AUT Journal of Modeling and Simulation

AUT J. Model. Simul., 49(1)(2017)43-56
DOI: 10.22060/miscj.2016.835

Integrated production-inventory model with price-dependent demand, imperfect quality, and investment in quality and inspection

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ABSTRACT: In practice, manufacturing systems are never perfect and may have low quality outputs. Therefore, different decisions such as reprocessing, sale at lower prices or diminishing are made according to industry and market. This paper investigates the importance of supply chain coordination through developing two models in centralized decision-making for an imperfect quality manufacturing system with probabilistic defect rate. Moreover, two types of errors in inspection process are considered: Type I error (classifying perfect products as defective ones) and Type II error (classifying defective products as perfect ones). Moreover, a cost function for investment on products quality as well as selling price is considered. The algorithms to find the optimal solution for both models are suggested. Numerical results show that even by less consumer prices; more profit, more satisfied customer, and improved quality can be achieved through coordination in supply chain.

Review History:

Received: 1 March 2016
Revised: 9 May 2016
Accepted: 21 May 2016
Available Online: 25 May 2016

Keywords:

Joint economic lot sizing
imperfect quality
inspection errors
investment in quality
price dependent demand

1- Introduction

Nowadays, due to more specialized roles, competition extension and more demanding customers, companies in a supply chain (SC) have no choice but coordination in decision making to satisfy the needs of customers more efficiently and stay ahead of competitors. The effectiveness of coordination in a supply chain could be studied in two areas: reducing costs (increasing profits) of supply chain and improving the services provided. Thus, the concept of coordination in supply chains are one of the areas that researchers are interested in.
Decision-making in SCs can be done in two ways: centralized and decentralized. Ideally, decision-making in a supply chain is done by a decision maker that has full access to information for all members. This is possible when the entire chain is under the control of one decision maker or that the benefits from this integration are divided fairly between the members. This type of coordination is known as centralized. However, usually each member only has access to its own information and tends to maximize its own profit. These cases in which the members are making decisions independently are known as decentralized systems. [1]
Inventory management in SCs also was an isolated activity done by only one of the actors for many years. After Goyal [2], a number of studies are formed in recent years that focused on coordinating the inventory replenishment decisions between the units to maximize the entire SC's benefit. In order to achieve coordinated inventory replenishment, researchers usually focus on minimizing total system cost. These classes of problems are known as joint economic lot sizing (JELS) problems or integrated production-inventory models. These problems are subsets of coordinated models and are usually useful in the cases in which suppliers are in a long-term relationship with their customers. These long-term relationships are very common in automotive industry. In these cases, SC members are encouraged to corporate together in order to reduce the entire system's cost.
On the other hand, in the cases SC members make the inventory replenishment decisions independently, vendor usually suffers more costs because the buyer specifies the amount of order just based on its own costs. However, if the buyer increases its order quantity, vendor's costs reduce. If the vendor be able to compensate the buyer to order more, it can reduce the total cost of the system and achieve the optimal solution [3]. Decision variables of these problems in the simplest case are the buyer's order lot size and the number of shipments in a production period. By adding more assumptions to the basic model, decision variables may increase.

1- 1- Literature Review

One of the first models to determine the joint economic lot size in a coordinated SC is provided by Goyal in 1976; that is a system consisting of a vendor and a buyer [3]. In this paper, the supplier is only the vendor of products, and inventory replenishment rate is infinite. Moreover, the products sent from supplier to buyer are in equal sized shipments. Later, by considering the production rate for the vendor, Goyal in 1988, proposed a new model [4]. In the literature, these simple single-vendor single-buyer models have been developed in two dimensions:

- Depth: models that have more stages at SC or more actors on each stage.
- Scope: models with considering more assumptions such as the quality of products, learning in production, etc.

A classification made by Glock in his review paper is as follows:

- Basic models: including two-level and multi-level models with the assumptions of basic model.

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- Developed models according to some assumption such as random demand or lead time, investment in order to reduce the delay in delivery or setup costs, imperfect quality of products, products decay, learning in production and etc. [3]

One of the first papers considering the imperfect quality in the integrated production-inventory model is Huang [5]. Affisco et al. also considered the fraction of perfect products as a function of quality improvement investment [6]. Moreover, Goyal et al. studied a system that in its process of production, both perfect and poor quality products are produced. Buyer sells defective items at a price lower than the price of the normal items. The amount of defect production is considered as a random variable in this model, while inspection is not incorporated [7]. Later, Liu and Cetinkaya, amended the investment function introduced by Affisco [8]. Lin then developed the problem by adding the errors in the inspection of products. He incorporated two types of errors with specific rate for inspection. Moreover, the rate of defective products is assumed to be random variable with specific distribution [9]. Hsu and Hsu expanded Lin's model for items with imperfect quality and inspection errors [10]. Then, Lin suggested an integrated production-inventory model with lacking quality for the random demand and adjustable lead time, with backlogging demand [11]. Li and Chen proposed a model that optimizes the number of shipments, in which the buyer returns the defective product to the vendor [12]. Chuang et al. introduced a model in which the investment factor on quality and delays in payments is evaluated [13].

Ahmadi Rad et al. combined the pricing of the product delivered to the buyer in a joint economic lot sizing problem with imperfect quality products [14]. Hsu and Hsu then added the assumption of demand backlog to their previous paper [15]. Khan et al. investigated the impact of learning in quality of production on integrated production-inventory model with defective quality and inspection with errors [16].

In the table below we review the main assumptions of models considering the quality factor in JELS literature as well as our two models.

Table 1 shows that the models considering defective products have been developed in different areas in the literature including inspection operations, errors in inspection and investment on production quality to reduce defect rates. As can be seen, the models considering the probabilistic defect rate, has not considered the option of investment in quality and/or price-dependent demand. However, in reality, companies may invest to improve the quality of their finished products. Moreover, customer demands are usually correlated to the selling price. Considering these two assumptions makes the models more realistic and applicable. That is out motivation to develop a general model incorporating four mentioned assumptions simultaneously, i.e. the quality of products with random defective rate, inspection operations with two-type random errors, investment on production quality, and the impact of price on demand.

The rest of this paper is organized as follows. In Section 2, the modeling assumptions and notation are provided. The mathematical development of the model is introduced in Section 3. We develop solution approaches for both coordinated and independent decision-making models in Section 4. Section 5 uses numerical examples and sensitivity analysis to compare the models. Conclusions and further research directions are presented in Section 6.

Table 1. Review of the models with imperfect quality.

	Defect rate	Inspection	Investment in quality	Demand
[3]	Probabilistic	Without error	-	Deterministic and fixed
[4]	Deterministic variable	-	yes	Deterministic and fixed
[5]	Probabilistic	-	-	Deterministic and fixed
[7]	Probabilistic	Deterministic error type I and type II	-	Deterministic and fixed
[8]	Probabilistic	Probabilistic error type I and type II	-	Deterministic and fixed
[9]	Probabilistic	Without error	-	Probabilistic with variable lead time
[11]	Deterministic variable	Without error	yes	Deterministic and fixed
[12]	Deterministic	Without error	-	Price-dependent
[13]	Probabilistic	Probabilistic error type I and type II	-	Deterministic and fixed
1st proposed model	Probabilistic variable	Probabilistic error type I and type II	yes	Deterministic and fixed
2nd proposed model	Probabilistic variable	Probabilistic error type I and type II	yes	Price-dependent

2- Problem Definition

A model of JELS problem with regard to errors in the inspection process was proposed by Khan et al, in 2011 [17] that Hsu wrote a note on it in 2012 [18]. In 2012 Hsu and Hsu developed that problem to an integrated production-inventory model for a SC with a vendor and a buyer. Our research is based on this model [10].

2- 1- Assumptions

The main assumptions of the problem are as follows:

- A two-stage supply chain composed of a vendor (manufacturer) and a buyer (retailer) is considered.
- One product type is produced and its demand depends on the final price.
- System output consists of two categories: perfect and defective. The production rate of defective products is a random variable.
- Defective average rate decreases by investment on quality. A logarithmic investment function model is used for this purpose.
- Given that the rate of defective products follows uniform distribution, with investment on quality, the range of this uniform distribution tightens.
- A 100% inspection of buyer is along with errors type I and type II that both are of uniform distribution.
- After each inspection period, the defective products are returned to the vendor, and the vendor sells them at a

lower price in a secondary market. This statement is true about the defective product that has been sold instead of a non-defective product to customers and has been returned to the buyer by them.

2- 2- Notation

The notation used through the paper are as follows:

Indices

b: This index specifies the parameters and functions related to the buyer.

v: This index specifies the parameters and functions related to the vendor.

j: This index determines the functions of the whole system, as a joint system.

Parameters

P: Annual production rate of vendor

x: Annual inspection rate of buyer

S: Production set up costs for a manufacturing period of vendor

F: Fixed costs per shipment for buyer

e_1: Error type I (recognition of a perfect product as defective one)

$f(e_1)$: Error type I probability distribution function, with mean $\alpha/2; e_1 \sim U[0,\alpha]$

e_2: Error type II (recognition of a defective product as perfect one)

$f(e_2)$: Error type II probability distribution function, with mean $\beta/2; e_2 \sim U[0,\beta]$

c_j: Inspection cost of each unit

c_w: The cost of producing one unit of a defective product for vendor

c_{ab}: The cost of selling a defective product unit instead of perfect product for the buyer

c_{av}: The cost of selling a defective product unit instead of perfect product for the vendor

c_a: The cost of selling a defective product unit instead of perfect product for the system, $c_a=c_{ab}+c_{av}$

c_r: Cost of recognizing a perfect product unit as defective product

h_v: Annual holding cost of an item by the vendor

h_b: Annual holding cost of an item by the buyer

Decision variables

Q: Size of shipments to the buyer

n: Total number of shipments sent in a production cycle time

D: Annual demand rate

γ: Probability of producing a defective product

f(γ): Distribution function of γ with mean q/2, where q is of the potential decision variable, γ~U[0,q]

T: The time interval between two consecutive shipments

3- Mathematical Modelling

In this section, we first assume constant demand. Then a new model is proposed assuming price-dependent demand.

3- 1- Buyer's costs

Each time products arrive to the buyer as size of Q, a full inspection is carried out. A fraction of each shipment is assigned to defective products; γ that follows a distribution function f(γ). During the inspection, the inspector may detect a perfect product as defective with the probability of e_1. He may also detect a defective product as perfect with the probability of e_2. All products detected defective by buyer and also the defective products returned from the market to the buyer, returns to the vendor at the end of inspection process in a shipment, and vendor pays back the entire money buyer paid for them. Thus, any defective product imposes a cost of c_w to the vendor, i.e. the difference between the cost of producing one unit of output and the final price for selling defective product in the secondary market. Moreover, each unit of defective products returned from the market imposes the cost of c_{av} to the vendor and c_{ab} to the buyer, which includes the loss of trade credit costs. Figure 1 shows the inventory level of the buyer.

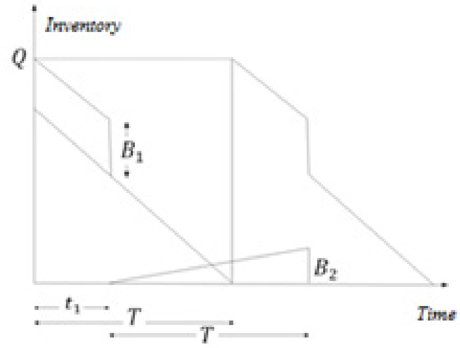

Fig. 1. The buyer's inventory level.

The number of defective items detected in each lot, and the number of products returned by customers in any lot are $B_1=Q(1-\gamma)e_1+Q\gamma(1-e_2)$ and $B_2=Q\gamma e_2$, respectively. Time required to inspect every shipment is also $t_1=Q/x$. For each returned product from market, there must be a perfect product to be replaced. The buyer is then facing two types of demand; a common demand and a replacement demand. So the effective demand rate will be $D`=D+B_2/T$, where

$$T = \frac{Q-B_1}{D'} = \frac{Q-B_1-B_2}{D} = \frac{Q(1-\gamma)(1-e_1)}{D} \tag{1}$$

The inventory holding cost for one period of shipment delivery is:

$$HC_b = h_b\left(B_1 t_1 + \frac{(Q-B_1)T}{2} + \frac{B_2 T}{2}\right) = h_b\left(\begin{array}{l}\frac{Q^2}{x}\left((1-\gamma)e_1+\gamma(1-e_2)\right)+\\ \frac{Q^2\left(1-(e_1+\gamma)+\gamma(e_1+2e_2)\right)(1-\gamma)(1-e_1)}{2D}\end{array}\right) \tag{2}$$

Thus, buyer's cost per production period is as follows:

$$C_b = nF + nc_i Q + nc_{ab} Q\gamma e_2 + nHC_b \tag{3}$$

To obtain the expected total cost of the buyer, we have: $ETC_b=E(C_b)/E(T_c)$. As the production period is a random variable, $T_c=nT$, by replacing the expected values of the random variables in Equation (3), we get Equation (4).

3- 2- Vendor's costs

The behavior of inventory at the vendor is according to Figure 2. As can be seen, vendor responds to buyer demand by producing in time period T1. We use Figure 3 to calculate the cost of producing and holding products:

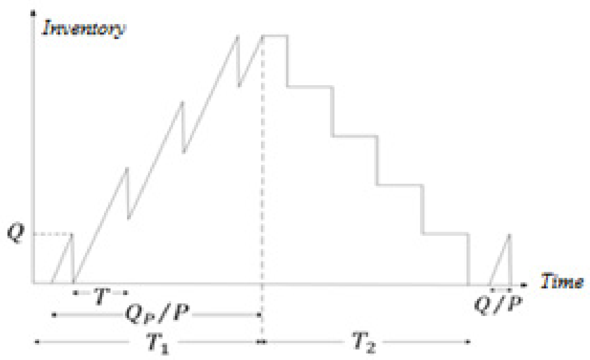

Fig. 2. The vendor's inventory.

$$ETC_b(n,q,Q)=\frac{\left(\frac{FD}{Q}\right)+D\left(c_i+c_{ab}\frac{q}{2}\times\frac{\beta}{2}\right)}{\left(1-\frac{q}{2}\right)\left(1-\frac{\alpha}{2}\right)}+$$

$$h_bQ\left\{\begin{array}{l}\left(\frac{D\left(\left(1-\frac{q}{2}\right)\frac{\alpha}{2}+\frac{q}{2}\left(1-\frac{\beta}{2}\right)\right)}{x\left(1-\frac{q}{2}\right)\left(1-\frac{\alpha}{2}\right)}\right)+\\ \frac{1-q-\alpha+\alpha q+\frac{\beta}{2}\left(1-\frac{\alpha}{2}\right)q}{2\left(1-\frac{q}{2}\right)\left(1-\frac{\alpha}{2}\right)}+\\ \left(\frac{\left(1-\alpha-\beta+\frac{\alpha\beta}{2}+\frac{\alpha^2}{3}\right)\frac{q^2}{3}+\frac{\alpha^2}{3}(1-q)}{2\left(1-\frac{q}{2}\right)\left(1-\frac{\alpha}{2}\right)}\right)\end{array}\right\} \quad (4)$$

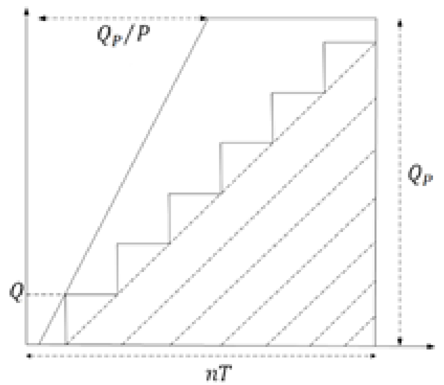

Fig. 3 Cumulative behavior of the vendor's inventory.

In Figure 3, the trapezoid represents the cumulative inventory of manufactured products and the shaded area represents the amount of inventory decreased over time in a production period. Thus, inventory holding cost of the vendor in a production period is calculated using the following expression;

$$HC_v=h_v\left\{\begin{array}{l}\frac{nQ^2}{P}-\frac{n^2Q^2}{2P}+\\ \frac{n(n-1)Q^2(1-\gamma)(1-e_1)}{2D}\end{array}\right\} \quad (5)$$

Other costs of vendor are production set up costs, cost of producing each unit of a perfect product, and costs of producing defective products. The total cost of vendor during a production period is then:

$$C_v=S+nc_wQ\gamma+nc_rQ(1-\gamma)e_1+nc_{av}Q\gamma e_2+$$
$$h_v\left\{\frac{nQ^2}{P}-\frac{n^2Q^2}{2P}+\frac{n(n-1)Q^2(1-\gamma)(1-e_1)}{2D}\right\} \quad (6)$$

Therefore, to calculate the total annual cost of the vendor, we have:

$$ETC_v(n,q,Q)=\frac{S}{nQ\left(1-\frac{q}{2}\right)\left(1-\frac{\alpha}{2}\right)}+$$

$$\frac{D\left(c_w\frac{q}{2}+c_r\left(1-\frac{q}{2}\right)\frac{\alpha}{2}+c_{av}\frac{q}{2}\times\frac{\beta}{2}\right)}{\left(1-\frac{q}{2}\right)\left(1-\frac{\alpha}{2}\right)}+$$

$$h_v\left\{\begin{array}{l}\frac{QD}{P\left(1-\frac{q}{2}\right)\left(1-\frac{\alpha}{2}\right)}-\\ \frac{nQD}{2P\left(1-\frac{q}{2}\right)\left(1-\frac{\alpha}{2}\right)}+\frac{(n-1)Q}{2}\end{array}\right\} \quad (7)$$

3- 3- Cost of the system in the coordinated case

In coordinated case, objective function is equal to the total costs of vendor and buyer, as well as the investment in quality. According to the investment function introduced in [8], $a=\ln(q_0)/\delta$ and $b=1/\delta$. Thus, the objective function is:

$$ETC(n,q,Q)=ETC_b(n,q,Q)+$$
$$ETC_v(n,q,Q)+ia-ib\times\ln(q) \quad (8)$$

The model for coordinated case is then as follows:

$$Min\ z=ETC(n,q,Q) \quad (9)$$

s.t.

$$0\le q\le q_0 \quad (10)$$
$$Q\ge 0 \quad (11)$$

n positive integer

Total cost of buyer is then obtained from the following equation:

$$ETC_b(n,T,Q)=\frac{F}{T}+\frac{c_iQ}{T}+$$
$$\frac{c_{ab}Q}{T}\frac{\beta}{2}-c_{ab}D\times\frac{\beta}{2}/(1-\alpha/2)+$$
$$h_b\left(\begin{array}{l}\frac{QD}{x}\left(\frac{\alpha/2}{1-\alpha/2}\right)+\\ \frac{Q^2}{xT}(1-\beta/2)-\\ \frac{DQ}{x}\left(\frac{1-\beta/2}{1-\alpha/2}\right)+\\ \frac{DT}{2}+\frac{Q\beta}{2}-\frac{DT\beta}{2(1-\alpha/2)}\end{array}\right) \quad (12)$$

Total cost of vendor is also obtained from the following equation:

$$ETC_v(n,T,Q)=\frac{S}{nT}+\frac{c_w Q}{T}-c_w\frac{D}{\left(1-\frac{\alpha}{2}\right)}+$$
$$c_r\frac{D}{\left(1-\frac{\alpha}{2}\right)}\frac{\alpha}{2}+\frac{c_{av}Q}{T}\frac{\beta}{2}-c_{av}\frac{D}{\left(1-\frac{\alpha}{2}\right)}\frac{\beta}{2}+ \tag{13}$$
$$h_v\left\{\frac{Q^2}{PT}\left(1-\frac{n}{2}\right)+\frac{(n-1)Q}{2}\right\}$$

The total system cost is then as follows:

$$ETC_j(n,T,Q)=ETC_b(n,T,Q)+$$
$$ETC_v(n,T,Q)+ia-ib\times ln\left(2-2\frac{DT}{Q(1-\alpha/2)}\right) \tag{14}$$

As q=2-2DT/(Q(1-α/2)), it can be rewritten as:

$$Min\ z=ETC_j(n,T,Q) \tag{15}$$

s.t.

$$DT-Q(1-\alpha/2)\le 0 \tag{16}$$
$$Q(1-\alpha/2)(1-q_0/2)-DT\le 0 \tag{17}$$
$$Q,T\ge 0 \tag{18}$$

n positive integer

3- 4- The price-dependent demand model

In this case, inventory and cost function are same as the first model. However, the objective function here is total system benefit. In this model, final demand has a linear relationship with consumer price (*D=l-mp*) where *l* and *m* are the parameters and *p* represents the consumer price. Given these changes, the vendor benefit function is:

$$ETP_v=cD-ETC_v=$$
$$cD-\frac{S}{nQ\left(1-\frac{q}{2}\right)\left(1-\frac{\alpha}{2}\right)}-$$
$$\frac{D\left(c_w\frac{q}{2}+c_r\left(1-\frac{q}{2}\right)\frac{\alpha}{2}+c_{av}\frac{q}{2}\times\frac{\beta}{2}\right)}{\left(1-\frac{q}{2}\right)\left(1-\frac{\alpha}{2}\right)}- \tag{19}$$
$$h_v\left\{\frac{QD}{P\left(1-\frac{q}{2}\right)\left(1-\frac{\alpha}{2}\right)}-\frac{nQD}{2P\left(1-\frac{q}{2}\right)\left(1-\frac{\alpha}{2}\right)}+\frac{(n-1)Q}{2}\right\}$$

Where, *cD* is the vendor income from selling products to the buyer and ETC_v is same as Equation (7).
The buyer profit function can also be obtained as:

$$ETP_b=pD-cD-ETC_b=$$
$$pD-cD-\frac{FD}{Q\left(1-\frac{q}{2}\right)\left(1-\frac{\alpha}{2}\right)}-\frac{D\left(c_i+c_{ab}\frac{q}{2}\times\frac{\beta}{2}\right)}{\left(1-\frac{q}{2}\right)\left(1-\frac{\alpha}{2}\right)}-$$
$$h_bQ\left\{\frac{D\left(\left(1-\frac{q}{2}\right)\frac{\alpha}{2}+\frac{q}{2}\left(1-\frac{\beta}{2}\right)\right)}{x\left(1-\frac{q}{2}\right)\left(1-\frac{\alpha}{2}\right)}+\frac{1-q-\alpha+\alpha q+\frac{\beta}{2}\left(1-\frac{\alpha}{2}\right)q}{2\left(1-\frac{q}{2}\right)\left(1-\frac{\alpha}{2}\right)}+\frac{\left(1-\alpha-\beta+\frac{\alpha\beta}{2}+\frac{\alpha^2}{3}\right)\frac{q^2}{3}+\frac{\alpha^2}{3}(1-q)}{2\left(1-\frac{q}{2}\right)\left(1-\frac{\alpha}{2}\right)}\right\} \tag{20}$$

where *cD* is the cost of buying product from vendor and ETC_b is also the same as Equation (4). Moreover, *pD* is the buyer income of selling products to consumers. As *D* is a function of p, it can be placed by *p=l/m-D/m* in Equation (20):

$$ETP_b=-\frac{D^2}{m}+\frac{l}{m}D-cD-$$
$$\frac{FD}{Q\left(1-\frac{q}{2}\right)\left(1-\frac{\alpha}{2}\right)}-\frac{D\left(c_i+c_{ab}\frac{q}{2}\times\frac{\beta}{2}\right)}{\left(1-\frac{q}{2}\right)\left(1-\frac{\alpha}{2}\right)}-$$
$$h_bQ\left\{\frac{D\left(\left(1-\frac{q}{2}\right)\frac{\alpha}{2}+\frac{q}{2}\left(1-\frac{\beta}{2}\right)\right)}{x\left(1-\frac{q}{2}\right)\left(1-\frac{\alpha}{2}\right)}+\frac{1-q-\alpha+\alpha q+\frac{\beta}{2}\left(1-\frac{\alpha}{2}\right)q}{2\left(1-\frac{q}{2}\right)\left(1-\frac{\alpha}{2}\right)}+\frac{\left(1-\alpha-\beta+\frac{\alpha\beta}{2}+\frac{\alpha^2}{3}\right)\frac{q^2}{3}+\frac{\alpha^2}{3}(1-q)}{2\left(1-\frac{q}{2}\right)\left(1-\frac{\alpha}{2}\right)}\right\} \tag{21}$$

The system benefit function is:

$$\begin{aligned} ETP_j &= -\frac{D^2}{m} + \frac{l}{m}D - ETC_b - \\ & ETC_v - \left(ia - ib \times ln\left(q\right)\right) = \\ & ETP_v + ETP_b - \left(ia - ib \times ln\left(q\right)\right) \end{aligned} \tag{22}$$

Thus, the mathematical model for coordinated case is as follows:

$$Max\ z = ETP_j\left(n, q, Q\right) \tag{23}$$

s.t.

$$0 \le q \le q_0 \tag{24}$$

$$0 \le D \le l \tag{25}$$

$$Q \ge 0 \tag{26}$$

n positive integer

Considering another model for the second problem, the vendor benefit function is:

$$ETP_v = cD - \left\{ \begin{aligned} & \frac{S}{nT} + \frac{c_w Q}{T} - c_w \frac{D}{1-\alpha/2} \\ & + c_r \frac{D}{(1-\alpha/2)} \frac{\alpha}{2} + \frac{c_{av} Q}{T}\frac{\beta}{2} \\ & - c_{av} \frac{D}{1-\alpha/2}\frac{\beta}{2} + \\ & h_v \left\{ \frac{Q^2}{PT}\left(1-\frac{n}{2}\right) + \frac{(n-1)Q}{2} \right\} \end{aligned} \right\} \tag{27}$$

Also, in this model buyer's profit function is:

$$\begin{aligned} ETP_b &= pD - \frac{F + c_i Q + \beta c_{ab} Q}{T} - \frac{c_{ab} D \beta}{\left(1-\frac{\alpha}{2}\right)} - \\ & h_b \left(\begin{aligned} & \frac{QD}{x}\left(\frac{\frac{\alpha}{2}}{1-\frac{\alpha}{2}}\right) + \frac{Q^2}{xT}\left(1-\frac{\beta}{2}\right) - \\ & \frac{DQ}{x}\left(\frac{1-\frac{\beta}{2}}{1-\frac{\alpha}{2}}\right) + \frac{DT}{2} + \frac{Q\beta}{2} - \frac{DT\beta}{2\left(1-\frac{\alpha}{2}\right)} \end{aligned} \right) \end{aligned} \tag{28}$$

By replacement of $p=l/m-D/m$ in Equation (28), the function will be:

$$\begin{aligned} ETP_b &= -\frac{D^2}{m} + \frac{l}{m}D - cD - \\ & \frac{F + \left(c_i + c_{ab}\frac{\beta}{2}\right)Q}{T} + \frac{c_{ab}\beta D}{2\left(1-\frac{\alpha}{2}\right)} - \\ & h_b \left(\begin{aligned} & \frac{QD}{x}\left(\frac{\frac{\alpha}{2}}{1-\frac{\alpha}{2}}\right) + \frac{Q^2}{xT}\left(1-\frac{\beta}{2}\right) - \\ & \frac{DQ}{x}\left(\frac{1-\frac{\beta}{2}}{1-\frac{\alpha}{2}}\right) + \frac{DT}{2} + \frac{Q\beta}{2} - \frac{DT\beta}{2\left(1-\frac{\alpha}{2}\right)} \end{aligned} \right) \end{aligned} \tag{29}$$

For simplicity, instead of obtaining the value of *p*, we specify the value of *D*. The profit function of the whole system is achieved as follows:

$$\begin{aligned} ETP_j &= -\frac{D^2}{m} + \frac{l}{m}D - \frac{S}{nT} - \frac{c_w Q}{T} + c_w \frac{D}{\left(1-\frac{\alpha}{2}\right)} \\ & - c_r \frac{D}{\left(1-\frac{\alpha}{2}\right)}\frac{\alpha}{2} - \frac{c_a Q}{T}\frac{\beta}{2} + c_a \frac{D}{\left(1-\frac{\alpha}{2}\right)}\frac{\beta}{2} - \\ & h_v \left\{ \frac{Q^2}{PT}\left(1-\frac{n}{2}\right) + \frac{(n-1)Q}{2} \right\} - \frac{F + c_i Q}{T} - \\ & h_b \left(Q\left(\frac{D}{x}\left(\frac{\frac{\alpha}{2}+\frac{\beta}{2}-1}{1-\frac{\alpha}{2}}\right) + \frac{Q}{xT}\left(1-\frac{\beta}{2}\right) + \frac{\beta}{2} \right) + \frac{DT\left(1-\frac{\alpha}{2}-\beta\right)}{2\left(1-\frac{\alpha}{2}\right)} \right) \\ & - ia + ib \times ln\left(2 - 2\frac{DT}{Q\left(1-\frac{\alpha}{2}\right)}\right) \end{aligned} \tag{30}$$

Thus, another mathematical form of the second model is:

$$Max\ z = ETP_j\left(n, T, Q, D\right) \tag{31}$$

$s.t.$

$$DT - Q\left(1-\alpha/2\right) \le 0 \tag{32}$$

$$Q\left(1-\alpha/2\right)\left(1-q_0/2\right) - DT \le 0 \tag{33}$$

$$0 \le D \le l \tag{34}$$

$$Q, T \ge 0 \tag{35}$$

n positive integer

4- Solution Method

In this section, we provide solution approaches for coordinated and independent decision-making for both proposed models.

4- 1- The coordinated case of first model

As n is a discrete variable, we first find upper bound and lower bound for its possible values. Then, in each of algorithm iteration for fixed value of n, we optimize the objective function in terms of Q and q. For simplicity, we define the following new functions:

$$\Psi(q)=\frac{D\left(\left(1-\frac{q}{2}\right)\frac{\alpha}{2}+\frac{q}{2}\left(1-\frac{\beta}{2}\right)\right)}{x\left(1-q/2\right)\left(1-\alpha/2\right)}+\frac{1-q-\alpha+\alpha q+\frac{\beta}{2}\left(1-\frac{\alpha}{2}\right)q}{2\left(1-q/2\right)\left(1-\alpha/2\right)}+\frac{\left(1-\alpha-\beta+\frac{\alpha\beta}{2}+\frac{\alpha^2}{3}\right)\frac{q^2}{3}+\frac{\alpha^2}{3}\left(1-q\right)}{2\left(1-q/2\right)\left(1-\alpha/2\right)} \tag{36}$$

$$\Omega(n,q)=\frac{D}{P\left(1-q/2\right)\left(1-\alpha/2\right)}-\frac{nD}{2P\left(1-q/2\right)\left(1-\alpha/2\right)}+\frac{(n-1)}{2} \tag{37}$$

The objective function can then be paraphrased as:
However, considering the specific values for n and q, we have:

$$ETC(n,q,Q)=\frac{S+nFD}{nQ\left(1-\frac{q}{2}\right)\left(1-\frac{\alpha}{2}\right)}+\frac{D\left(c_i+c_a\frac{q}{2}\times\frac{\beta}{2}+c_w\frac{q}{2}+c_r\left(1-\frac{q}{2}\right)\frac{\alpha}{2}\right)}{\left(1-\frac{q}{2}\right)\left(1-\frac{\alpha}{2}\right)}+Q\left(h_b\Psi(q)+h_v\Omega(n,q)\right)+ia-ib\times ln(q) \tag{38}$$

$$\frac{\partial ETC}{\partial Q}=-\frac{S+nFD}{nQ^2\left(1-q/2\right)\left(1-\alpha/2\right)}+h_b\Psi(q)+h_v\Omega(n,q)\rightarrow \frac{\partial^2 ETC}{\partial Q^2}=\frac{2\left(S+nFD\right)}{nQ^3\left(1-q/2\right)\left(1-\alpha/2\right)}>0 \tag{39}$$

For the specified value of n and q, the minimum happens in the root of first derivative of function:

$$\frac{\partial ETC}{\partial Q}=0\rightarrow Q^*=\sqrt{\frac{S+nFD}{n\left(h_b\Psi(q)+h_v\Omega(n,q)\right)\left(1-\frac{q}{2}\right)\left(1-\frac{\alpha}{2}\right)}} \tag{40}$$

By substituting Q^* in the objective function, the it can be stated in terms of n and q:

$$ETC\left(n,q,Q^*\right)=ETC^{'}(n,q)=2\sqrt{\frac{\left(S+nFD\right)\left(h_b\Psi(q)+h_v\Omega(n,q)\right)}{n\left(1-q/2\right)\left(1-\alpha/2\right)}}+\frac{D\left(c_i+c_a\frac{q}{2}\times\frac{\beta}{2}+c_w\frac{q}{2}+c_r\left(1-\frac{q}{2}\right)\frac{\alpha}{2}\right)}{\left(1-q/2\right)\left(1-\alpha/2\right)}+ia-ib\times ln(q) \tag{41}$$

At first, we consider n to be continuous. For any fixed values of q, we find its optimal value. Due to the bounded nature of q, upper bounds and lower bounds can be found for n. Minimization of Equation (41) is equivalent to the minimization of the following equation:

$$\Delta=\frac{\left(S+nFD\right)\left(h_b\Psi(q)+h_v\Omega(n,q)\right)}{n\left(1-q/2\right)\left(1-\alpha/2\right)}=\frac{\left(S+nFD\right)}{n}\left(\Gamma+n\Pi\right) \tag{42}$$

where

$$\Gamma=h_b\Psi(q)/\left(\left(1-q/2\right)\left(1-\alpha/2\right)\right)+\left(h_v/\left(\left(1-q/2\right)\left(1-\alpha/2\right)\right)\right)\left\{D/P\left(1-q/2\right)\left(1-\alpha/2\right)-0.5\right\}$$

and

$$\Pi=h_v/\left(\left(1-\frac{q}{2}\right)\left(1-\frac{\alpha}{2}\right)\right)\left\{-D/2P\left(1-\frac{q}{2}\right)\left(1-\frac{\alpha}{2}\right)+0.5\right\}.$$

It should be mentioned that, as $D\le P\left(1-q/2\right)\left(1-\alpha/2\right)$, Π is always greater than or equal to zero, but Γ`s sign cannot be determined. First and second derivatives of Δ with respect to n are:

$$\frac{\partial\Delta}{\partial n}=-\frac{S\Gamma}{n^2}+FD\Pi \tag{43}$$

$$\frac{\partial^2\Delta}{\partial n^2}=\frac{2S\Gamma}{n^3} \tag{44}$$

Now with respect to the sign of Γ, we have:
1- Γ<0: In this case, Δ respect to n is ascending and the minimum is placed at the least possible value for *n*; i.e. 1.
2- Γ>0: In this case, the minimum happens in the root of the first derivative of the Δ with respect to n; i.e. $\sqrt{S\Gamma / FD\Pi}$
However, to find the upper bound and lower bound of n, we use the following equation as an approximation:

$$\frac{S}{FD}\times\frac{min\left(h_b\Psi(q)+h_v\left(\frac{D}{P(1-q/2)(1-\alpha/2)}-\frac{1}{2}\right)\right)}{max\left(h_v\left\{-\frac{D}{2P(1-q/2)(1-\alpha/2)}+\frac{1}{2}\right\}\right)}\sin^{-1}\theta \le \frac{S\Gamma}{FD\Pi} \le \frac{S}{FD}\times\frac{max\left(h_b\Psi(q)+h_v\left(\frac{D}{P(1-q/2)(1-\alpha/2)}-\frac{1}{2}\right)\right)}{min\left(h_v\left\{-\frac{D}{2P(1-q/2)(1-\alpha/2)}+\frac{1}{2}\right\}\right)} \quad (45)$$

To calculate the minimum and maximum of Ψ(q), we have:

$$\frac{\partial\left(\left(1-\alpha+\frac{\alpha^2}{3}\right)\left(1-q+\frac{q^2}{3}\right)+\left(\beta-\frac{\alpha\beta}{2}\right)\left(q-\frac{q^2}{3}\right)\right)}{\partial q} = \left(1-\alpha+\frac{\alpha^2}{3}-\beta+\frac{\alpha\beta}{2}\right)\times\left(-1+\frac{2}{3}q\right)<0 \quad (46)$$

We conclude that the mentioned function is descending in the range of possible values for *q*. According to the specified minimum and maximum of function in the numerator of the fractions and the denominator, we have:

$$upperbound\left(\frac{S\Gamma}{FD\Pi}\right)=\frac{S}{FD}\times\frac{h_b\left\{\frac{D\frac{\alpha}{2}}{x\left(1-\frac{\alpha}{2}\right)}+\frac{D\left(1-\frac{\beta}{2}\right)q_0}{2x\left(1-\frac{\alpha}{2}\right)\left(1-\frac{q_0}{2}\right)}+\frac{1-\alpha+\frac{\alpha^2}{3}}{2\left(1-\frac{\alpha}{2}\right)\left(1-\frac{q_0}{2}\right)}\right\}}{h_v\left\{-\frac{D}{2P\left(1-\frac{q_0}{2}\right)\left(1-\frac{\alpha}{2}\right)}+\frac{1}{2}\right\}} + \frac{S}{FD}\times\frac{\left(\frac{D}{P\left(1-\frac{\alpha}{2}\right)\left(1-\frac{q_0}{2}\right)}-\frac{1}{2}\right)}{\left(\frac{1}{2}-\frac{D}{2P\left(1-\frac{q_0}{2}\right)\left(1-\frac{\alpha}{2}\right)}\right)} \quad (47)$$

$$lowerbound\left(\frac{S\Gamma}{FD\Pi}\right)=\frac{S}{FD}\times\frac{h_b\left\{\frac{D\frac{\alpha}{x}+\left(1-\alpha+\frac{\alpha^2}{3}\right)\left(1-q_0+\frac{q_0^2}{3}\right)+\left(\beta-\frac{\alpha\beta}{2}\right)\left(q_0-\frac{q_0^2}{3}\right)}{2\left(1-\frac{\alpha}{2}\right)}\right\}}{h_v\left\{-\frac{D}{2P\left(1-\frac{\alpha}{2}\right)}+\frac{1}{2}\right\}} + \frac{S}{FD}\times\frac{\left\{\frac{D}{P\left(1-\frac{\alpha}{2}\right)}-\frac{1}{2}\right\}}{\left\{-\frac{D}{2P\left(1-\frac{\alpha}{2}\right)}+\frac{1}{2}\right\}} \quad (48)$$

So we have:

$$n_{min}=\begin{cases}\sqrt{lowerbound\left(\frac{S\Gamma}{FD\Pi}\right)}, & lowerbound\left(\frac{S\Gamma}{FD\Pi}\right)>1\\ 1, & otherwise\end{cases} \quad (49)$$

$$n_{max}=\begin{cases}\sqrt{upperbound\left(\frac{S\Gamma}{FD\Pi}\right)}, & upperbound\left(\frac{S\Gamma}{FD\Pi}\right)>0\\ 1, & otherwise\end{cases} \quad (50)$$

To solve the problem for each specified *n*, the first and second derivatives of the function compared to *T* and *Q* are calculated. Therefore, we have:

$$\begin{bmatrix}Q & T\end{bmatrix}\times\begin{bmatrix}\frac{\partial^2 ETC_j}{\partial Q^2} & \frac{\partial^2 ETC_j}{\partial Q\partial T}\\ \frac{\partial^2 ETC_j}{\partial Q\partial T} & \frac{\partial^2 ETC_j}{\partial T^2}\end{bmatrix}\times\begin{bmatrix}Q\\ T\end{bmatrix} = \frac{2\left(F+\frac{S}{n}\right)}{T}>0 \quad (51)$$

So the total cost is concave respect to *T* and *Q*. Moreover, as the constraints are linear, the problem is a convex programming. To solve the problem, primal-dual interior point method was used. We introduce the algorithm as follows:

Solution algorithm
Step 0: Calculate n_{min}, n_{max} from Equations (49) and (50), respectively. Set k to 1 and n_k to n_{min}.
Step 1: For the specified n_k, find the optimal values for T_k

and Q_k, using Equation (40) and employing primal-dual interior point method, considering $q=q_0$, and $T_0=Q_0(1-q_0/2)(1-\alpha/2)/D$ as starting point. Save the optimum values obtained (n_k, Q^*_k, T^*_k, ETC_j).
Step 2: Set k=k+1.
Step 3: If $n_k \le n_{max}$ go to step 1, otherwise go to Step 4.
Step 4: Find the minimum of $ETC_j(n_k, Q^*_k, T^*_k)$ for all values of k, and report (n_k, Q^*_k, T^*_k) as the optimal values of decision variables.

4- 2- The independent decision-making case of first model

In this case, first buyer determines the optimal quantity of order, and then the vendor obtains the optimal number of shipments. Therefore, due to the lack of coordination between vendor and buyer, vendor does not investment on the quality.

Optimizing the buyer's cost

First, considering fixed q, we must optimize the cost of the buyer with respect to Q. According to the definition of Ψ(q) in Equation (36) and cost function, we have:

$$ETC_b\left(n,q,Q\right) = \frac{FD}{Q\left(1-\frac{q_0}{2}\right)\left(1-\frac{\alpha}{2}\right)} + \frac{D\left(c_i + c_{ab}\frac{q_0}{2}\times\frac{\beta}{2}\right)}{\left(1-\frac{q_0}{2}\right)\left(1-\frac{\alpha}{2}\right)} + h_b Q \Psi\left(q_0\right) \tag{52}$$

So:

$$\frac{\partial ETC_b}{\partial Q} = -\frac{FD}{Q^2\left(1-\frac{q_0}{2}\right)\left(1-\frac{\alpha}{2}\right)} + h_b \Psi\left(q_0\right)$$
$$\rightarrow \frac{\partial^2 ETC_b}{\partial Q^2} = \frac{2\left(FD\right)}{Q^3\left(1-\frac{q_0}{2}\right)\left(1-\frac{\alpha}{2}\right)} > 0 \tag{53}$$

The minimum of cost function occurs in the root of the first derivative;

Optimizing the vendor's cost function

$$Q_b^* = \sqrt{FD \Big/ \left(h_b \Psi\left(q_0\right)\left(1-\frac{q_0}{2}\right)\left(1-\frac{\alpha}{2}\right)\right)}$$

The vendor cost function is:

$$ETC_v\left(n\right) = \frac{S}{nQ_b^*\left(1-\frac{q_0}{2}\right)\left(1-\frac{\alpha}{2}\right)} + \frac{D\left(c_w\frac{q_0}{2} + c_r\left(1-\frac{q_0}{2}\right)\frac{\alpha}{2} + c_{av}\frac{q_0}{2}\times\frac{\beta}{2}\right)}{\left(1-\frac{q_0}{2}\right)\left(1-\frac{\alpha}{2}\right)} + h_v\left\{\frac{Q_b^* D\left(1-n/2\right)}{P\left(1-\frac{q_0}{2}\right)\left(1-\frac{\alpha}{2}\right)} + \frac{\left(n-1\right)Q_b^*}{2}\right\} \tag{54}$$

First and second derivatives are as follows:

$$\frac{\partial ETC_v}{\partial n} = -\frac{S}{n^2 Q_b^*\left(1-\frac{q_0}{2}\right)\left(1-\frac{\alpha}{2}\right)} + h_v\left(\frac{1}{2} - \frac{Q_b^* D}{2P\left(1-\frac{q_0}{2}\right)\left(1-\frac{\alpha}{2}\right)}\right)$$
$$\rightarrow \frac{\partial^2 ETC_v}{\partial n^2} = \frac{2\left(S\right)}{n^3 Q_b^*\left(1-\frac{q_0}{2}\right)\left(1-\frac{\alpha}{2}\right)} > 0 \tag{55}$$

The minimum of function occurs in the root of the first derivative:

$$n^* = \sqrt{S \Big/ \left(h_v Q_b^*\left(0.5 - \frac{D}{2P\left(1-\frac{q_0}{2}\right)\left(1-\frac{\alpha}{2}\right)}\right) Q_b^*\left(1-\frac{q_0}{2}\right)\left(1-\frac{\alpha}{2}\right)\right)}$$

As the root of the first derivative may not be integer; we calculate $ETC_v(n)$ function for the nearest integer values.

4- 3- The price-dependent model in the coordinated case

First, for the specified values of n we evaluate the behavior of total profit function. Thus, we calculate the first and second derivatives of cost function with respect to T, Q and D. Then, to examine the function behavior, we calculate $\partial^2 ETP_j / \partial Q \partial T$, $\partial^2 ETP_j / \partial D \partial Q$ and $\partial^2 ETP_j / \partial D \partial T$ So, we have:

$$\begin{bmatrix} Q & T & D \end{bmatrix} \times \begin{bmatrix} \frac{\partial^2 ETP_j}{\partial Q^2} & \frac{\partial^2 ETP_j}{\partial Q \partial T} & \frac{\partial^2 ETP_j}{\partial D \partial Q} \\ \frac{\partial^2 ETP_j}{\partial Q \partial T} & \frac{\partial^2 ETP_j}{\partial T^2} & \frac{\partial^2 ETP_j}{\partial D \partial T} \\ \frac{\partial^2 ETP_j}{\partial D \partial Q} & \frac{\partial^2 ETP_j}{\partial D \partial T} & \frac{\partial^2 ETP_j}{\partial D^2} \end{bmatrix} \times \begin{bmatrix} Q \\ T \\ D \end{bmatrix}$$
$$= -\frac{2\left(F + \frac{S}{n}\right)}{T} - \frac{2}{m} D^2\left(1 - \frac{\alpha}{2-\alpha}\right) - ib \times \frac{D^2 T^2}{\left(DT - \left(1-\alpha/2\right)Q\right)^2} - 2h_b D\left(\frac{Q}{x}\left(-1 + \frac{\frac{\beta}{2}}{\left(1-\alpha/2\right)}\right) + T\left(\frac{1}{2} - \frac{\beta/2}{\left(1-\alpha/2\right)}\right)\right) \tag{56}$$

As the above function is negative, the benefit function of system is concave with respect to T, Q and D. As in most papers and empirical studies, inspected type I and type II errors are usually less than 5 and 20 percent, respectively (see for instance: [19]); we have $1-\alpha/2-4\beta \ge 0$, and so

$T\left(1/8-\left(\beta/2\right)/\left(1-\alpha/2\right)\right)\geq 0$ For the possibility of the problem we need to assume that the inspection rate is greater than the effective rate of demand. With regard to the establishment of the conditions $x\geq 3.5D$ and $q\leq q_0\leq 0.3$, the function is concave, because:

$$\frac{8}{3}\times\frac{D}{\left(1-\frac{q}{2}\right)\left(1-\frac{\alpha}{2}\right)}\leq\frac{8}{3}\times\frac{D}{\left(0.85\right)\left(0.90\right)}\leq 3.5D \tag{57}$$

$$x\geq 3.5D\geq\frac{8}{3}\times\frac{D}{\left(1-\frac{q}{2}\right)\left(1-\frac{\alpha}{2}\right)}\rightarrow$$
$$\frac{3}{8}\times\frac{Q\left(1-\frac{q}{2}\right)\left(1-\frac{\alpha}{2}\right)}{D}-\frac{Q}{x}\geq 0\rightarrow\frac{3}{8}T-\frac{Q}{x}\geq 0 \tag{58}$$

The assumption of $x\geq 3.5D$ is reasonable, as in the literature and in practice, in numerical examples the ratio x/D is far greater than of 3.5. Also in some papers (such as [5]), inspection rate is assumed to be infinite. The assumption of $q\leq q_0\leq 0.3$ is also reasonable as the maximum defective rate accepted in standard tables ANSI / ASQ Z1.4-2008 is 15 percent. Also according to [20], the most defective rate accepted for retailers and buyers is 6.5 percent. Therefore, the assumption of the defective rate in the range of 0 to 30%, is reasonable. So, by considering the above assumptions, the objective function is concave with respect to T, Q and D. Considering these three variables as constant, we examine the behavior of the function with respect to n.

$$\frac{\partial ETP_j}{\partial n}=\frac{S}{n^2T}+h_v\frac{Q^2}{2PT}-h_v\frac{Q}{2}$$
$$\rightarrow\frac{\partial^2 ETP_j}{\partial n^2}=-\frac{2S}{n^3T} \tag{59}$$

According to the second derivative's sign, the objective function is concave with respect to n. The objective function of problem is consequently concave with respect to T, Q and D. Also, we show that the objective function is concave with respect to n. However, the constraints are not convex.

If the latter constraint was also convex, we could have a convex programming problem for any fixed value of n and resolve it using algorithms such as the primary-dual interior point. According to concave objective function with respect to n, its optimal value can be found searching from the lowest possible value, i.e. 1 until the optimal value of the objective function in term of T, Q and D faced with decline. Due to the fact that the second constraint is not convex, the following rule helps us:

To maximize a concave function with several constraints, the local optimum is the global optimum (if this point satisfies all the constraints) or the global optimum is located exactly on the constraint border.

Thus, for any value of n, once the maximum of objective function is found with exact algorithms; regardless of constraints, if the answer satisfies the second constraint, the solution is the optimal for the specified value of n. Otherwise; to find the optimal solution we should search the answer on the border of second constraint. The answer cannot be on limit values of $0\leq D\leq l$, because products that are sold at a price of zero and also products that there is no demand for them, would not be cost-effective. To find the answer, we insert the second constraint to the objective function, i.e. $D=Q\left(1-\alpha/2\right)\left(1-q_0/2\right)/T$. So we have:

$$\begin{aligned}ETP'_j=&-\frac{Q^2\left(1-\frac{\alpha}{2}\right)^2\left(1-\frac{q_0}{2}\right)^2}{mT^2}+\frac{lQ\left(1-\frac{\alpha}{2}\right)\left(1-\frac{q_0}{2}\right)}{mT}\\&-\frac{S}{nT}-\frac{c_wQ}{T}\left(\frac{q_0}{2}\right)-\frac{c_rQ}{T}\left(1-\frac{q_0}{2}\right)\frac{\alpha}{2}-\frac{c_aQ}{T}\left(\frac{q_0}{2}\right)\frac{\beta}{2}\\&-h_vQ\left\{\frac{Q}{PT}\left(1-\frac{n}{2}\right)+\frac{\left(n-1\right)}{2}\right\}-\frac{F+c_iQ}{T}\\&-h_bQ\left(\begin{array}{l}\frac{Q}{xT}\left\{\left(1-\frac{q_0}{2}\right)\left(\frac{\alpha}{2}\right)+\left(1-\frac{\beta}{2}\right)\left(\frac{q_0}{2}\right)\right\}+\\\frac{\left(1-\frac{\alpha}{2}\right)\left(1-\frac{q_0}{2}\right)}{2}+\frac{\beta q_0}{4}\end{array}\right)\end{aligned} \tag{60}$$

According to the following equation, the function is concave, and by using primary-dual interior point the global maximum could be found.

$$\begin{aligned}&\left[T\quad Q\right]\times\begin{bmatrix}\frac{\partial^2 ETP'_j}{\partial T^2} & \frac{\partial^2 ETP'_j}{\partial T\,\partial Q}\\ \frac{\partial^2 ETP'_j}{\partial T\,\partial Q} & \frac{\partial^2 ETP'_j}{\partial Q^2}\end{bmatrix}\times\begin{bmatrix}T\\Q\end{bmatrix}\\&=-\frac{2S}{nT}-\frac{2F}{T}\leq 0\end{aligned} \tag{61}$$

Thus, the solution algorithm can be summarized as follows:

Solution algorithm

Step zero: Set k to 1 and n_k to 1.

Step 1: Calculate the initial point $D_0=l/2$ using Equation (22), Q_0 by replacement of q= q_0 from Equation (40), and $T_0=Q_0\left(1-q_0/2\right)\left(1-\alpha/2\right)/D_0$ .Using primal-dual interior point method, find the optimal values for T_k and Q_k for function ETP_j according to Equation (30) with regard to sign constraints and beginning point of T_0, Q_0 and D_0. If obtained answer satisfies the constraints (33) and (34), save four variable values as well as the objective, $\left(n_k,Q_k^*,T_k^*,D_k^*,ETP_j\right)$ and go to Step 3. Otherwise, go to Step 2.

Step 2: Using primal-dual interior point method, find the optimal values of T_k and Q_k using Equation (60). Then replace the answers obtained for T^*_k and Q^*_k in relation $D=Q(1-q_0/2)(1-\alpha/2)/T$. Save four variable values and the objective ($n_k,Q^*_k,T^*_k,D^*_k,ETP_j$).

Step 3: If $k\geq 2$ and $ETP_j\left(n_k,Q_k^*,T_k^*,D_k^*\right)\leq ETP_j\left(n_{k-1},Q_{k-1}^*,T_{k-1}^*,D_{k-1}^*\right)$, the final answer will be $ETP_j(n_{k-1},Q^*_{k-1},T^*_{k-1},D^*_{k-1})$ and go to Step 4. Otherwise, set k=k+1 and $n_k=n_{k-1}+1$ and go to Step 1.

Step 4: If the final answer ETP_j is negative, it would not be cost-effective. Otherwise, report the answer as the output and the algorithm stops.

4- 4- The independent decision-making case of the price-dependent demand model

In this case because of the lack of coordination between vendor and buyer, the vendor does not invest on quality. First, we obtain the optimal values of D, T and Q.

The Buyer's profit optimization

In the absence of investment on quality, the following equation is obtained by replacement of $D = Q\left(1-\alpha/2\right)\left(1-q_0/2\right)/T$ in Equation (29):

$$ETP_b^{'} = -\frac{Q^2\left(1-\frac{\alpha}{2}\right)^2\left(1-\frac{q_0}{2}\right)^2}{mT^2} + \frac{lQ\left(1-\frac{\alpha}{2}\right)\left(1-\frac{q_0}{2}\right)}{mT} - \frac{cQ\left(1-\frac{\alpha}{2}\right)\left(1-\frac{q_0}{2}\right)}{T} - \frac{c_{ab}Q}{T}\frac{\beta}{2}\left(\frac{q_0}{2}\right) + \frac{F+c_iQ}{T} - h_bQ\left(\frac{Q}{xT}\left\{\left(1-\frac{q_0}{2}\right)\left(\frac{\alpha}{2}\right)+\left(1-\frac{\beta}{2}\right)\left(\frac{q_0}{2}\right)\right\} + \frac{\left(1-\frac{\alpha}{2}\right)\left(1-\frac{q_0}{2}\right)}{2} + \frac{\beta q_0}{4}\right) \tag{62}$$

Based on the following equation, the function is concave and its global maximum can be found with primary-dual interior point method.

$$\begin{bmatrix} T & Q \end{bmatrix} \times \begin{bmatrix} \frac{\partial^2 ETP_b^{'}}{\partial T^2} & \frac{\partial^2 ETP_b^{'}}{\partial T\,\partial Q} \\ \frac{\partial^2 ETP_b^{'}}{\partial T\,\partial Q} & \frac{\partial^2 ETP_b^{'}}{\partial Q^2} \end{bmatrix} \times \begin{bmatrix} T \\ Q \end{bmatrix} = -\frac{2F}{T} \le 0 \tag{63}$$

To find the starting point of the interior point algorithm, we use DT/(1-α/2)(1-q_0/2) instead of Q. By substituting the above expression in Equation (29), we have:

$$ETP_b^{''} = -\frac{D^2}{m} + \frac{l}{m}D - cD - \frac{F}{T} - \frac{c_iD}{\left(1-\frac{\alpha}{2}\right)\left(1-\frac{q_0}{2}\right)} - \frac{c_{ab}D}{\left(1-\frac{\alpha}{2}\right)\left(1-\frac{q_0}{2}\right)}\frac{\beta}{2} + c_{ab}\frac{D}{\left(1-\frac{\alpha}{2}\right)}\frac{\beta}{2} - h_bDT\left(\frac{D}{x\left(1-\frac{\alpha}{2}\right)\left(1-\frac{q_0}{2}\right)}\left(\frac{1}{1-\frac{\alpha}{2}}-\frac{\beta}{2}\right)+\frac{1}{2} - \frac{D\left(1-\frac{\beta}{2}\right)}{x\left(1-\frac{\alpha}{2}\right)^2\left(1-\frac{q_0}{2}\right)} + \frac{\beta}{2\left(1-\frac{\alpha}{2}\right)}\left(\frac{\frac{q_0}{2}}{\left(1-\frac{q_0}{2}\right)}\right)\right) \tag{64}$$

By calculating the first and second partial derivatives, we have:

$$\frac{\partial ETP_b^{'}}{\partial T} = +\frac{F}{T^2} - h_bD\left(\frac{D}{x\left(1-\frac{\alpha}{2}\right)\left(1-\frac{q_0}{2}\right)}\left(\frac{1}{1-\frac{\alpha}{2}}-\frac{\beta}{2}\right)+\frac{1}{2} - \frac{D\left(1-\frac{\beta}{2}\right)}{x\left(1-\frac{\alpha}{2}\right)^2\left(1-\frac{q_0}{2}\right)} + \frac{\beta}{2\left(1-\frac{\alpha}{2}\right)}\left(\frac{\frac{q_0}{2}}{\left(1-\frac{q_0}{2}\right)}\right)\right) \tag{65}$$

$$\frac{\partial^2 ETP_b^{'}}{\partial T^2} = -\frac{2F}{T^3} \tag{66}$$

As Equation (66) is negative, we obtain the root of (65) by considering $D=D_0=l/2$ as the optimal value of T for the specified demand rate. So we use it as T_0.

$$\frac{\partial ETP_b^{'}}{\partial T} = 0 \rightarrow T_0 = \sqrt{\frac{F\left(1-\frac{\alpha}{2}\right)}{h_bD_0\left(\frac{D_0}{x\left(1-\frac{q_0}{2}\right)}\left(\frac{1}{1-\frac{\alpha}{2}}-\frac{\beta}{2}\right)+\frac{1}{2} - \frac{D_0\left(1-\frac{\beta}{2}\right)}{x\left(1-\frac{\alpha}{2}\right)\left(1-\frac{q_0}{2}\right)} + \frac{q_0\beta}{4\left(1-\frac{q_0}{2}\right)}\right)}} \tag{67}$$

To calculate Q_0, we substitute the values obtained for D_0 and T_0 in DT/(1-α/2)(1-q_0/2).

Vendor's profit optimization

The only variable that the vendor should optimize is n. The following equation represents vendor cost function:

$$ETP_v\left(n,Q^*,D^*,T^*\right) = cD^* - \frac{S}{nT^*} - \frac{c_wQ^*}{T^*} + c_w\frac{D^*}{\left(1-\frac{\alpha}{2}\right)} - c_r\frac{D^*}{\left(1-\frac{\alpha}{2}\right)}\frac{\alpha}{2} - \frac{c_{av}Q^*}{T^*}\frac{\beta}{2} + c_{av}\frac{D^*}{\left(1-\frac{\alpha}{2}\right)}\frac{\beta}{2} - h_v\left\{\frac{Q^{*2}}{PT^*}\left(1-\frac{n}{2}\right)+\frac{(n-1)Q^*}{2}\right\} \tag{68}$$

We first assume n is continuous and calculate the first and second derivatives of the function with respect to n.

$$\frac{\partial ETP_v}{\partial n} = \frac{S}{n^2T^*} - h_v\left\{\frac{-Q^{*2}}{2PT^*} + \frac{Q^*}{2}\right\}$$
$$\rightarrow \frac{\partial^2 ETP_v}{\partial n^2} = -\frac{2S}{n^3T^*} < 0 \tag{69}$$

According to Equation (69), first derivative's root is the global maximum. If the root is not integer, we calculate its nearest integer values.

$$n^* = \sqrt{\frac{S}{T^*h_v\left\{\frac{-Q^{*2}}{2PT^*} + \frac{Q^*}{2}\right\}}} \tag{70}$$

5- Numerical Study And Sensitivity Analysis

Using the standard data in the literature, some numerical studies are made. In the table below, the input parameters are extracted from the literature.

Table 2. Parameters of the numerical example.

Parameter	value
D	5000
P	6000
x	175200
S	50000
F	50
q_0	0.1
a	0.05
b	0.05
c_i	0.5
c_w	10
c_{ab}	50
c_{av}	20
c_r	100
h_v	2
h_b	20
d	5×10^{-4}

5- 1- First model numerical example

The results of solving the example are presented in Table 3 for coordinated and independent decision making cases, as well as independent decision-making with investment on quality. The value obtained for the investment on quality and the defect rate in coordinated case are considered as the amount of investment and the defect rate in the case of independent decision-making.

Table 3. Results of the first model numerical example.

	Coordinated case	Independent decisions case	Independent decisions with investment case
Q^*	2151.02	167.86	160.41
n^*	5	5	4
q^*	0.00551	q_0=0.1	0.00551
Total cost	64114.38	343671.65	419646.29

The results show the importance of coordination, as even with investment on quality in the third case, costs not only have not decreased, but also have been increased. Thus, investing in quality does not necessarily lead to reduced costs, while the coordination of decision making leads to effectiveness of such policies.

5- 2- First model sensitivity analysis

In this section, the sensitivity analysis of proposed model for average defect rate before investment is studied, keeping the rest of parameters fixed. For this purpose, we need to define $PI_1 = TC_I - TC_J / TC_I$,which represents the ratio of cost reduction in coordinated case compared to independent decision-making case. In the above equation, TC_I represents the total cost of system in the independent decision-making case, with investment in quality. Also, TC_J represents the total cost of the system in the coordinated case. It should be noted that to compare two cases, we use the optimal value of investment in quality and the defect rate of the coordinated case in the independent decision-making case. Table 4 represents the results of different values for two levels of q_0.

Table 4. some results of first model sensitivity analysis.

	q_0=0.05		q_0=0.25	
	Independent	Cooperated	Independent	Cooperated
Q^*	160.41	2151.02	160.45	1402.19
n^*	4	5	4	11
q^*	0.00552		0.00609	
Total cost	419646.72	64114.38	49779.10	419710.94
PI_1	0.881		0.847	

We can conclude that:

- As shown in Figure 4, by increasing q_0, the importance of coordination is increased.
- Increasing q_0 has a little incremental effect on Q^* and q^* in independent case, while it has a decremental effect on the values of Q^* in the coordinated state.
- Increasing q_0 has an incremental effect on n^* in the coordinated state, but has no effect on the independent case.

5- 3- Second model numerical example

Using the price dependent demand function introduced by [21] and the parameters in Table 5, we optimize all decision variables in the second model.
The result of solving this example is shown in Table 6.

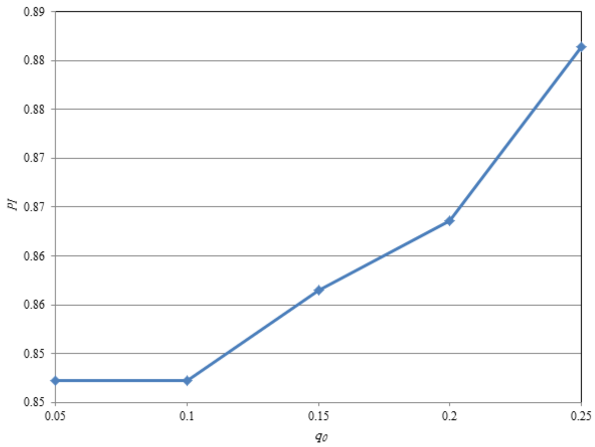

Fig. 4. First model sensitivity analysis with respect to q_0.

Table 5. Parameters of the second model numerical example.

Parameter	value
l	5000
m	25
c	120

Table 6. Parameters of the second model numerical example.

	Coordinated case	Independent decisions case	Independent decisions with investment case
Q^*	111.73	80.13	76.74
n^*	41	34	35
q^*	0.00652	q_0=0.1	0.00652
D^*	2327.79	1105.77	1112.94
Total profit	210824.57	150955.46	152467.98

Unlike the previous example, the objective function has been improved in the third case, i.e. it confirms the fact that even without centralized coordination we can take advantage of improvement in quality. Another finding is that we can achieve more profit with even less selling price under coordination in SC. In fact, with coordination, we gain more profits, more satisfied customer and improved quality. Moreover, the defect rate will decrease, and we can keep our brand reputation safe.

5- 4- Second model sensitivity analysis

For the second model we define $PI_2 = \left(TP_J - TP_I\right)/TP_I$,where TP_I and TP_J represent total system cost in the third case and in the coordinated case, respectively. Table 7 shows the results for the different values of parameter m in the second model.

According to the values of Table 6, PI_2 is equal to 0.38 for m=25. So, we can conclude that:

- By increasing m, the importance of coordination increases as the profit of the system increases with high speed. Figure 5 shows the effect of changing the value of parameter m on PI_2.
- Increasing in m has little increasing effect on q^*, but it has a decremental effect on the values of Q^* and D^*.

Table 7. Parameters of the second model numerical example.

	m=5		m=40	
	Independent	Cooperated	Independent	Cooperated
Q^*	108.40	114.52	37.57	110.08
n^*	40	42	32	40
q^*	0.00681		0.00618	
D^*	2223.08	2465.82	266.69	2222.73
Total cost	1198120.08	1209882.88	24666.69	117790.80
PI_2	3.78		0.01	

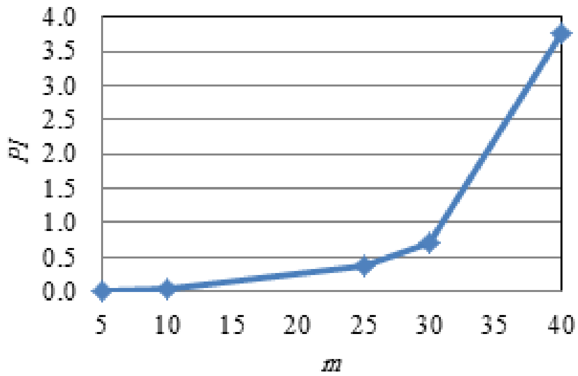

Fig. 5 Second model sensitivity analysis with respect to *m*.

6- Conclusion And Directions For Future Research

This paper investigates the importance of coordination in supply chains through developing two models in centralized decision making for a manufacturing system with imperfect quality. As one of the most important goals of coordination is improving quality, we also considered a cost function for investment on products quality. Moreover, it is assumed that the selling price to the final consumer affects the demand rate. We find the optimal solution for both independent and coordinated cases.

Numerical results show that coordination is more beneficial if the defective rate is high. Moreover, it can be concluded that even by less consumer price, we can achieve more profit through coordination in supply chains. In fact, we gain more profits, more satisfied customer, and also improved quality. The defect rate will also decrease, and we can keep our brand reputation safe.

To study more realistic and practical cases, following subjects can be suggested for future researches:

- Incorporating the investment on the production set up cost reduction.
- Considering budget constraints or limited warehouse space.
- Developing the model to extended SCs with more stages and members.

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Please cite this article using:

M. Barzegar Astanjin and M. S. Sajadieh,"Integrated production-inventory model with price-dependent demand, imperfect quality, and investment in quality and inspection", *AUT J. Model. Simul.*, 49(1) (2017)43-56.

DOI: 10.22060/miscj.2016.835