787-1806.

[22] Abu-Nada, Eiyad. "Investigation of entropy generation over a backward facing step under bleeding conditions." Energy Conversion and Management 49.11 (2008): 3237-3242.

[23] Atashafrooz, Meysam, and Seyyed Abdolreza Gandjalikhan Nassab. "Numerical analysis of laminar forced convection recess flow with two inclined steps considering gas radiation effect." Computers & Fluids 66 (2012): 167-176.

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