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Adaptive Leader-Following and Leaderless Consensus of a Class of Nonlinear Systems Using Neural Networks

B. Karimi[1,*], H. Ghiti-Sarand[2]

1- Associated Professor, Department of Electrical Engineering, Malek-e Ashtar University of Technology
2- Assistant Professor, Department of Electrical Engineering, Malek-e Ashtar University of Technology

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ABSTRACT

This paper deals with leader-following and leaderless consensus problems of high-order multi-input/multi-output (MIMO) multi-agent systems with unknown nonlinear dynamics in the presence of uncertain external disturbances. The agents may have different dynamics and communicate together under a directed graph. A distributed adaptive method is designed for both cases. The structures of the controllers simplify their implementation and reduce computational cost.Unknown nonlinearities are estimated by a radial basis function neural network (RBFNN). The ultimate boundness of the closed-loop system is guaranteed through Lyapunov stability analysis by introducing a suitably driven adaptive rule. Finally, the simulation results verify the performance of the proposed control method.

KEYWORDS:

Adaptive Control, Consensus, MIMO Systems, Neural Networks, Multi-Agent Systems

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*Corresponding Author, Email: bkarimi@mut-es.ac.ir

1- Introduction

The consensus of multi-agent systems (MASs) has been intensively studied in recent years due to its potential applications in broad areas such as sensor networks, biology systems, rendezvous tasks, focking, swarming and mobile robots [1-3]. Multi-agent systems usually consist of a group of agents cooperating to complete certain tasks for the group. Therefore, their coordination control protocol is designed such that the states of the agents converge into a consistent value.

The cooperation of agents with linear dynamics has been developed broadly (see, e.g., [1-6] and references therein). Since, in many real-world multi-agent systems, the agents have nonlinear dynamics, research on cooperation of nonlinear MASs with known dynamics has received much attention in recent years [7-17]. However, it is difficult to precisely describe a nonlinear system by known nonlinear functions. Therefore, the control of systems with unknown nonlinear dynamics is a challenging task. Furthermore, some methods are required to approximate these nonlinearities.

Using neural networks (NNs) capability in the approximation of nonlinear functions, recently, some works have been dedicated to deal with distributed control of unknown multi-agent systems. In [18], synchronization problem for a group of uncertain Lagrangian systems was studied using an adaptive backstepping method under a directed communication topology. Hou et al. [19] proposed a decentralized robust adaptive control of unknown MASs for undirected graphs. Then, the leader following problem of these systems was investigated in [20]. In [21], a leader following synchronization of unknown single input-single output (SISO) nonlinear networked systems was established. The dynamics of agents in [19-21] were first-order and unknown nonlinearities were approximated by NNs. Cooperation of high-order nonlinear SISO multi-agent systems with unknown dynamics was studied in [22-24]. In [22], an NN adaptive protocol for an undirected connected graph was considered and in [23,24] the same method was employed for consensus tracking under a directed graph containing a spanning tree. The work in [25-27] addressed a distributed consensus tracking of second-order unknown nonlinear MASs. In [25], an adaptive control for synchronization of MASs under directed graphs was considered. In [26], an output feedback formation has been proposed for a topology of an undirected connected graph using terminal sliding mode control method, and in [27] the same method was applied for MASs. Peng et al. [28] proposed a distributed NN adaptive control method using state feedback and then extending to output feedback for an undirected connected communication topology.

In summary, all the mentioned works address systems with first-order [19-21] and second-order dynamics [18,25-27] or SISO systems with higher order dynamics [22-24]. Besides, there are a few studies that discuss the cooperation of nonlinear MIMO agents [18-20,26-28]. Moreover, in these studies, input gain (matrix gain) is assumed to be unity [19-20,26] or constant [28]. While in practical applications, the matrix gain is a nonlinear vector function of system's states and unknown. Therefore, the control problem of such systems becomes more complicated.

According to the best of the authors' knowledge, there is no work on cooperation of high-order MIMO nonlinear MASs with unknown dynamics. On the other hand, the study of consensus problems for directed graphs is more challenging and more general than that of undirected graphs [2]. In this paper, a distributed adaptive control method is proposed for leaderless and leader following problems of unknown high-order MIMO nonlinear MASs under directed graphs. Also, the gain matrix of each agent is assumed to be unknown. The agents' dynamics are in affine form and subject to uncertain external disturbances. The designed control protocol is based on local error and utilizes a radial basis function neural network (RBFNN) to estimate unknown nonlinearities. Compared to the methods proposed to the control of unknown single systems which employ two NNs (see, e.g. [29-31]), the new distributed NN adaptive control is designed with only one NN which simplifies the structure of the controller and its implementation. In order to cancel out approximation error and external disturbances, a distributed robust term is designed in the local control of each agent.

Compared with the existing methods applied to the consensus of unknown nonlinear MASs, the main contributions of this paper are summarized as follows: (1) Gain matrix of each agent is assumed to be function of states and unknown while in the most of the aforementioned methods, the input gain (matrix gain) is assumed to be unity [24-31]. (2) The considered class for agent dynamics leads to taking into account the cooperation of more general class of nonlinear systems. (3) Generally, dynamics of agents are assumed to be non-identical. (4) Instead of using two separate NNs to approximate the unknown dynamics and input gain matrix, only one NN is utilized in the proposed method which leads to simplifying the structure of the controller and reducing computational load. By using Lyapunov stability analysis, the stability of the overall system is achieved which shows all the signals are uniformly ultimately bounded.

The rest of this paper is organized as follows: problem statement, including graph theory, and the derivation of the error dynamics for consensus problem are introduced in section 2. In Section 3, first, a controller is designed for the leader-following problem. Then, a stability analysis using Lyapunov method is presented to guarantee the performance and the stability of the designed NN distributed adaptive controller. In section 4, the leaderless problem is discussed for the networked systems using the similar designing procedure of section 3. Simulation results are given in section 5. Section 6 concludes the paper.

2- Preliminaries

A. Graph Theory

Let $G=(V,E,A)$ be a directed graph of order n, where $V=\{v_1,...,v_n\}$ is the nonempty set of nodes, $E \subseteq V\times V$ is the set of edges and $A=[a_{ij}]$ is a weighted adjacency matrix. $e_{ij}=\{v_i,v_j\}$ is an edge of G and implies that node v_j can receive information from the node v_i. The adjacency matrix is defined as $a_{ii}=0$ and $a_{ij}>0$ if $e_{ji}\{v_j,v_i\}\in E$. A directed path from node j to node i is a sequence of distinct edges which starts from node j and ends to node i. The set of neighbors of the node v_i is denoted by $N_i=\{v_j\in E:(v_j,v_i)\in E\}$. The in-degree matrix is defined as $D=\mathrm{diag}(d_i)$ with $d_i=\sum_{j\in N_i} a_{ij}$ (i.e. ith row sum of A) [21].The Laplacian matrix of a graph is defined as $L=D–A$. The all row sums of the Laplacian matrix are equal to zero. Then, for the Laplacian matrix $L=[l_{ij}]$, we have $l_{ii}=d_i$ and $l_{ij}=-a_{ij}, i\neq j$. A directed graph is strongly connected if there is a directed path from every node to every other node. A directed graph contains a spanning tree if there is a node which can reach all the other nodes through a directed path [21,24].

B. Dynamics of Agents

The class of n-th-order MIMO nonlinear systems considered for i-th agent, and termed the companion form or controllability canonical form [32] is given by:

$$\mathbf{x}_i^{(n)} = \mathbf{f}_i(\underline{\mathbf{x}}_i) + \mathbf{g}_i(\underline{\mathbf{x}}_i)\mathbf{u}_i + \mathbf{w}_i(t), \quad i = 1,...,N \tag{1}$$

where $\mathbf{x}_i=(x_{i1},...,x_{im})^T\in\mathbb{R}^m$, $\mathbf{u}_i,\mathbf{w}_i\in\mathbb{R}^m$ are the states, the inputs, and disturbances of the agent, respectively. $\underline{\mathbf{x}}_i=(x_{i1},...,x_{i1}^{(n_1-1)},...,x_{im},...,x_{im}^{(n_m-1)})^T\in\mathbb{R}^n$ and $\mathbf{x}_i^{(n)}=(x_{i1}^{(n)},...,x_{im}^{(n)})^T\in\mathbb{R}^m$. $n=\sum_{i=1}^m n_i$ is the state vector available for measurement [30] and N is the number of agents. $\mathbf{f}_i(\underline{\mathbf{x}}_i)\in\mathbb{R}^m$ is unknown continuous dynamics. $\mathbf{g}_i(\underline{\mathbf{x}}_i)\in\mathbb{R}^{m\times m}$ is called gain matrix which is bounded, nonsingular and unknown.

3- Leader-Following Problem

A. Problem Formulation

The local tracking error of i-th agent is defined as:

$$\mathbf{e}_i = \sum_{j=1}^N a_{ij}(\mathbf{x}_j - \mathbf{x}_i) + b_i(\mathbf{x}_d - \mathbf{x}_i) \tag{2}$$

where $\mathbf{e}_i=(e_{i1},...,e_{im})^T\in\mathbb{R}^m$, $b_i\ge 0$, defined as pinning gain [21] and $b_i>0$ for at least one i. $\mathbf{x}_d\in\mathbb{R}^m$ is the state of leader/reference trajectory that should be followed by agents.

Remark 1: In this paper, $\mathbf{1}_n\in\mathbb{R}^n$ is a vector with each entry being 1. $\mathbf{0}_n\in\mathbb{R}^n$ and I_n are the n-dimensional zero and identity matrix, respectively. $\otimes$ is the Kronecker product. $\underline{\sigma}(.)$ and $\bar{\sigma}(.)$ denotes the minimum and maximum singular values of a matrix and $\|.\|_F$ denotes Frobenius norm of a given matrix. $tr\{.\}$ represents the trace of a matrix [21].

Remark 2: In this paper, L denotes the Laplacian matrix and $B=diag\{b_i\}$.

Lemma 1 [33]**:** Let L be irreducible and B have at least one diagonal entry $b_i>0$. Then $L+B$ is a nonsingular M-matrix. Define $q=[q_1,...,q_N]^T=(L+B)^{-1}\mathbf{1}_N$ and $P=diag(q_i)$. Then $P>0$ and matrix Q defined as:

$$P(L+B)+(L+B)^T P = Q \tag{3}$$

is positive definite.

By introducing $\mathbf{E}_i=(\mathbf{e}_i^T,\dot{\mathbf{e}}_i^T,...,\mathbf{e}_i^{(n-1)^T})^T\in\mathbb{R}^{nm}$, the error dynamics is defined as:

$$\dot{\mathbf{E}}_i = \mathbf{A}_1\mathbf{E}_i + \mathbf{B}_1\Big(\sum_{j=1}^N a_{ij}(\mathbf{x}_j^{(n)} - \mathbf{x}_i^{(n)}) + b_i(\mathbf{x}_d^{(n)} - \mathbf{x}_i^{(n)})\Big) \tag{4}$$

where $\dot{\mathbf{E}}_i=(\dot{\mathbf{e}}_i^T,...,\mathbf{e}_i^{(n)^T})^T\in\mathbb{R}^{nm}$, $\mathbf{A}_1=\begin{bmatrix}0 & I_{n-2}\\ 0 & 0\end{bmatrix}\otimes I_m$ and $\mathbf{B}_1=\Big[\underbrace{\mathbf{0}_m \; ... \; \mathbf{0}_m}_{n-1} \; I_m\Big]^T$. The overall dynamics, tracking error and error dynamics for the networked system are given by:

$$\mathbf{x}^{(n)} = \mathbf{f}(\underline{\mathbf{x}}) + \mathbf{g}(\underline{\mathbf{x}})\mathbf{u} + \mathbf{w}(t) \tag{5}$$

$$\mathbf{e} = -((L+B)\otimes I_m)(\mathbf{x} - \underline{\mathbf{x}}_d) \tag{6}$$

$$\dot{\mathbf{E}} = (I_N \otimes \mathbf{A}_1)\mathbf{E} - (M \otimes \mathbf{B}_1)(\mathbf{x}^{(n)} - \underline{\mathbf{x}}_d^{(n)}) \tag{7}$$

where $\mathbf{g} = diag(\mathbf{g}_1,...,\mathbf{g}_m) \in \mathbb{R}^{Nm\times Nm}$, $\mathbf{u} = (\mathbf{u}_1^T,...,\mathbf{u}_N^T)^T \in \mathbb{R}^{Nm}$, $\mathbf{w} = (\mathbf{w}_1^T,...,\mathbf{w}_N^T)^T \in \mathbb{R}^{Nm}$, $\mathbf{f} = (\mathbf{f}_1^T,...,\mathbf{f}_N^T)^T \in \mathbb{R}^{Nm}$, $\mathbf{e} = (\mathbf{e}_1^T,...,\mathbf{e}_N^T)^T \in \mathbb{R}^{Nm}$, $\mathbf{E} = (\mathbf{E}_1^T,...,\mathbf{E}_N^T)^T \in \mathbb{R}^{Nnm}$, $\underline{\mathbf{x}}_d = \mathbf{x}_d \otimes \mathbf{1}_N$ and $M = L + B$. Substituting (5) into (6) leads to:

$$\dot{\mathbf{E}} = (I_N \otimes \mathbf{A}_1)\mathbf{E} - (M \otimes \mathbf{B}_1)(\mathbf{f} + \mathbf{g}\mathbf{u} + \mathbf{w} - \underline{\mathbf{x}}_d^{(n)}) \tag{8}$$

Let $P_1 \in \mathbb{R}^{mn\times mn}$ be a positive definite matrix and solution to the following Riccati inequality:

$$P_1\mathbf{A}_1 + \mathbf{A}_1^T P_1 + Q_1 - P_1\mathbf{B}_1\mathbf{B}_1^T P_1 < 0 \tag{9}$$

where Q_1 is a positive definite matrix.

B. CONTROLLER DESIGN

The multi-agent system (1) is said to achieve consensus if for any initial conditions and by some appropriate controllers.

$$\lim_{t\to\infty}\left\|\mathbf{x}_i^{(j)} - \mathbf{x}_0^{(j)}\right\| \to \varepsilon_u,\ \forall i = 1,...,N,\ j = 1,...,n \tag{10}$$

where $\varepsilon_u > 0$. At the same time, all the closed-loop signals are to be kept bounded. In control engineering, NNs are usually employed to approximate unmodelled functions. For unknown systems with the dynamics presented in Eq. (1), the traditional controller based on feedback linearization includes two separate NNs (see, e.g. [29-31]). In this paper, based on the method proposed in [34], a new distributed NN adaptive control is designed with only one NN which results in the simplifying structure of the controller and its implementation. Due to "linear-in-the-weight" property and the universal approximation capability of any continuous function [35-36], the radial basis function neural network (RBFNN) is a good candidate for this purpose. Assume that the unknown nonlinearities in Eq. (1) are continuous and thus can be approximated on a compact set $\Omega_i \in \mathbb{R}$ by:

$$\mathbf{f}_i(\underline{\mathbf{x}}_i) = \mathbf{W}_i^T\phi_i(\underline{\mathbf{x}}_i) + \boldsymbol{\varepsilon}_i \tag{11}$$

where $\mathbf{W}_i \in \mathbb{R}^{n_{ri}\times m}$ is an ideal weight matrix, n_{ri} denotes the number of neurons and $\phi_i(\underline{\mathbf{x}}_i) = \exp(-\frac{\left\|\underline{\mathbf{x}}_i - \zeta_i\right\|^2}{\sigma_i^2}) \in \mathbb{R}^{n_{ri}}$ is the activation function with ζ_i as the center and σ_i as the influence size of the neurons. $\boldsymbol{\varepsilon}_i \in \mathbb{R}^m$ is the bounded approximation error. Typically, values of the center ζ_i and the influence size σ_i are held fixed. The ideal weight matrix $\mathbf{W}_i$ is unknown. Subsequently, for real applications, its approximation, $\hat{\mathbf{W}}_i$ is utilized. Estimation of Eq. (11) is defined as:

$$\hat{\mathbf{f}}_i(\underline{\mathbf{x}}_i) = \hat{\mathbf{W}}_i^T\phi_i(\underline{\mathbf{x}}_i) \tag{12}$$

Considering Eqs. (11) and (12), the overall graph nonlinearities and their estimation are written as:

$$\mathbf{f}(\underline{\mathbf{x}}) = \mathbf{W}^T\boldsymbol{\Phi}(\underline{\mathbf{x}}) + \boldsymbol{\varepsilon} \tag{13}$$

$$\hat{\mathbf{f}}(\underline{\mathbf{x}}) = \hat{\mathbf{W}}^T\boldsymbol{\Phi}(\underline{\mathbf{x}}) \tag{14}$$

with $\hat{\mathbf{f}} = (\hat{\mathbf{f}}_1^T, ..., \hat{\mathbf{f}}_N^T) \in \mathbb{R}^{Nm}$, $\mathbf{W} = diag\{\mathbf{W}_i\} \in \mathbb{R}^{n_r\times Nm}$, $\boldsymbol{\Phi} = (\phi_1^T, ..., \phi_N^T) \in \mathbb{R}^{n_r}$, $\hat{\mathbf{W}} = diag\{\hat{\mathbf{W}}_i\} \in \mathbb{R}^{n_r\times Nm}$, $n_r = \sum_{i=1}^{N} n_{ri}$ and $\boldsymbol{\varepsilon} = (\boldsymbol{\varepsilon}_1^T,...,\boldsymbol{\varepsilon}_N^T)^T \in \mathbb{R}^{Nm}$. The local control law for every agent is given by:

$$\mathbf{u}_i = \mathbf{g}_{i0}^{-1}[-\hat{\mathbf{f}}_i(\underline{\mathbf{x}}_i) + \beta\boldsymbol{\lambda}\mathbf{E}_i + \mathbf{u}_{si}] \tag{15}$$

where $\boldsymbol{\lambda} \in \mathbb{R}^{m\times mn}$ is a feedback gain and coupling gain $\beta > 0$. $\mathbf{g}_{i0}^{-1}$ is the inverse of a constant and symmetric positive definite matrix $\mathbf{g}_{i0}$. $\mathbf{u}_{si} = -\hat{\rho}_i \tanh(\mathbf{B}_1^T P_1(p_i(b_i + d_i))\mathbf{E}_i) \in \mathbb{R}^m$ is a robust term to counter approximation error and external disturbances. Then, taking Eqs. (13) to (15) into account, the global control input of the follower agents is:

$$\mathbf{u} = \mathbf{G}_0^{-1}[-\hat{\mathbf{f}}(\underline{\mathbf{x}}) + \beta\boldsymbol{\Lambda}\mathbf{E} + \mathbf{u}_s] \tag{16}$$

where $\boldsymbol{\Lambda} = I_N \otimes \boldsymbol{\lambda}$, $\mathbf{u}_s = (\mathbf{u}_{s1}^T,...,\mathbf{u}_{sN}^T)^T \in \mathbb{R}^{Nm}$, $\mathbf{G}_0^{-1} = diag\{\mathbf{g}_{i0}^{-1}\}$. Adding and subtracting $\mathbf{G}_0$ to the gain matrix in error dynamics, Eq. (7) and substituting it into Eq. (16) yields to:

$$\begin{aligned}\dot{\mathbf{E}} = (I_N \otimes \mathbf{A}_1)\mathbf{E} - (M \otimes \mathbf{B}_1)(\tilde{\mathbf{W}}^T\boldsymbol{\Phi} + (\mathbf{g} - \mathbf{G}_0)\mathbf{u} \\ + \mathbf{u}_s + \beta\boldsymbol{\Lambda}\mathbf{E} + \mathbf{w} + \boldsymbol{\varepsilon} - \underline{\mathbf{x}}_d^{(n)})\end{aligned} \tag{17}$$

where $\mathbf{G}_0 = diag\{\mathbf{g}_{i0}\}$ and $\tilde{\mathbf{W}} = \mathbf{W} - \hat{\mathbf{W}}$ is the NN weights estimation error.

C. STABILITY ANALYSIS

First, in this subsection, the how of tuning the NN weights and the gain of the robust term in a distributed manner is shown. Next, stability analysis is presented. The following standard assumptions are required.

Assumption 1: The uncertain disturbance, $\mathbf{w}_i$ and the vector of approximation error, $\boldsymbol{\varepsilon}_i$ are bounded by constants w_{iM} and ε_{iM}, i.e. $\left\|\mathbf{w}_i\right\| \le w_{iM}$ and $\left\|\boldsymbol{\varepsilon}_i\right\| \le \varepsilon_{iM}$.

Assumption 2: The state of the leader agent (reference trajectory) and its time-derivatives up to order n are given and bounded. Especially $\mathbf{x}_d^{(n)}$ is bounded as $\left\|\mathbf{x}_d^{(n)}\right\| \le x_M$.

Assumption 3: Unknown ideal NN weight matrix and NN activation functions are bounded by $\|\mathbf{W}\| \le W_M$ and $\|\ddot{\mathbf{O}}\| \le \Phi_M$, respectively.

The following adaptive rules are proposed to update the parameters $\hat{\mathbf{W}}_i$ and $\hat{\rho}_i$, $i = 1,...,N$.

$$\dot{\hat{\mathbf{W}}}_i = -k_w \phi_i \mathbf{E}_i^T p_i (d_i + b_i) - k_w \eta_1 \hat{\mathbf{W}}_i \tag{18}$$

$$\dot{\hat{\rho}}_i = k_s \left\| \mathbf{E}_i^T p_i (b_i + d_i) P_1 B_1 \right\| - k_s \eta_2 \hat{\rho}_i \tag{19}$$

where $k_w > 0, k_s > 0, \eta_1 > 0$ and $\eta_2 > 0$ are real constants. For the total system, the NN parameters' update rule is obtained as:

$$\dot{\hat{\mathbf{W}}} = -k_w \mathbf{\Phi} \mathbf{E}^T ((P(D+B)) \otimes P_1 B_1) - k_w \eta_1 \hat{\mathbf{W}} \tag{20}$$

Lemma 2 [8]**:** The Kronecker product has the following properties: for matrices $\mathbf{C}_1, \mathbf{C}_2, \mathbf{C}_3$ and $\mathbf{C}_4$ of appropriate dimensions:

1) $(\mathbf{C}_1 + \mathbf{C}_2) \otimes \mathbf{C}_3 = \mathbf{C}_1 \otimes \mathbf{C}_3 + \mathbf{C}_2 \otimes \mathbf{C}_3$
2) $(\mathbf{C}_1 \otimes \mathbf{C}_2)(\mathbf{C}_3 \otimes \mathbf{C}_4) = (\mathbf{C}_1 \mathbf{C}_3) \otimes (\mathbf{C}_2 \mathbf{C}_4)$

Theorem 1: consider the *i*-th system (1) with the adaptive protocol (15), the updating laws (18-19) and Assumptions 1–3. If the communication directed graph *G* is strongly connected and the feedback gain and the coupling gain satisfy

$$\boldsymbol{\lambda} = \mathbf{B}_1^T P_1 \tag{21}$$

$$\beta > p_{i\max} / \lambda_{\min}, \qquad i = 1,...,N \tag{22}$$

then, all agents synchronize to the leader and all the signals of the closed-loop system are uniformly ultimately bounded. $p_{i\max}$ and $\lambda_{\min}$ are the maximum and the minimum eigenvalues of matrices *P* and *Q*, respectively.

Proof: Consider the following Lyapunov function candidate:

$$V = V_1 + V_2 + V_3 \tag{23}$$

$$V_1 = \frac{1}{2}\mathbf{E}^T (P \otimes P_1)\mathbf{E}, \quad V_2 = \frac{tr}{2k_w}\{\tilde{\mathbf{W}}^T \tilde{\mathbf{W}}\}, \quad V_3 = \frac{\tilde{\boldsymbol{\rho}}^T \tilde{\boldsymbol{\rho}}}{2k_s} \tag{24}$$

where $\tilde{\mathbf{n}} = \tilde{\mathbf{n}} - \hat{\mathbf{n}}$, $\tilde{\mathbf{n}} = (\rho_1,...,\rho_N)^T$ and $\hat{\mathbf{n}} = (\hat{\rho}_1,...,\hat{\rho}_N)^T$. The ideal gain, $\rho_i, i = 1,...,N$, will be determined during the stability analysis. First, the time derivative of V_1 along the error dynamics (17) is calculated:

$$\begin{aligned}\dot{V}_1 &= \mathbf{E}^T (P \otimes P_1 \mathbf{A}_1)\mathbf{E} - \mathbf{E}^T (PM \otimes P_1 \mathbf{B}_1) \\ &(\tilde{\mathbf{W}}^T \mathbf{\Phi} + (\mathbf{g} - \mathbf{G}_0)\mathbf{u} + \mathbf{u}_s + \beta \mathbf{\Lambda} \mathbf{E} + \mathbf{w} + \boldsymbol{\varepsilon} - \underline{\mathbf{x}}_d^{(n)})\end{aligned} \tag{25}$$

In light of Lemma 1, inequality (9) and applying (21), (25) yields:

$$\begin{aligned}\dot{V}_1 &= \frac{\mathbf{E}^T}{2}[P \otimes (P_1 \mathbf{A}_1 + \mathbf{A}_1^T P_1) \\ &-\beta (Q \otimes P_1 \mathbf{B}_1 \mathbf{B}_1^T P_1)]\mathbf{E}\, \mathbf{E}^T (PM \otimes P_1 \mathbf{B}_1) \\ &(\tilde{\mathbf{W}}^T \mathbf{\Phi} + (\mathbf{g} - \mathbf{G}_0)\mathbf{u} + \mathbf{u}_s + \mathbf{w} + \boldsymbol{\varepsilon} - \underline{\mathbf{x}}_d^{(n)})\end{aligned} \tag{26}$$

Let $\mathbf{U} \in \mathbb{R}^{N \times N}$ be a unitary matrix such that $\mathbf{U}^T Q \mathbf{U} = \Lambda_q = diag(\lambda_1,...,\lambda_N)$ and $\lambda_i > 0, \forall i$. By choosing state transformation $\mathbf{E} = (\mathbf{U} \otimes I_{mn})\mathbf{y}$, where $\mathbf{y} = (y_1^T,...,y_N^T)^T$, one obtains:

$$\begin{aligned}\dot{V}_1 &= \frac{\mathbf{E}^T}{2}[P \otimes (P_1 \mathbf{A}_1 + \mathbf{A}_1^T P_1)]\mathbf{E} \\ &-\beta \frac{\mathbf{y}^T}{2}(\Lambda_q \otimes P_1 \mathbf{B}_1 \mathbf{B}_1^T P_1)\mathbf{y} - \mathbf{E}^T (PM \otimes P_1 \mathbf{B}_1) \\ &(\tilde{\mathbf{W}}^T \mathbf{\Phi} + (\mathbf{g} - \mathbf{G}_0)\mathbf{u} + \mathbf{u}_s + \mathbf{w} + \boldsymbol{\varepsilon} - \underline{\mathbf{x}}_d^{(n)}) \\ &\le \frac{\mathbf{E}^T}{2}[P \otimes (P_1 \mathbf{A}_1 + \mathbf{A}_1^T P_1)]\mathbf{E} \\ &-\beta \lambda_{\min} \frac{\mathbf{y}^T}{2}(I_N \otimes P_1 \mathbf{B}_1 \mathbf{B}_1^T P_1)\mathbf{y} - \mathbf{E}^T (PM \otimes P_1 \mathbf{B}_1) \\ &(\tilde{\mathbf{W}}^T \mathbf{\Phi} + (\mathbf{g} - \mathbf{G}_0)\mathbf{u} + \mathbf{u}_s + \mathbf{w} + \boldsymbol{\varepsilon} - \underline{\mathbf{x}}_d^{(n)})\end{aligned} \tag{27}$$

where $\lambda_{\min} = \min(\lambda_1,...,\lambda_N)$. Substituting $\mathbf{y} = (\mathbf{U}^T \otimes I_{mn})\mathbf{E}$ into (27), one obtains

$$\begin{aligned}\dot{V}_1 &\le \frac{\mathbf{E}^T}{2}[P \otimes (P_1 \mathbf{A}_1 + \mathbf{A}_1^T P_1) - \beta \lambda_{\min} (I_N \otimes P_1 \mathbf{B}_1 \mathbf{B}_1^T P_1)]\mathbf{E} \\ &-\mathbf{E}^T (PM \otimes P_1 \mathbf{B}_1)(\tilde{\mathbf{W}}^T \mathbf{\Phi} + (\mathbf{g} - \mathbf{G}_0)\mathbf{u} + \mathbf{u}_s + \mathbf{w} + \boldsymbol{\varepsilon} - \underline{\mathbf{x}}_d^{(n)}) \\ &\le \sum_{i=1}^{N} \frac{\mathbf{E}_i^T}{2} p_i [P_1 \mathbf{A}_1 + \mathbf{A}_1^T P_1 - \beta \frac{\lambda_{\min}}{p_i} P_1 \mathbf{B}_1 \mathbf{B}_1^T P_1)]\mathbf{E}_i \\ &-\mathbf{E}^T (PM \otimes P_1 \mathbf{B}_1)(\tilde{\mathbf{W}}^T \mathbf{\Phi} + (\mathbf{g} - \mathbf{G}_0)\mathbf{u} + \mathbf{u}_s + \mathbf{w} + \boldsymbol{\varepsilon} - \underline{\mathbf{x}}_d^{(n)})\end{aligned} \tag{28}$$

Using property (22), the fact $\bar{\sigma}(\mathbf{B}) = 1$ and Riccati inequality (9), (28) yields:

$$\begin{aligned}\dot{V}_1 &\le -\frac{p_{i\max} \underline{\sigma}(Q_1)}{2}\|\mathbf{E}\|^2 - \mathbf{E}^T (PM \otimes P_1 \mathbf{B}_1) \\ &(\tilde{\mathbf{W}}^T \mathbf{\Phi} + (\mathbf{g} - \mathbf{G}_0)\mathbf{u} + \mathbf{u}_s + \mathbf{w} + \boldsymbol{\varepsilon} - \underline{\mathbf{x}}_d^{(n)})\end{aligned} \tag{29}$$

Considering (16) and assumptions 2-3 $\|\mathbf{u}\|$ is given:

$$\|\mathbf{u}\| \le \underline{\sigma}_0^{-1}(c_1 + \Phi_M \left\|\tilde{\mathbf{W}}\right\|_F + \beta \bar{\sigma}(P_1)\|\mathbf{E}\| + m\|\tilde{\boldsymbol{\rho}}\|) \tag{30}$$

where $c_1 = \Phi_M W_M + m\|\tilde{\mathbf{n}}\|$, $\underline{\sigma}_0$ is the minimum singular value of $\mathbf{G}_0$ and $\|\tanh(.)\| = 1$ applied to (30). Let $d_g = \max_{\underline{\mathbf{x}} \in \mathbb{R}^{Nn}} \|\mathbf{g}(\underline{\mathbf{x}}) - \mathbf{G}_0\| > 0$. Using the definition *L*=*D*-*A*, the second property of Lemma 2 and adding time derivatives of V_2 and V_3, (30) can be rewritten as:

$$\dot{V} \le -\frac{p_{i\min}\underline{\sigma}(Q_1)}{2}\|\mathbf{E}\|^2 - \mathbf{E}^T(P(D+B)\otimes P_1\mathbf{B}_1)$$
$$[\mathbf{u}_s + \mathbf{w} + \boldsymbol{\varepsilon} - \underline{\mathbf{x}}_d^{(n)}] + \mathbf{E}^T(PA\otimes P_1\mathbf{B}_1)$$
$$[\tilde{\mathbf{W}}^T\boldsymbol{\Phi} + \mathbf{u}_s + \mathbf{w} + \boldsymbol{\varepsilon} - \underline{\mathbf{x}}_d^{(n)}]$$
$$+d_g\bar{\sigma}(P)\bar{\sigma}(M)\bar{\sigma}(P_1\mathbf{B}_1)\|\mathbf{E}\|\|\mathbf{u}\| + \frac{\tilde{\boldsymbol{\rho}}^T\dot{\tilde{\boldsymbol{\rho}}}}{k_s} \tag{31}$$
$$+tr\{\tilde{\mathbf{W}}^T(\frac{\dot{\tilde{\mathbf{W}}}}{k_w} - \boldsymbol{\Phi}\mathbf{E}^T(P(D+B)\otimes P_1\mathbf{B}_1))\}$$

Property of the trace operator $\mathbf{x}^T\mathbf{y} = tr\{\mathbf{y}\mathbf{x}^T\}$, $\forall\,\mathbf{x},\mathbf{y}\in\mathbb{R}^m$ applied to (31). According to the definition of the NN weight estimation error matrix, its derivative is $\dot{\tilde{\mathbf{W}}} = -\dot{\hat{\mathbf{W}}}$. By using this derivative to the updated law of NN weight (20) and substituting them into (31) and employing (22) and (30), one obtains:

$$\dot{V} \le -k_1\|\mathbf{E}\|^2 + k_2\|\mathbf{E}\|\|\tilde{\mathbf{W}}\|_F + k_3\|\mathbf{E}\|\|\tilde{\boldsymbol{\rho}}\|$$
$$+k_4\|\mathbf{E}\| + \frac{\tilde{\boldsymbol{\rho}}^T\dot{\tilde{\boldsymbol{\rho}}}}{k_s} - \mathbf{E}^T(P(D+B)\otimes P_1\mathbf{B}_1) \tag{32}$$
$$[\mathbf{u}_s + \mathbf{w} + \boldsymbol{\varepsilon} - \underline{\mathbf{x}}_d^{(n)}] - \eta_1 tr\{\tilde{\mathbf{W}}^T\hat{\mathbf{W}}\}$$

where $k_4 = c_1c_2 + \bar{\sigma}(P)\bar{\sigma}(A)\bar{\sigma}(P_1)N_M$, $k_3 = mc_2 + m\bar{\sigma}(P)\times\bar{\sigma}(A)\bar{\sigma}(P_1)$, $k_2 = \Phi_M(\bar{\sigma}(P)\bar{\sigma}(A)\bar{\sigma}(P_1)) + c_2)$, $k_1 = p_{i\max}(\frac{\underline{\sigma}(Q_1)}{2} - \frac{c_2\bar{\sigma}(P_1)}{\lambda_{\min}})$ and $c_2 = \underline{\sigma}_0^{-1}d_g\bar{\sigma}(P)\bar{\sigma}(M)\bar{\sigma}(P_1)$ with $N_M = \sqrt{N}x_M + m\|\bar{\mathbf{n}}\| + [\sum_{i=1}^{N} w_{iM}^2 + \varepsilon_{iM}^2]^{0.5}$. Since:

$$-\mathbf{E}^T(P(D+B)\otimes P_1\mathbf{B}_1)[\mathbf{u}_s + \mathbf{w} + \boldsymbol{\varepsilon} - \underline{\mathbf{x}}_d^{(n)}] \le$$
$$\sum_{i=1}^{N}[\|\mathbf{E}_i^T p_i(d_i + b_i)P_1\mathbf{B}_1\|(\boldsymbol{\varepsilon}_{iM} + \mathbf{w}_{iM} + x_M)$$
$$-\mathbf{E}_i^T p_i(d_i + b_i)P_1\mathbf{B}_1\hat{\rho}_i\tanh((\mathbf{B}_1^T P_1)p_i(d_i + b_i)\mathbf{E}_i)]$$

Therefore, by using inequality $|y| - y\tanh(y) \le k_u$ for a given a variable y, $k_u = 0.2785$, choosing $\rho_i = \mathbf{å}_{iM} + \mathbf{w}_{iM} + x_M$ and replacing $\tilde{\rho}_i = \rho_i - \hat{\rho}_i$, (32) can be rewritten as:

$$\dot{V} \le -k_1\|\mathbf{E}\|^2 + k_2\|\mathbf{E}\|\|\tilde{\mathbf{W}}\|_F + k_3\|\mathbf{E}\|\|\tilde{\boldsymbol{\rho}}\| + k_4\|\mathbf{E}\|$$
$$+\sum_{i=1}^{N}\tilde{\rho}_i(\frac{\dot{\tilde{\rho}}_i}{k_s} - \|\mathbf{E}_i^T p_i(d_i + b_i)P_1\mathbf{B}_1\|) - \eta_1 tr\{\tilde{\mathbf{W}}^T\hat{\mathbf{W}}\} \tag{33}$$
$$+mk_u(\|\tilde{\boldsymbol{\rho}}\| + \|\boldsymbol{\rho}\|) + \bar{\sigma}(P)\bar{\sigma}(D+B)\bar{\sigma}(P_1)\|\mathbf{E}\|\|\tilde{\boldsymbol{\rho}}\|$$

Replacing the updated law of robust term gain (19) and $\hat{\mathbf{W}} = \mathbf{W} - \tilde{\mathbf{W}}$ with (33), we have:

$$\dot{V} \le -k_1\|\mathbf{E}\|^2 + k_2\|\mathbf{E}\|\|\tilde{\mathbf{W}}\|_F + k_3'\|\mathbf{E}\|\|\tilde{\boldsymbol{\rho}}\| + k_4\|\mathbf{E}\|$$
$$-\eta_1\|\tilde{\mathbf{W}}\|_F^2 + \eta_1 W_M\|\tilde{\mathbf{W}}\|_F - \eta_2\|\tilde{\rho}\|^2 + k_5\|\tilde{\rho}\| + mk_u\|\boldsymbol{\rho}\| \tag{34}$$

where $k_3' = k_3 + \bar{\sigma}(P)\bar{\sigma}(D+B)\bar{\sigma}(P_1)$ and $k_5 = (\eta_2 + mk_u)\|\bar{\mathbf{n}}\|$. Let $\mathbf{r} = [k_4 \quad \eta_1 W_M \quad k_5]$ and:

$$\mathbf{y} = [\|\mathbf{E}\| \quad \|\tilde{\mathbf{W}}\|_F \quad \|\tilde{\boldsymbol{\rho}}\|]^T,$$
$$\mathbf{R} = \begin{bmatrix} k_1 & -k_2/2 & -k_3'/2 \\ -k_2/2 & \eta_1 & 0 \\ -k_3'/2 & 0 & \eta_2 \end{bmatrix} \tag{35}$$

then the following inequality is obtained:

$$\dot{V} \le -\mathbf{y}^T\mathbf{R}\mathbf{y} + \mathbf{r}\mathbf{y} + mk_u\|\boldsymbol{\rho}\| \tag{36}$$

In order to satisfy $\dot{V} \le 0$, $\mathbf{R}$ must be positive definite and:

$$\|\mathbf{y}\| > \frac{\sqrt{k_6}}{\underline{\sigma}(\mathbf{R})} + \frac{\|\mathbf{r}\|}{2\underline{\sigma}(\mathbf{R})} \tag{37}$$

with $k_6 = mk_u\|\bar{\mathbf{n}}\| + \frac{\|\mathbf{r}\|^2}{4\underline{\sigma}^2(\mathbf{R})}$. Therefore, the overall system is ultimately bounded and according to [37, Sec. 4.8], it is proved that all the signals are uniformly ultimately bounded. Since $\mathbf{R} > 0$ using (22), the following inequalities are obtained:

$$\underline{\sigma}(Q_1) > \frac{1}{p_{i\max}}(\frac{k_2^2}{2\eta_1} + \frac{k_3'^2}{2\eta_2} + 2\frac{c_2\bar{\sigma}(P_1)}{\lambda_{\min}}) \tag{38}$$

This completes the proof.

4- Leaderless Problem

A. Problem Formulation

In the previous section, agents make an agreement to follow the desired trajectory. In this section, it is shown how all the agents are eventually driven to an unprescribed common value.

Lemma 3 [11]**:** Suppose that L is irreducible. Then, $L\mathbf{1}_N = 0$ and there is a positive vector $\xi = (\xi_1, ..., \xi_N)^T$ such that $\xi^T L = 0$, $\sum_{i=1}^{N}\xi_i = 1$. In addition, there exists a positive definite diagonal matrix $\Xi = diag(\xi_1, ..., \xi_N)$ such that:

$$\hat{L} = \frac{\Xi L + L^T\Xi}{2} \tag{39}$$

Similar to [11], for a strongly connected network with the Laplacian matrix L, the general algebraic connectivity is defined by:

$$a(L) = \min_{x^T\xi=0,x\ne0}\frac{x^T\hat{L}x}{x^T\Xi x} > 0 \tag{40}$$

Based on Lemma 2 of [38], we have $x^T\hat{L}x \ge a(L)(x^T\Xi x) > 0$.

The controller design follows the design procedure of section 3. Thus, the definition of variables is similar to those of section 3. Define the local tracking error of i-th agent as:

$$\mathbf{e}_i = \sum_{j=1}^{N} a_{ij}(\mathbf{x}_j - \mathbf{x}_i) = -\sum_{j=1}^{N} l_{ij}\mathbf{x}_j \quad (41)$$

By introducing $\mathbf{E}_i = (\mathbf{e}_i^T, \dot{\mathbf{e}}_i^T, ..., \mathbf{e}_i^{(n-1)^T})^T \in \mathbb{R}^{nm}$, the error dynamics is defined as:

$$\dot{\mathbf{E}}_i = \mathbf{A}_1\mathbf{E}_i - \mathbf{B}_1\sum_{j=1}^{N} l_{ij}\mathbf{x}_j^{(n)} \quad (42)$$

The overall tracking error and error dynamics for the networked system are described by:

$$\mathbf{e} = -(L \otimes I_m)\mathbf{x} \quad (43)$$

$$\dot{\mathbf{E}} = (I_N \otimes \mathbf{A}_1)\mathbf{E} - (L \otimes \mathbf{B}_1)\mathbf{x}^{(n)} = (I_N \otimes \mathbf{A}_1)\mathbf{E} \\ -(L \otimes \mathbf{B}_1)(\mathbf{f} + \mathbf{gu} + \mathbf{w}) \quad (44)$$

B. Controller Design

The control objective here is to find some appropriate controllers such that for any initial conditions, an agreement being reached by all agents in the controlled network (1) in the sense that:

$$\lim_{t\to\infty}\left\|\mathbf{x}_i^{(j)} - \mathbf{x}_k^{(j)}\right\| \to \varepsilon_u,\ \forall i = 1,...,N,\ j = 1,...,n \quad (45)$$

where $\varepsilon_u > 0$ and k=1,…,N. At the same time, all closed-loop signals are to be kept bounded. The local control law is given by:

$$\mathbf{u}_i = \mathbf{g}_{i0}^{-1}[-\hat{\mathbf{f}}_i(\underline{\mathbf{x}}_i) + \beta\boldsymbol{\lambda}\mathbf{E}_i + \mathbf{u}_{si}] \quad (46)$$

where $\mathbf{u}_{si} = -\hat{\rho}_i \tanh(\mathbf{B}_1^T P_1(\xi_i + d_i))\mathbf{E}_i) \in \mathbb{R}^m$ is the robust term. Then, the global control input is:

$$\mathbf{u} = \mathbf{G}_0^{-1}[-\hat{\mathbf{f}}(\underline{\mathbf{x}}) + \beta\boldsymbol{\Lambda}\mathbf{E} + \mathbf{u}_s] \quad (47)$$

Adding and subtracting $\mathbf{G}_0$ to the gain matrix in error dynamics, (43), and substituting (47) yields to:

$$\dot{\mathbf{E}} = (I_N \otimes \mathbf{A}_1)\mathbf{E} - (L \otimes \mathbf{B}_1)(\tilde{\mathbf{W}}^T\boldsymbol{\Phi} + (\mathbf{g} - \mathbf{G}_0)\mathbf{u} + \mathbf{u}_s \\ + \beta\boldsymbol{\Lambda}\mathbf{E} + \mathbf{w} + \boldsymbol{\varepsilon}) \quad (48)$$

C. Stability Analysis

The following updating rules are proposed for the parameters $\hat{\mathbf{W}}_i$ and $\hat{\rho}_i$, $i = 1,...,N$:

$$\dot{\hat{\mathbf{W}}}_i = -k_w\phi_i\mathbf{E}_i^T\xi_i d_i - k_w\eta_1\hat{\mathbf{W}}_i \quad (49)$$

$$\dot{\hat{\rho}}_i = k_s\left\|\mathbf{E}_i^T\xi_i d_i P_1 B_1\right\| - k_s\eta_2\hat{\rho}_i \quad (50)$$

For the total system, the NN parameters' update rule is obtained as:

$$\dot{\hat{\mathbf{W}}} = -k_w\boldsymbol{\Phi}\mathbf{E}^T(\Xi D \otimes P_1 B_1) - k_w\eta_1\hat{\mathbf{W}} \quad (51)$$

Theorem 2: Consider the i-th system (1) with the adaptive protocol (46), the updating laws (49) and (50) and Assumptions 1–3. If the communication graph G is a strongly connected digraph and the feedback gain and the coupling gain satisfy:

$$\boldsymbol{\lambda} = \mathbf{B}_1^T P_1 \quad (52)$$

$$\beta > \xi_{\max} / a(L) \quad (53)$$

then, the consensus objective (45) can be achieved and all the signals of the closed-loop system are uniformly ultimately bounded. $\xi_{\max}$ is the minimum eigenvalues of the matrix Ξ.

Proof: consider the following Lyapunov function candidate:

$$V = V_1 + V_2 + V_3 \quad (54)$$

$$V_1 = \frac{1}{2}\mathbf{E}^T(\Xi \otimes P_1)\mathbf{E},\quad V_2 = \frac{tr}{2k_w}\{\tilde{\mathbf{W}}^T\tilde{\mathbf{W}}\}, \\ V_3 = \frac{\tilde{\boldsymbol{\rho}}^T\tilde{\boldsymbol{\rho}}}{2k_s} \quad (55)$$

The time derivative of V_1 is given by:

$$\dot{V}_1 = \mathbf{E}^T(\Xi \otimes P_1\mathbf{A}_1)\mathbf{E} - \mathbf{E}^T(\Xi L \otimes P_1\mathbf{B}_1) \\ (\tilde{\mathbf{W}}^T\boldsymbol{\Phi} + (\mathbf{g} - \mathbf{G}_0)\mathbf{u} + \mathbf{u}_s + \beta\boldsymbol{\Lambda}\mathbf{E} + \mathbf{w} + \boldsymbol{\varepsilon}) \quad (56)$$

Using Lemma 3, inequality (9) and applying (52), yields:

$$\dot{V}_1 = \frac{\mathbf{E}^T}{2}[\Xi \otimes (P_1\mathbf{A}_1 + \mathbf{A}_1^T P_1) - \beta(\hat{L} \otimes P_1\mathbf{B}_1\mathbf{B}_1^T P_1)]\mathbf{E} \\ -\mathbf{E}^T(\Xi L \otimes P_1\mathbf{B}_1)(\tilde{\mathbf{W}}^T\boldsymbol{\Phi} + (\mathbf{g} - \mathbf{G}_0)\mathbf{u} + \mathbf{u}_s + \mathbf{w} + \boldsymbol{\varepsilon}) \quad (57)$$

In light of (40), (57) can be rewritten:

$$\dot{V}_1 \le \frac{\mathbf{E}^T}{2}[\Xi \otimes (P_1\mathbf{A}_1 + \mathbf{A}_1^T P_1) - \beta a(L) \\ (I_N \otimes P_1\mathbf{B}_1\mathbf{B}_1^T P_1)]\mathbf{E} - \mathbf{E}^T(\Xi L \otimes P_1\mathbf{B}_1) \\ (\tilde{\mathbf{W}}^T\boldsymbol{\Phi} + (\mathbf{g} - \mathbf{G}_0)\mathbf{u} + \mathbf{u}_s + \mathbf{w} + \boldsymbol{\varepsilon}) \\ \le \sum_{i=1}^{N}\frac{\mathbf{E}_i^T}{2}\xi_i[P_1\mathbf{A}_1 + \mathbf{A}_1^T P_1 - \beta\frac{a(L)}{\xi_i}P_1\mathbf{B}_1\mathbf{B}_1^T P_1)]\mathbf{E}_i \\ -\mathbf{E}^T(\Xi L \otimes P_1\mathbf{B}_1)(\tilde{\mathbf{W}}^T\boldsymbol{\Phi} + (\mathbf{g} - \mathbf{G}_0)\mathbf{u} + \mathbf{u}_s + \mathbf{w} + \boldsymbol{\varepsilon}) \quad (58)$$

Using property (53) and Riccati inequality (9), (58) yields:

$$\dot{V}_1 \le -\frac{\xi_{\max}\sigma(Q_1)}{2}\|\mathbf{E}\|^2 - \mathbf{E}^T(\Xi L \otimes P_1\mathbf{B}_1) \\ (\tilde{\mathbf{W}}^T\boldsymbol{\Phi} + (\mathbf{g} - \mathbf{G}_0)\mathbf{u} + \mathbf{u}_s + \mathbf{w} + \boldsymbol{\varepsilon}) \quad (59)$$

where $\xi_{\max} = \min(\xi_1, ..., \xi_N)$. Considering (47), assumptions 2-3, definition L=D-A, the second property of Lemma 2 and adding time derivatives of V_2 and V_3, (59) can be rewritten as:

$$\begin{aligned}\dot{V} \le & -\frac{\xi_{\max}\underline{\sigma}(Q_1)}{2}\|\mathbf{E}\|^2 - \mathbf{E}^T(\Xi D \otimes P_1\mathbf{B}_1)[\mathbf{u}_s + \mathbf{w} + \boldsymbol{\varepsilon} - \underline{\mathbf{x}}_d^{(n)}] \\ & + \mathbf{E}^T(\Xi A \otimes P_1\mathbf{B}_1)[\tilde{\mathbf{W}}^T\mathbf{\Phi} + \mathbf{u}_s + \mathbf{w} + \boldsymbol{\varepsilon}] + \frac{\tilde{\boldsymbol{\rho}}^T\dot{\tilde{\boldsymbol{\rho}}}}{k_s} + \\ & tr\{\tilde{\mathbf{W}}^T(\frac{\dot{\tilde{\mathbf{W}}}}{k_w} - \mathbf{\Phi}\mathbf{E}^T(\Xi D \otimes P_1\mathbf{B}_1))\} \\ & + d_g\bar{\sigma}(\Xi)\bar{\sigma}(L)\bar{\sigma}(P_1)\|\mathbf{E}\|\|\mathbf{u}\|\end{aligned} \tag{60}$$

By employing (30), property (53) and using the property $\dot{\tilde{\mathbf{W}}} = -\dot{\hat{\mathbf{W}}}$ to the updating law of NN weight (51) and substituting them into (60), one obtains:

$$\begin{aligned}\dot{V} \le & -k_1\|\mathbf{E}\|^2 + k_2\|\mathbf{E}\|\|\tilde{\mathbf{W}}\|_F + k_3\|\mathbf{E}\|\|\tilde{\boldsymbol{\rho}}\| + k_4\|\mathbf{E}\| \\ & + \frac{\tilde{\boldsymbol{\rho}}^T\dot{\tilde{\boldsymbol{\rho}}}}{k_s} - \mathbf{E}^T(PD \otimes P_1\mathbf{B}_1)[\mathbf{u}_s + \mathbf{w} + \boldsymbol{\varepsilon}] - \eta tr\{\tilde{\mathbf{W}}^T\hat{\mathbf{W}}\}\end{aligned} \tag{61}$$

where $k_4 = c_1c_2 + \bar{\sigma}(\Xi)\bar{\sigma}(A)\bar{\sigma}(P_1)N_M$, $k_3 = mc_2 + m\bar{\sigma}(\Xi)\times\bar{\sigma}(A)\bar{\sigma}(P_1)$, $k_1 = \xi_{\max}(\frac{\underline{\sigma}(Q_1)}{2} - \frac{c_2\bar{\sigma}(P_1)}{a(L)})$, $k_2 = \Phi_M(c_2 + \bar{\sigma}(\Xi)\bar{\sigma}(A)\bar{\sigma}(P_1))$ and $c_2 = \underline{\sigma}_0^{-1}d_g\bar{\sigma}(\Xi)\bar{\sigma}(L)\bar{\sigma}(P_1)$ with $N_M = \sqrt{N}x_M + m\|\tilde{\mathbf{n}}\| + [\sum_{i=1}^{N} w_{iM}^2 + \varepsilon_{iM}^2]^{0.5}$. Applying the similar procedure of section 3 to ρ_i, replacing $\tilde{\rho}_i = \rho_i - \hat{\rho}_i$ and the updated law of the robust term gain (50), one obtains:

$$\begin{aligned}\dot{V} \le & -k_1\|\mathbf{E}\|^2 + k_2\|\mathbf{E}\|\|\tilde{\mathbf{W}}\|_F + k_3'\|\mathbf{E}\|\|\tilde{\boldsymbol{\rho}}\| + k_4\|\mathbf{E}\| \\ & -\eta_1\|\tilde{\mathbf{W}}\|_F^2 + \eta_1 W_M\|\tilde{\mathbf{W}}\|_F - \eta_2\|\tilde{\rho}\|^2 + k_5\|\tilde{\rho}\| + mk_u\|\boldsymbol{\rho}\|\end{aligned} \tag{62}$$

where $k_3' = k_3 + \bar{\sigma}(\Xi)\bar{\sigma}(D)\bar{\sigma}(P_1)$ and $k_5 = (\eta_2 + mk_u)\|\tilde{\mathbf{n}}\|$. Similar to the proof of Theorem 1, it is finally derived that:

$$\dot{V} \le -\mathbf{y}^T\mathbf{R}\mathbf{y} + \mathbf{r}\mathbf{y} + mk_u\|\boldsymbol{\rho}\| \tag{63}$$

In order to satisfy $\dot{V} \le 0$, $\mathbf{R}$ must be positive definite and:

$$\|\mathbf{y}\| > \frac{\sqrt{k_6}}{\underline{\sigma}(\mathbf{R})} + \frac{\|\mathbf{r}\|}{2\underline{\sigma}(\mathbf{R})} \tag{64}$$

with $k_6 = mk_u\|\tilde{\mathbf{n}}\| + \frac{\|\mathbf{r}\|^2}{4\underline{\sigma}^2(\mathbf{R})}$. Therefore, it is proved that all the signals are uniformly ultimately bounded. In order to satisfying $\mathbf{R} > 0$, the following inequalities are derived:

$$\underline{\sigma}(Q_1) > \frac{1}{\xi_{\max}}(\frac{k_2^2}{2\eta_1} + \frac{k_3^2}{2\eta_2} + 2\frac{c_2\bar{\sigma}(P_1)}{a(L)}) \tag{65}$$

This completes the proof.

5- Simulations

Leader-following problem: In this case, the consensus protocol given in Theorem 1 is applied to a multiple of nonlinear autonomous surface vehicles (ASVs) governed by the 3 degrees-of-freedom (3DOF) model [39]. The ASVs communicate together through a strongly connected directed graph shown in Fig. 1. The graph includes 3 agents with a leader node connected to agent 1. The *i*-th ASV dynamics are represented by the following equations:

$$\dot{\boldsymbol{\eta}}_i = \mathbf{R}_i(\psi_i)\mathbf{v}_i \tag{66}$$

$$\mathbf{M}_i\dot{\mathbf{v}}_i = \mathbf{h}_i(\boldsymbol{\eta}_i, \mathbf{v}_i) + \boldsymbol{\tau}_i + \boldsymbol{\tau}_{di} \tag{67}$$

where $\boldsymbol{\eta}_i = [x_i, y_i, \psi_i]^T \in \mathbb{R}^3$ is the position vector in the earth-fixed reference frame and $\boldsymbol{v}_i = [u_i, v_i, r_i]^T \in \mathbb{R}^3$ is the velocity vector in the body-fixed reference frame. $\mathbf{M}_i = \mathbf{M}_i^T \in \mathbb{R}^3$ and $\mathbf{R}_i = \mathbf{R}_i(\psi_i) \in \mathbb{R}^3$ denote the inertia matrix and the transformations matrix from the body-fixed to the earth-fixed reference frame, respectively. $\boldsymbol{\tau}_i = [\tau_{ui}, \tau_{vi}, \tau_{ri}]^T \in \mathbb{R}^3$ represents the generalized control input and $\boldsymbol{\tau}_{di} = [\tau_{udi}, \tau_{vdi}, \tau_{rdi}]^T \in \mathbb{R}^3$ is environment disturbances.The ASVs parameters are given in [39]. The Laplacian and adjacency matrices of the considered directed graph are defined by:

$$L = \begin{bmatrix} 2 & -1 & -1 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix}, B = diag(1,0,0) \tag{68}$$

The initial conditions of agents are $\boldsymbol{\eta}_1(0)=[1,2,\pi/6]^T$, $\boldsymbol{\eta}_2(0)=[-1,1,-\pi/10]^T$, $\boldsymbol{\eta}_3(0)=[0,-2,-\pi/6]^T$ and $\boldsymbol{v}_i=0$ for i=1,2,3. $\boldsymbol{\eta}_d=[x_d, y_d, \psi_d]^T = [0.2t, 3\cos(\frac{t}{50}), \arctan(\frac{\dot{y}_d}{\dot{x}_d})]^T$ is the reference trajectory and t denotes time. The external disturbances are chosen as:

$$\begin{aligned}\boldsymbol{\tau}_{di} = & 0.25[0.75 + 0.75\sin(\frac{t}{50}) + 0.6\sin(\frac{t}{10}) \\ & -0.3 + 0.75\sin(\frac{t}{50} - \frac{\pi}{6}) + 0.6\sin(0.3t) \\ & -0.3\sin(0.09t + \frac{\pi}{3}) - 1.5\sin(0.01t)]^T\end{aligned} \tag{69}$$

The proposed control protocol (15) is compared to the sliding mode control (SMC) protocol (70).

$$\mathbf{u}_i = \mathbf{g}_{i0}^{-1}[\beta_s\boldsymbol{\lambda}\mathbf{E}_i + k_r sign(\mathbf{B}_1^T P_1(p_i(b_i + d_i))\mathbf{E}_i)] \tag{70}$$

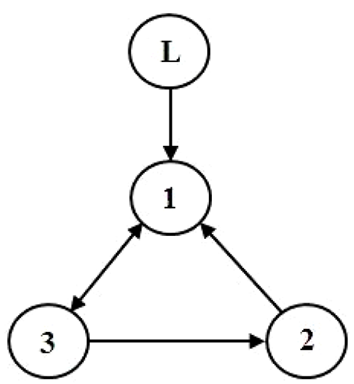

Fig 1. Strongly connected graph of 3 agents and a leader

with $\beta_s > 0$ and $k_r > 0$. The design parameters are set as $\beta = 9$, $\beta_s = 4$, $k_r = 0.01$, the gains of the updating laws $k_w = 1$, $k_s = 0.1$, $\eta_1 = \eta_2 = 1$ and the gain matrix $\mathbf{g}_{i0} = 0.05I_2$. The centers of the applied RBFNN, ζ_i, evenly spaced in $[-1,1]\times[-1,1]\times[-1,1]\times[-\pi,\pi]$ with spreads $\sigma_i = 0.825$ and number of neurons at each node $n_{ri} = 16$ were utilized. Figs. 2-4 illustrate the simulation's results. The results for the proposed method (15) and the SMC (70) are shown by the blue lines and black line, respectively. In the figures, NNA stands for the proposed NN adaptive controller.

The movements of the ASVs in the plane and their heading tracking curve are shown in Fig. 2. Figs. 3 and 4 show the applied control forces and norms of the tracking errors during consensus process of the ASVs, respectively. It is seen from Fig. 2 that the agents realized the coordinated tracking task. But, the fluctuations in the motion of the ASVs using the SMC protocol (70) imply that the proposed method (15) has a better performance against environment disturbances compared to the SMC protocol.In order to see the transients and the speed of convergence clearly, in Figs. 3 and 4, the results are illustrated for 10 seconds.

Leaderless problem: The proposed control law in (46) is simulated for three non-identical two-link robots. The model of a two-link robot is shown in Fig. 5. The dynamics of ith robot (i=12,3) is described by:

$$\mathbf{M}_i(\mathbf{q}_i)\ddot{\mathbf{q}}_i + \mathbf{C}_i(\mathbf{q}_i,\dot{\mathbf{q}}_i)\dot{\mathbf{q}}_i + \mathbf{g}_i(\mathbf{q}_i) = \boldsymbol{\tau}_i \qquad (71)$$

where $\mathbf{q}_i = [\phi_{1i},\phi_{2i}]^T \in \mathbb{R}^2$ is the state vector of configuration coordinates. $\boldsymbol{\tau}_i \in \mathbb{R}^2$ is the input vector, $\mathbf{M}_i(\mathbf{q}_i) \in \mathbb{R}^{2\times 2}$ denotes the symmetric and bounded positive definite inertia matrix and $\mathbf{C}_i(\mathbf{q}_i,\dot{\mathbf{q}}_i)$ represents the symmetric Coriolis matrix and centripetal

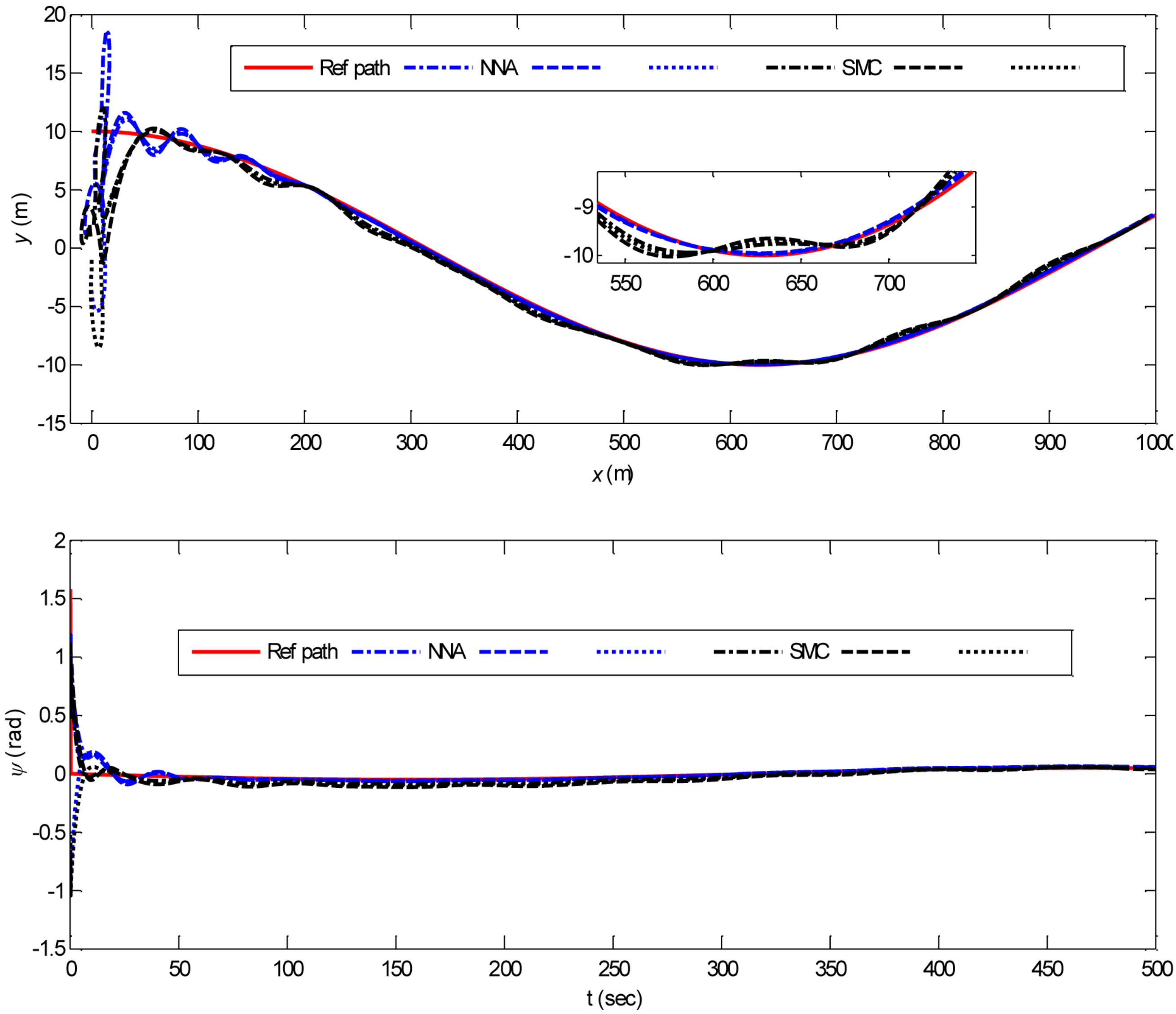

Fig 2. Trajectory tracking in horizontal plane

torques. $\mathbf{g}_i(\mathbf{q}_i)$ is the vector of gravitational torques. The physical parameters of the robots are taken from Table 5.1 of [40].

The Laplacian matrix of the communication topology between the robots is defined by:

$$L = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \tag{72}$$

Initial conditions of the agents are set to $\mathbf{q}_1(0) = [0.3, 0.4]^T$, $\mathbf{q}_2(0) = [0, -0.2]^T$, $\mathbf{q}_3(0) = [0.5, 0]^T$, $\dot{\mathbf{q}}_1(0) = [0.5, 0]^T$ and $\dot{\mathbf{q}}_2(0) = \dot{\mathbf{q}}_3(0) = \mathbf{0}$. The RBFNN parameters are chosen by the similar procedure of Leader-following problem. The other parameters are set as: $\beta = 10$, $k_w = 0.1$, $k_s = 0.1$, $\eta_1 = \eta_2 = 1$ and the gain matrix $\mathbf{g}_{i0} = 2I_2$. The results of applying the proposed control protocol (46) are illustrated in Fig.s 6-8. Synchronization of the robots is shown in Fig. 6. Figs. 7 and 8 show control inputs and local errors during consensus process, respectively. As seen from Fig. 6, the robots reach consensus on a common value. The simulation results illustrate that the estimation error and all the closed-loop signals are ultimately uniformly bounded.

6- CONCLUSIONS

In this paper, the leader-following and leaderless consensus problems of high-order MIMO multi-agent systems with nonlinear affine dynamics have been studied. In addition to the system dynamics, the gain matrix was assumed to be unknown. For both cases, a distributed neuro-adaptive method was proposed under

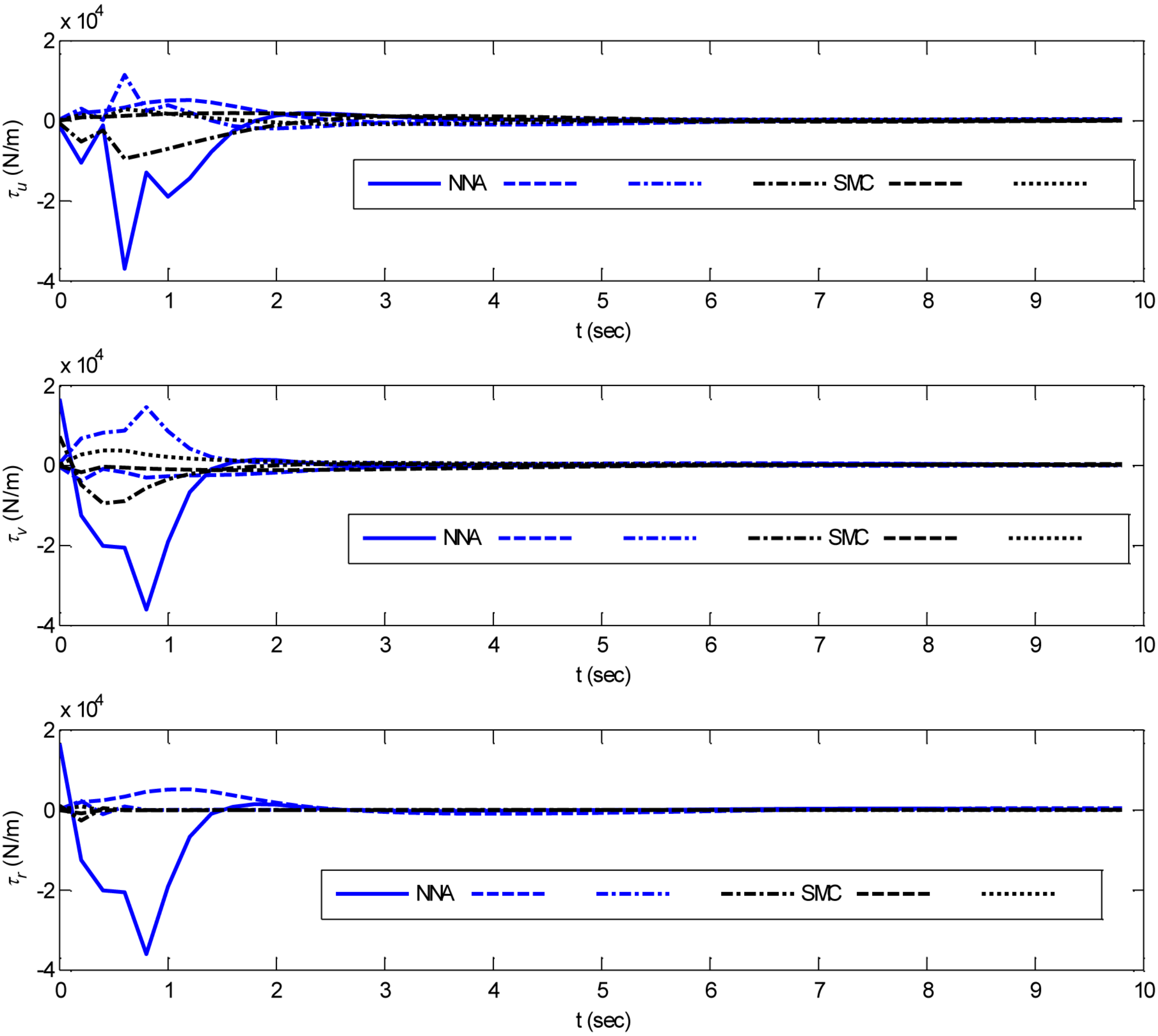

Fig 3. Control inputs of ASVs during consensus tracking

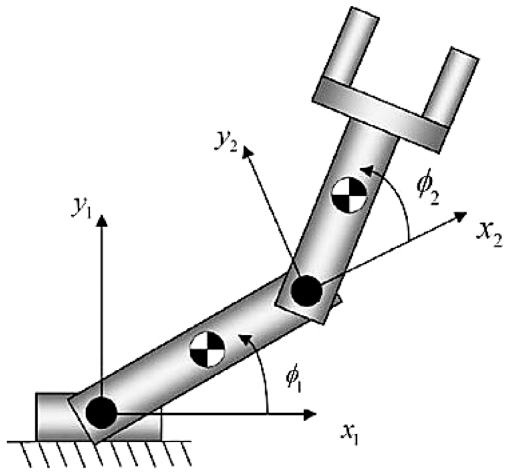

Fig 5. A two-link robot

directed graphs. The methods were constructed based on the local error. To estimate unknown nonlinearities, the RBFNNs were employed. Lyapunov method was utilized for stability analysis of the overall system. The update laws of unknown parameters of NNs and the robust term gain were determined from Lyapunov stability analysis. In particular, it was proved that the local errors are uniformly ultimately bounded, and converge into a neighborhood of the origin. The validity of the proposed protocol controls and their efficacy against disturbances and estimation error were verified through the simulation results.

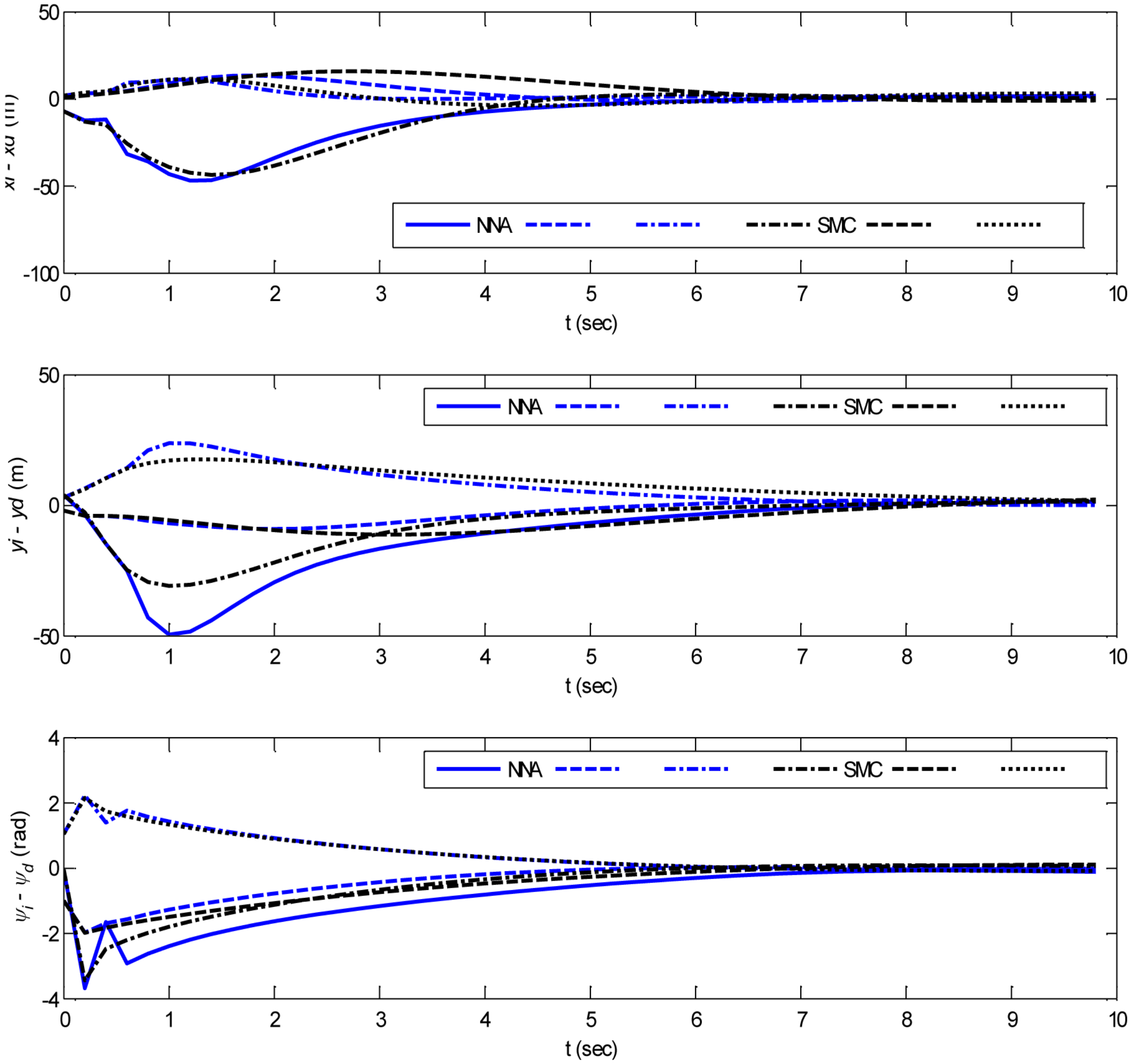

Fig 4. Tracking errors during consensus process

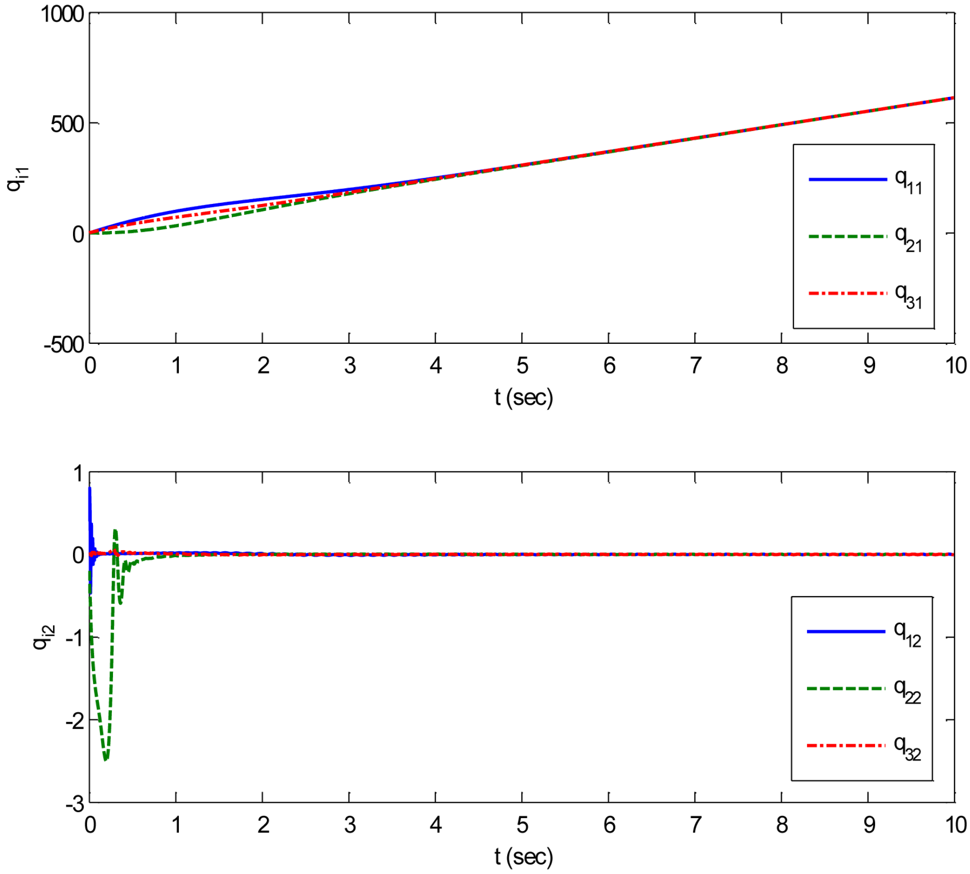

Fig 6. Consensus of joints of the robots

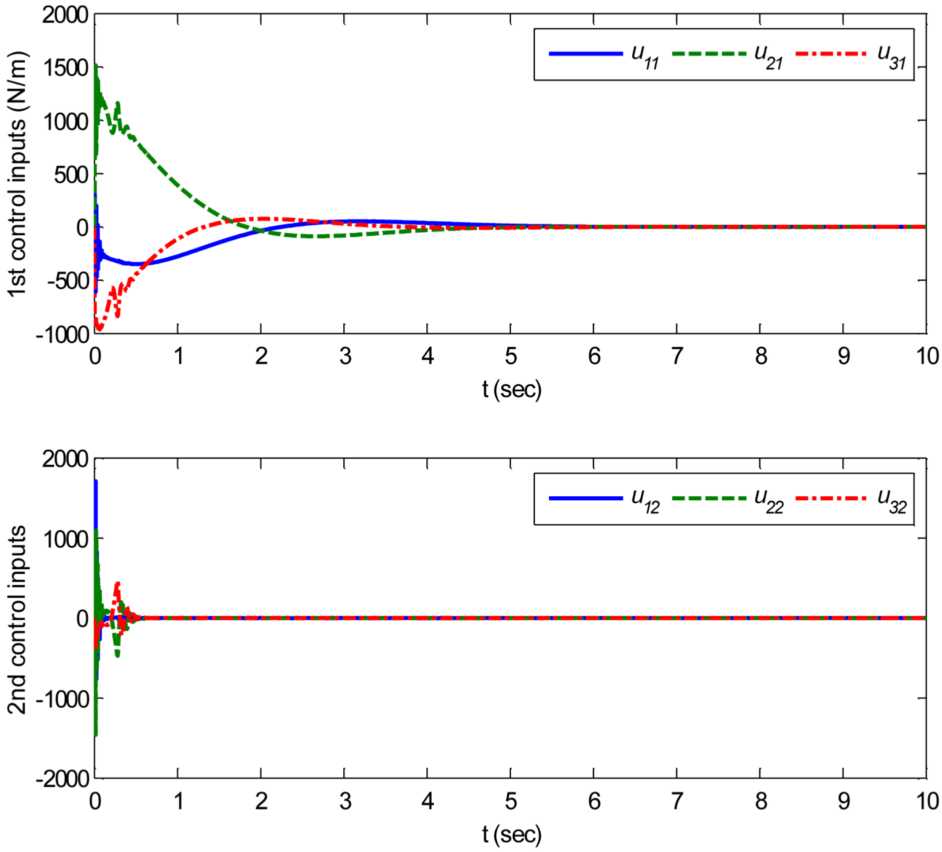

Fig 7. Control inputs of the robots

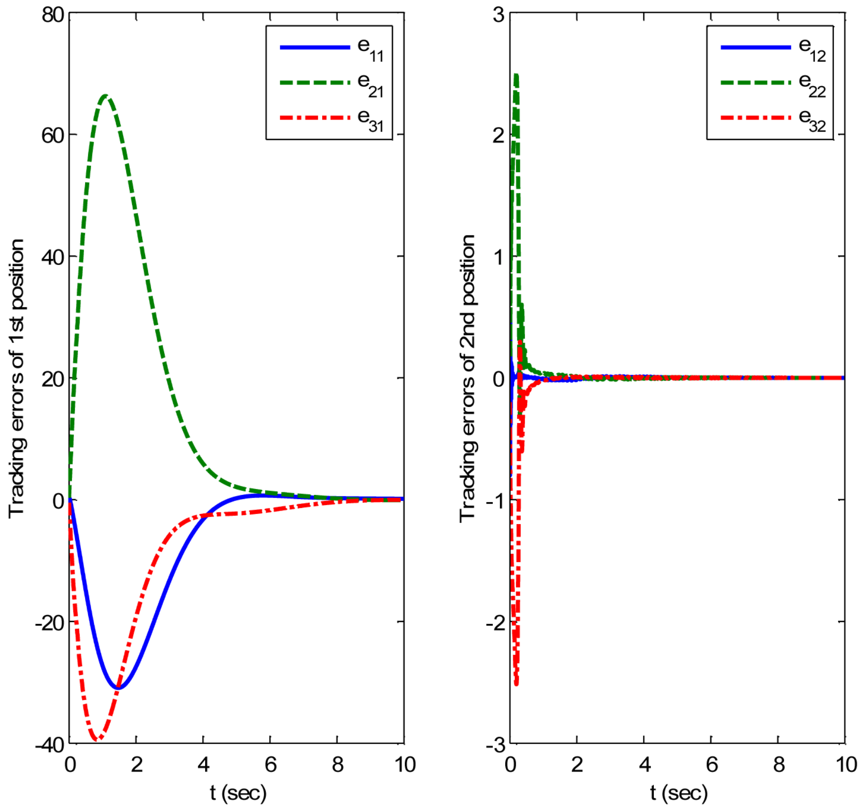

Fig 8. Local errors during consensus process

References

[1] Ren, W.; Beard, R. W. and Atkins, E. M.; "Information Consensus in Multivehicle Cooperative Control," *IEEE Control Syst. Mag.*, Vol. 27, No. 2, pp. 71–82, 2007.

[2] Olfati-Saber, R. and Murray, R. M.; "Consensus Problems in Networks of Agents with Switching Topology and Time-Delays," *IEEE Trans. Autom. Control*, Vol. 49, No. 9, pp. 1520–1533, 2004.

[3] Ren, W. and Beard, R. W.; "Distributed Consensus in Multi-Vehicle Cooperative Control: Theory and Applications," *Springer-Verlag*, London, 2008.

[4] Ren, W.; Beard, R. W. and Chen, Y.; "High-order and Model Reference Consensus Algorithms in Cooperative Control of Multivehicle Systems," *J. Dyn. Syst., Meas., Control*, Vol. 129, No. 5, pp. 678–688, 2007.

[5] Ni, W.; Wang, X. and Xiong, C.; "Consensus Controllability, Observability and Robust Design for Leader-Following Linear Multi-Agent Systems," *Automatica*, Vol. 49, No. 7, pp. 2199–2205, 2013.

[6] Li, Z.; Duan, Z. and Chen, G.; "Dynamic Consensus of Linear Multi-Agent Systems," *IET Control Theory Appl.*, Vol.5, No.1, pp.19–28, 2011.

[7] Wang, L. and Wang, Q. G.; "A General Approach for Synchronization of Nonlinear Networked Systems with Switching Topology," *In. J. Syst. Sci.*, Vol. 44, No. 12, pp. 2199–2210, 2013.

[8] Yu, W.; Chen, G. and Cao, M.; "Consensus in Directed Networks of Agents with Nonlinear Dynamics," *IEEE Trans. Autom. Control*, Vol. 56, No. 6, pp. 1436–1441, 2011.

[9] Wen, G.; Rahmani, A. and Yu, Y.; "Consensus Tracking for Multi-Agent Systems with Nonlinear Dynamics under Fixed Communication Topologies," *Proc. World Congr. Eng. Comput. Sci. (WCECS)*, San Francisco, USA, Vol. 1, pp. 19–21, 2011.

[10] Li, Z.; Liu, X. and Fu, M.; "Global Consensus Control of Lipschitz Nonlinear Multi-Agent Systems," *18th IFAC World Congr.*, Milano, Italy, pp. 10056–10061, 2011.

[11] Yu, W.; Chen, G.; Cao, M. and Kurths, J.; "Second-Order Consensus for Multiagent Systems with Directed Topologies and Nonlinear Dynamics," *IEEE Trans. Syst., Man, Cybern., Syst., Part B: Cybernetics*, Vol. 40, No. 3, pp. 881–891, 2010.

[12] [12] Ren, W.; "Distributed Leaderless Consensus Algorithms for Networked Euler–Lagrange Systems," *Int. J. Control*, Vol. 82, pp. 2137–2149, 2009.

[13] Hui, Q.; Haddad, M. M. and Bhat, S.; "Finite-Time Semistability and Consensus for Nonlinear Dynamical Networks," *IEEE Trans. Autom. Control*, Vol. 53, No. 8, pp. 1887–1900, 2008.

[14] Suiyang, K.; Xie, L. and Man, Z.; "Robust Finite-Time Consensus Tracking Algorithm for Multirobot Systems," *IEEE/ASME Trans. Mechatronics*, Vol. 14, No. 2, pp. 219–228, 2009.

[15] Dong, Y. and Huang, J.; "Cooperative Global Output Regulation for a Class of Nonlinear Multi-Agent Systems," *IEEE Trans. Autom. Control*, Vol. 59, No. 5, pp. 1348–1354, 2014.

[16] Wen, G.; Duan, Z.; Chen, G. and Yu, W.; "Consensus Tracking of Multi-Agent Systems with Lipschitz-Type Node Dynamics and Switching Topologies," *IEEE Trans. Circuits Syst. I, Reg. Papers*, Vol. 61, No. 2, pp. 499–511, 2014.

[17] Wen, G.; Peng, Z.; Rahmani, A. and Yu, Y.; "Distributed Leader-Following Consensus for Second-Order Multi-Agent Systems with Nonlinear Inherent Dynamics," *Int. J. Systems Sci*., Vol. 45, No. 9, pp. 1892–1901, 2014.

[18] Zhang, W.; Wang, Z. and Guo, Y.; "Backstepping-Based Synchronisation of Uncertain Networked Lagrangian Systems," *Int. J. Systems Sci.*, Vol. 45, No. 2, pp. 145–158, 2014.

[19] Hou, Z.; Cheng, L.; and Tan, M.; "Decentralized Robust Adaptive Control for the Multiagent System Consensus Problem Using Neural Networks," *IEEE Trans. Syst., Man, Cybern.,Syst., Part B: Cybernetics*, Vol. 39, No. 3, pp. 636–647, 2009.

[20] Cheng, L.; Hou, Z. G.; Tan, M.; Lin, Y. and Zhang, W.; "Neural-Network Based Adaptive Leader-Following Control for Multiagent Systems with Uncertainties," *IEEE Trans. Neural Netw.*, Vol. 21, No. 8, pp. 1351–1358, 2010.

[21] Das, A. and Lewis, F. L.; "Distributed Adaptive Control for Synchronization of Unknown Nonlinear Networked Systems," *Automatica*, Vol. 46, No. 2, pp. 2014–2021, 2010.

[22] Liu, Y. and Jia, Y.; "Adaptive Consensus Protocol for Networks of Multiple Agents with Nonlinear Dynamics Using Neural Networks," *Asian J. Control*, Vol. 14, No. 5, pp. 1328–1339, 2012.

[23] Cui, R.; Ren, B. and Ge, S. S.; "Synchronised Tracking Control of Multi-Agent System with High-Order Dynamics," *IET Control Theory Appl.*, Vol. 6, No. 5, pp. 603–614, 2012.

[24] Zhang, H. and Lewis, F. L.; "Adaptive Cooperative Tracking Control of Higher-Order Nonlinear Systems with Unknown Dynamics," *Automatica*, Vol. 48, No. 7, pp. 1432–1439, 2012.

[25] Das, A. and Lewis, F. L.; "Cooperative Adaptive Control for Synchronization of Second-Order Systems with Unknown Nonlinearities," *Int. J. Robust Nonlinear Control*, Vol. 21, No. 13, pp. 1509–1524, 2011.

[26] Zouand, A. M. and Kumar, K. D.; "Neural Network-Based Adaptive Output Feedback Formation Control for Multi-Agent Systems," *J. Nonlinear Dynamics*, Vol. 70, No. 2, pp. 1283–1296, 2012.

[27] Zou, A. M.; Kumar, K. D. and Hou, Z. G.; "Distributed Consensus Control for Multi-Agent Systems Using Terminal Sliding Mode and Chebyshev Neural Networks," *Int. J. Robust Nonlinear Control*, Vol. 23, pp. 334–357, 2013.

[28] Peng, Z.; Wang, D.; Sun, G. and Wang, H.; "Distributed Cooperative Stabilisation of Continuous–Time Uncertain Nonlinear Multi-Agent Systems," *Int. J. Systems Sci.*, Vol. 45, No. ,10, pp. 2031–2041, 2013.

[29] Xu, H. and Ioannou, P. A.; "Robust Adaptive Control for a Class of MIMO Nonlinear Systems with Guaranteed Error Bounds," *IEEE Trans. Autom. Control*, Vol. 48, No. 5, pp. 728–742, 2003.

[30] Bechlioulis, C. P. and Rovithakis, G. A.; "Prescribed Performance Adaptive Control for Multi-Input Multi-Output Affine in the Control Nonlinear Systems," *IEEE Trans. Autom. Control*, Vol. 55, No.5, pp. 1220–1226, 2010.

[31] Frikha, S.; Djemel, M. and Derbel, N.; "Observer Based Adaptive Neuro-Sliding Mode Control for MIMO Nonlinear Systems," *Int. J. Control, Automation, Syst.*, Vol. 8, No. 2, pp. 257–265, 2010.

[32] Slotine, J. J. and Li, W.; "Applied Nonlinear Control," Upper Saddle River, Englewood Cliffs, NJ: Prentice–Hall, Vol. 199, No. 1, 1991.

[33] Qu, Z.; "Cooperative Control of Dynamical

Systems: Applications to Autonomous Vehicles," *Springer Science and Business Media*, London, 2009.

[34] Lian, J.; Hu, J. and Zak, S. H.; "Variable Neural Adaptive Robust Control: A Switched System Approach," *IEEE Trans. Neural Networks Learning Systems*, Vol. 26, No. 5, pp. 903–15, 2015.

[35] Karimi, B.; Menhaj, M. B.; Karimi–Ghartemani, M. and Saboori, I.; "Decentralized Adaptive Control of Large–Scale Affine and Nonaffine Nonlinear Systems," *IEEE Trans. Instrum. Meas.*, Vol. 58, No. 8, pp. 2459–2467, 2009.

[36] Kar, I. and Behera, J.; "Direct Adaptive Neural Control for Affine Nonlinear Systems," *Appl. Soft Computing*, Vol. 9, No. 2, pp. 756–764, 2008.

[37] Khalil, H. K.; "Nonlinear Systems," *Upper Saddle River, NJ: Prentice–Hall*, 3rd ed., 2002.

[38] Qin, J.; Yu, C.; Gao, H. and Wang, X.; "Leaderless Consensus Control of Dynamical Agents under Directed Interaction Topology," *50th IEEE Conference on Decision and Control and European Control Conference (CDC–ECC)*, Orlando, FL, USA, pp. 12–15, 2011.

[39] Almeida, J.; Silvestre, C.; Pascoal, A. M.; "Cooperative Control of Multiple Surface Vessels with Discrete–Time Periodic Communications," *Int. J. Robust and Nonlinear Control*, Vol. 22, No. 4, pp. 398–419, 2012.

[40] Chung, S. J.; "Nonlinear Control and Synchronization of Multiple Lagrangian Systems with Application to Tethered Formation Flight Spacecraft," *Ph.D. Dissertation*, Department of Aeronautics and Astronautics, MIT, 2007.