Amirkabir University of Technology
(Tehran Polytechnic)

Amirkabir International Jounrnal of Science & Research
Modeling, Identification, Simulation and Control
(AIJ-MISC)
Vol. 48, No. 2, Fall 2016, pp. 75-91

Potentials of Evolving Linear Models in Tracking Control Design for Nonlinear Variable Structure Systems

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(Received 15 May 2016, Accepted 3 September 2016)

ABSTRACT

Evolving models have found applications in many real world systems. In this paper, potentials of the Evolving Linear Models (ELMs) in tracking control design for nonlinear variable structure systems are introduced. At first, an ELM is introduced as a dynamic single input, single output (SISO) linear model whose parameters as well as dynamic orders of input and output signals can change through the time. Then, the potential of ELMs in modeling nonlinear time-varying SISO systems is explained. Next, the potential of the ELMs in tracking control of a minimum phase nonlinear time-varying SISO system is introduced. To this end, two tracking control strategies are proposed, respectively for (a) when the ELM is known perfectly and (b) when the ELM model has uncertainties but dynamic orders of the input and output signals are fixed. The methodology and superiority of the proposed tracking control systems are shown via some illustrative examples: speed control in a DC motor and link position control in a flexible joint robot.

KEYWORDS:

Evolving Linear Model, Nonlinear Time-Varying Systems, Tracking Control System

Please cite this article using:
Kalhor, A., Hojjatzadeh, N., and Golgouneh, A., 2016. “Potentials of Evolving Linear Models in Tracking Control Design for Nonlinear Variable Structure Systems”. *Amirkabir International Journal of Modeling, Identification, Simulation and Control*, 48(2), pp. 75–91.
DOI: 10.22060/miscj.2016.829
URL: http://miscj.aut.ac.ir/article_829.html
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1- Introduction

A. Motivation and Related Works

Nonlinearity and structural variability cause significant challenges in control design, particularly when the system is not affine or there are uncertainties. Robust control and adaptive control methods have been introduced and developed for such systems. Whereas robust control guarantees the stability and performance of the system under bounded uncertainties, adaptive control satisfies the required performance and stability by posing adaptation rules for the controller with regard to the new condition of the system.

Among known robust control strategies, H-infinity, μ-synthesis and sliding mode are more popular and applicable. Considering unstructured uncertainty in a control system, H-infinity based methods are used to minimize the closed loop impact of perturbation [1,2] and in recent years, new aspects of the H-infinity are studied [3,4]. μ-synthesis based methods unlike H-infinity are used to design robust control against certain structured uncertainties [5,6]. Sliding mode control is a variable structure control method providing a systematic approach to the problem of maintaining stability in the face of modeling uncertainty [7,8]. This approach has been also used and developed in many nonlinear control systems [9-11].

A wide range of methods have been introduced in the literature of adaptive control systems. Adaptive pole placement, different self-tuning regulators and iterative learning control are straightforward techniques using direct or indirect approaches to provide stability and performance in the system [12-14]. However, they are suggested principally for linear systems. Model Reference Adaptive Controllers often suggest direct solutions for adaptive control of continuous time systems based on a reference model. Regarding the structure of the system, the reference model can be chosen linear or nonlinear as well [15-17]. Gain scheduling based methods offer a set of control strategies corresponding to different operating regimes of the system prepared as a table [18,19]. Such methods are also developed for nonlinear and also time-varying systems. Also, for nonlinear systems, adaptive control systems such as self-oscillating adaptive systems [20,21], variable structure system [22,23] and duality controllers [24,25], have been proposed.

On the other hand, during last decade, evolving fuzzy and neuro-fuzzy models have been developed in modeling nonlinearity and time-varying structures. In an evolving model, the structure can evolve through the time based on observing samples of input and output signals. The structure of a nonlinear evolving model often can be described as an interpolated of locally linear models constructed from simple if-then fuzzy rules. There are two particularly influential works in this area of research [26,27]. Kasabov proposes an adaptive online learning algorithm as a dynamic evolving neural–fuzzy inference system (DENFIS). In this algorithm, fuzzy inference rules are created by using maximum distance clustering, which is utilized in partitioning of input space. Angelov and Filev introduce an online identification approach for the Takagi Sugeno (TS) model, where evolving clustering method along with a concept of potential is used to define the antecedent parts of the rules. This approach has been modified in [28] and [29]. In [30], to evolve a specific form of TS Fuzzy Model, Lughofer suggests to use a modified version of vector quantization for new rule generation. Although evolving nonlinear systems have been developed successfully in simulation, approximation, classification and prediction, they are not straightforward basis for analyzing and designing control systems, In recent years, by the author and its cooperators, there is a tendency toward using and developing Evolving Linear Models (ELMs) [31,32]. An ELM can adapt and follow the variations of the nonlinear and time-varying system with agility [33]. Moreover, it seems ELMs can fulfill facilities in control, analyze and design due to their linear forms.

B. Our Contributions

In this paper, some potentials of ELMs in tracking control of nonlinear variable structure systems are introduced. It is shown that if nonlinear models could be represented as ELMs, two significant challenges in tracking control systems will be solved. At first, the nonlinear systems do not have essentially an affine structure with regard to the input signal. This avoids using some given classic solutions like feedback linearization or sliding mode control [8] by which a stable differential equation of error is satisfied. ELM represents the system in an affine form which allows one to design a tracking control system easily for nonlinear non-affine systems. Also, when the dynamic order of a nonlinear system changes, computing its impulse effects is not straightforward and this causes that one cannot solve the tracking control problem even when a general solution like lyapunov method is existed. Using ELMs allows the input signal to be computed in order that the tracking control system in switching time remains stable. Also, it is proved that using ELMs allows us to use Sliding Mode Control (SMC) for when the system has some inherent uncertainties or the linearization of the nonlinear system causes some bounded uncertainties.

The rest of the paper is organized as follows: in section II, ELMs are introduced, then in section III the potential of ELM in modeling nonlinear models is explained and in section IV, the potential of ELM in tracking control systems are explained. In section V, the conclusion remarks are given.

2- Evolving Linear Models

An evolving linear model (ELM), in continuous time domain, is defined as a linear differential equation between input signal $u(t)\in R$ and output signal $y(t)\in R$ of a SISO system:

$$y^{[n_t]}(t)=\sum_{i=0}^{n_t-1}a_i(t)y^{[i]}(t)+\sum_{j=0}^{m_t}b_j(t)u^{[j]}(t)+c(t)$$
$$\forall t\in R,\ y^{[i]}(t)=\frac{\partial^i y}{\partial t^i}\quad u^{[j]}(t)=\frac{\partial^j u}{\partial t^j} \tag{1}$$

where n_t and m_t, respectively denote dynamic orders of output and input signals which are bounded to n_{max} and m_{max}. Also, $a_i(t)\in R$, $b_j(t)\in R$ denote respectively linear parameters for ith and jth derivatives of $y(t)$ and $u(t)$, and $c(t)\in R$ denotes the bias parameter. In this paper, $x^{[l]}(t)$ denotes the lth derivative of the signal $x(t)$. As it is seen in (1), linear parameters as well as dynamic orders of exogenous input and output can change through the time.

Assumption 1: In an ELM, the dynamic orders of input and output signals can change only at finite instants named as evolving instants (EIs). The set of EIs for the introduced ELM in (1) is defined as follows:

$$A=\{\forall t\in R\mid \lim_{(\tau\to t^+)}n_\tau\neq\lim_{(\tau\to t^-)}n_\tau\ or\ \lim_{(\tau\to t^+)}m_\tau\neq\lim_{(\tau\to t^-)}m_\tau\} \tag{2}$$

In another representation, an ELM can be supposed as a model which switches at each instancet to a new LTI model whose parameters and dynamic orders are fixed.

$$\begin{cases} y(t)=y_\tau(x)\ for\ t=x=\tau \\ y_\tau^{[n_\tau]}(x)=\sum_{i=0}^{n_\tau-1}a_i(\tau)y_\tau^{[i]}(x)+\sum_{j=0}^{m_\tau}b_j(\tau)u^{[j]}(x)+c(\tau) \\ Initial\ Conditions:y_\tau^{[g]}(\tau)=y^{[g]}(\tau)\quad g\in\{0,1,\dots,n_\tau-1\} \end{cases} \tag{3}$$

Fig. 1 shows a diagram representing ELM model as switching LTI models.

The above representation shows that ELMs is more capable of modeling nonlinearity than dynamic local linear models, such as ANFIS in [34], LOLIMOT in [35] and TS-SAMC in [36]. This is because, in such models the number of Local Linear Models (LLMs) is restricted and the output is defined as an interpolation of LLMs but in an ELM at each working point there is an independent LLM and the number of LLMs is not restricted; moreover, unlike most of the locally linear neuro-fuzzy models, in an ELM the dynamic order of the input or output can change at EIs.

3- Potential of ELM in Modeling Nonlinear Models

A. Time-Varying Nonlinear Model with Time-Varying Dynamic Order

Consider following input-output representation of a time-varying nonlinear model:

$$y^{[n_t]}(t)=F_{(n_t,m_t)}(y^{[n_t-1]}(t),\cdots,\dot{y}(t),y(t),u^{[m_t]}(t),\dots,\dot{u}(t),u(t),t) \tag{4}$$

where $F(n_t,m_t)$ denotes a time-varying nonlinear function of the input and output signals and their derivatives; n_t and m_t, respectively denote dynamic orders of the output and input signals at time t and their possible maximum values are n_{max} and m_{max}. Actually, the structure of a nonlinear dynamic model

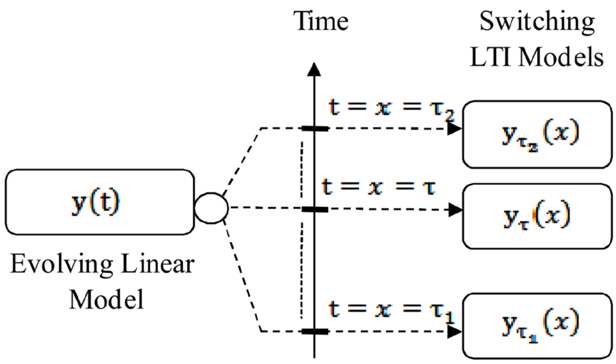


Fig. 1. An evolving linear model is represented as switching LTI models

may change through the time due to internal physical variations, erosions, delays and environmental factors.

Assumption 2: For the function $F(n_t,m_t)$–similar to the considered assumption for ELMs–the dynamic order of the input and output signal: n_t and m_t can change just at finite instancets, i.e. EIs. Consider that the set A={$\sigma_1,\sigma_2,\ldots,\sigma_F$} denotes EIs for the $F(n_t,m_t)$).

Assumption 3: The function $F(n_t,m_t)$) and its variables are continuously differentiable except at its EIs.

C. Potential of ELM in Modeling Control Systems

Consider T as sampling period of the input and output of the system. Considering variation variables: $\delta y^{[i]}(t)=y^{[i]}(t)-y^{[i]}(t-T)$, $\delta u^{[j]}(t)=u^{[j]}(t)-u^{[j]}(t-T)$ and $\delta t=T$, the Jacobian of the given nonlinear dynamic function in (4), for $\forall t\epsilon(R-A)$, can be stated as follows:

$$\begin{aligned}\delta y^{[n_t]}(t)&=\sum_{i=0}^{n_t-1}\left(\frac{\partial F_{(n_t,m_t)}}{\partial y^i(t)}\right)\delta y^{[i]}(t)\\&+\sum_{j=0}^{m_t}\left(\frac{\partial F_{(n_t,m_t)}}{\partial u^j(t)}\right)\delta u^{[j]}(t)+\frac{\partial F_{(n_t,m_t)}}{\partial t}\delta t+\gamma(t,T)\end{aligned}\tag{5}$$

where $\gamma(t,T)$ denotes all high order terms in Taylor series; one can suppose that if for at least one coefficient of the input variations was not zero, $\gamma(t,T)$ is negligible for small enough sampling period T. Here, we assume $\gamma(t,T)$ and its derivatives are restricted. The above model in (5) can be also rewritten as follows:

$$\begin{aligned}y^{[n_t]}(t)&=\sum_{i=0}^{n_t-1}\left(\frac{\partial F_{(\mathrm{n_t,m_t})}}{\partial y^i(t)}\right)y^{[i]}(t)\\&+\sum_{j=0}^{m_t}\left(\frac{\partial F_{(\mathrm{n_t,m_t})}}{\partial u^j(t)}\right)u^{[j]}(t)+r(t);\quad\forall t\epsilon(\mathrm{R-A})\\r(t)&=\frac{\partial F_{(n_t,m_t)}}{\partial t}T+y^{[n_t]}(t-T)\\&-\sum_{i=0}^{n_t-1}\left(\frac{\partial F_{(\mathrm{n_t,m_t})}}{\partial y^i(t)}\right)y^{[i]}(t-T)\\&-\sum_{j=0}^{m_t}\left(\frac{\partial F_{(\mathrm{n_t,m_t})}}{\partial u^j(t)}\right)u^{[j]}(t-T)+\gamma(t,T)\end{aligned}\tag{6}$$

Here, the Jacobian of the nonlinear model at EIs: σ_h, h=1,2,…,F is defined as the same Jacobian for $\{t|t\to\sigma_h^+\}$. Accordingly, we can define the Jacobian for all over the time. Considering $a_i(t)=(\partial F(n_t,m_t)/(\partial u^j(t))$, $b_j(t)=(\partial F(n_t,m_t))/(\partial u^j(t))$, $c(t)=r(t)$ and respecting to (1), the considered nonlinear function in (4) can be stated as an ELM:

$$y^{[n_t]}(t)=\sum_{i=0}^{n_t-1}a_i(t)y^{[i]}(t)+\sum_{j=0}^{m_t}b_j(t)u^{[j]}(t)+c(t),\tag{7}$$

$\forall t\epsilon\mathrm{R}$

Definition 1: Corresponding to each time-varying nonlinear SISO system represented in (4), if Assumption 2 and Assumption 3 are satisfied, an ELM can be defined as (6) and (7).

4- Potential of ELM in Tracking Control Systems

In this section, for tracking control system of an ELM, two different control strategies are proposed, respectively for two different states: (a) the ELM is known perfectly and (b) the ELM is known but with some uncertainties. In the first state, it is assumed that the dynamic order can change through the time but in the second state, for the sake of simplicity, it is assumed the dynamic orders for the input and output signals are fixed through the time.

A. Control Strategy for a Known ELM

In this section, it is supposed that the ELM in (7) is perfectly known and there is no uncertainty in the model. Considering some assumptions and using a control strategy, it is aimed that the output of the system asymptotically converges to the reference signal.

Assumption 4: For the ELM in (7), $\forall t:\exists j\epsilon\{0,1,\ldots,m_{max}\}|b_j(\mathrm{t})\neq 0$.

Assumption 5: The zero dynamic in (7) $\sum_{j=0}^{m_0}b_j(t)u^{[j]}(t)=\alpha(t)$ is stable for any bounded signal $\alpha(t)$.

Assumption 6: The reference signal, $y_r(t)$, is real, bounded and continuously differentiable for all $t>0$.

Theorem 1: Considering Assumptions 4-6 and applying the computed signal control from (8) to the system (4), the output $y(t)$, will converge to $y_r(t)+\pi(t)$:

$$\begin{aligned}&\sum_{j=0}^{m_t}b_j(t)u^{[j]}(t)=\mu(t)\\&-\left(\sum_{i=0}^{n_t-1}a_i(t)y^{[i]}(t)+\hat{c}(t)+y_r^{[n_t]}(t)-\sum_{l=1}^{n_{max}}d_l e^{[n_t-l]}(t)\right)\end{aligned}\tag{8}$$

$$e^{[l]}(t)=\begin{cases}\dfrac{\partial^l e(t)}{\partial t^l} & l\geq 0\\ \displaystyle\int_{\tau_l=\sigma_t}^{t}\cdots\int_{\tau_2=\sigma_t}^{\tau_3}\int_{\tau_1=\sigma_t}^{\tau_2} e(\tau_1)d\tau_1 d\tau_2\cdots d\tau_l & l<0\end{cases}$$

$$e(t)=y(t)-y_r(t);\ \hat{c}(t)=c(t)-\gamma(t,T)$$

$$\mu(t)=\sum_{k=1}^{n_{max}-n_t}\frac{1}{(k-1)!}e^{[k+n_t]}(\sigma_t)(t-\sigma_t)^{k-1}$$

where the roots of the characteristic function: $s^{n_{max}}+d_1 s^{n_{max}-1}+d_2 s^{n_{max}-2}+...+d_{n_{max}}=0$ are in the left part of the complex plane $s=\sigma+j\omega$ and σ_t denotes the last evolving instant before t.

$$\begin{aligned}&\pi(t)=\int_{\tau=\sigma_t}^{\tau=t}\varphi(\tau,T)\vartheta(t-\tau)d\tau\\&\vartheta(t)=L^{-1}\left([s^{n_{max}}+d_1 s^{n_{max}-1}+d_2 s^{n_{max}-2}+..+d_{n_{max}}]^{-1}\right)\\&\varphi(t,T)=\gamma^{[n_{max}-n_t]}(t,T)\\&\quad+\sum_{k=1}^{n_{max}-n_t}\frac{1}{(k-1)!}\gamma^{[k]}(\sigma_t,T)\delta^{(n_{max}-n_t-k)}(t-\sigma_t)\end{aligned}\tag{9}$$

where $\delta(.)$ denotes the Dirak delta function.

Proof: Assume the following error differential equation:

$$\begin{aligned}&e^{[n_{max}]}(t)+d_1 e^{[n_{max}-1]}(t)+d_2 e^{[n_{max}-2]}(t)\\&+\cdots+d_{n_{max}}e(t)=\varphi(t,T)\\&\varphi(t,T)=\gamma^{[n_{max}-n_t]}(t,T)+\\&\sum_{k=1}^{n_{max}-n_t}\frac{1}{(k-1)!}\gamma^{[k]}(\sigma_t,T)\delta^{(n_{max}-n_t-k)}(t-\sigma_t)\end{aligned}\tag{10}$$

We call the above differential equation Error Based Equation (EBE). Now, we compute the signal control which satisfies EBE. Regarding initial conditions for EBE at σ_t, by n_{max}-n_t times integral of the EBE, one can get the following equation:

$$\begin{aligned}&e^{[n_t]}(t)=\mu(t)-\sum_{l=1}^{n_{max}}d_l e^{[n_t-l]}(t)+\gamma(t,T)\\&\mu(t)=\sum_{k=1}^{n_{max}-n_t}\frac{1}{(k-1)!}e^{[k+n_t-1]}(\sigma_t)(t-\sigma_t)^{k-1}\end{aligned}\tag{11}$$

As it is understood, the considered Dirac delta functions in EBE eliminate the effects of initial conditions from $\gamma(t,T)$ and its derivatives. Now, by considering $e^{[n_t]}(t)=y^{[n_t]}(t)-y_r^{[n_t]}(t)$ we have:

$$y^{[n_t]}(t)=y_r^{[n_t]}(t)+\mu(t)-\sum_{l=1}^{n_{max}}d_l e^{[n_t-l]}(t)+\gamma(t,T)\tag{12}$$

Putting the ELM in (7) in the left hand of the above equation, we get the following differential equation:

$$\begin{aligned}&\sum_{j=0}^{m_t}b_j(t)u^{[j]}(t)=\mu(t)-(\sum_{i=0}^{n_t-1}a_i(t)y^{[i]}(t)+\hat{c}(t)\\&+y_r^{[n_t]}(t)-\sum_{l=1}^{n_{max}}d_l e^{[n_t-l]}(t))\end{aligned}\tag{13}$$

In fact, by computing the control signal from above equation the initially considered EBE will be satisfied. The response of the EBE is the sum of the response to initial condition and the response to the input signal. We can discount response to the initial condition because it decays through the time. However, we can compute the steady response by applying the Laplace Transform to the EBE:

$$\begin{aligned}&E(s)=[s^{n_{max}}+d_1 s^{n_{max}-1}+d_2 s^{n_{max}-2}\\&+\ldots+d_{n_{max}}]^{-1}\varphi(s,T)\end{aligned}\tag{14}$$

Now by applying the inverse of the Laplace Transform, we get:

$$\begin{aligned}&e(t)=\pi(t,T)=\int_{\tau=\sigma_t}^{\tau=t}\varphi(\tau,T)\vartheta(t-\tau)d\tau\\&\vartheta(t)=L^{-1}([s^{n_{max}}+d_1 s^{n_{max}-1}+d_2 s^{n_{max}-2}+\ldots+d_{n_{max}}]^{-1})\end{aligned}\tag{15}$$

Accordingly, the $e(t)$ will converge to $\pi(t,T)$ or $y(t)$ will converge to $y_r(t)+\pi(t,T)$.

Here, two important notes about the Theorem 1 are stated:

1) If T converges to zero, $\gamma(t,T)$, its derivatives and accordingly $\pi(t,T)$ converges to zero and $y(t)$ will follow $y_r(t)$, asymptotically.

2) The signal control $u(t)$ can be computed easily by solving the non-homogenous time-varying linear differential equation in (7) [37]. This differential equation must be solved separately for each interval in which the dynamic order m_t is fixed; also, when m_t changes, the given initial conditions are considered.

1) *Illustrative Example*

The Dynamics of a DC motor is stated as the following state model:

$$\begin{cases}\dot{x}_1=-\theta_1 x_1-\theta_2 x_2 u+\theta_3\\ \dot{x}_2=-\theta_4 x_2+\theta_5 x_1 u\\ y=x_2\end{cases}\tag{16}$$

where x_1 is the armature current, x_2 is the motor speed , u is the field current and θ_1 to θ_5 are positive constants [8,38]. Assume the domain of operations will be restricted to regimes that the system is minimum phase. It is desired to design a controller based on

Theorem 1 such that the output y asymptotically tracks the reference signal $y_r(t)$=2+0.5sin(2t). For this reason, three following cases are considered:

Case 1: T=0.001 seconds, $\theta_1=\theta_2=\theta_4=\theta_5=1$ and $\theta_3=5$ for $t\geq 0$.

Case 2: T=0.1 seconds, $\theta_1=\theta_2=\theta_4=\theta_5=1$ and $\theta_3=5$ for $t\geq 0$.

Case 3: T=0.001 seconds:

- $\theta_1=\theta_2=\theta_4=\theta_5=1$ and $\theta_3=5$: $0<t<5$ ($\sigma_1=0$)
- $\theta_1=\theta_2=\theta_3=0$ and $\theta_4=\theta_5=1$: $t\geq 5$ ($\sigma_2=5$)

Case 1

One can compute the following input-output differential equation (from (10)):

$\ddot{y}(t)=F_{(2,1)}(\dot{y},y,\dot{u},u)=(-\theta_2\theta_5 yu^2+\theta_5\theta_3 u)+[\dot{y}\dot{u}u^{-1}-(\theta_4+\theta_1)\dot{y}]+[\theta_4 y\dot{u}u^{-1}-\theta_4\theta_1 y]$

Considering the sampling period T=0.001 seconds, $\theta_1=\theta_2=\theta_4=\theta_5=1$ and $\theta_3=5$ the following evolving linear model will be computed:

$$\begin{aligned}
&\ddot{y}(t)=a_0(t)y(t)+a_1(t)\dot{y}(t)+b_0(t)u(t)\\
&+b_1(t)\dot{u}(t)+c(t)\\
&a_0(t)=\frac{\partial F_{(2,1)}}{\partial y}=-u^2+\dot{u}u^{-1}-1\\
&a_1(t)=\frac{\partial F_{(2,1)}}{\partial \dot{y}}=\dot{u}u^{-1}-2\\
&b_0(t)=\frac{\partial F_{(2,1)}}{\partial u}=-2yu+5-(\dot{y}+y)\dot{u}u^{-2}\\
&b_1(t)=\frac{\partial F_{(2,1)}}{\partial \dot{u}}=(\dot{y}+y)u^{-1}\\
&c(t)=\ddot{y}(t-T)-a_0(t)y(t-T)-a_1(t)\dot{y}(t-T)\\
&-b_0(t)u(t-T)-b_1(t)\dot{u}(t-T))+\gamma(t,T)
\end{aligned} \tag{17}$$

where $\gamma(t,T)$ denotes high order terms in the Taylor series.

We assume $s^2+4s+4=0$ as the characteristic function of the error signal $e(t)=y_r(t)-y(t)$; then, the proposed signal control is computed by solving the following time-varying linear differential equation:

$$\begin{aligned}
&b_0(t)u(t)+b_1(t)\dot{u}(t)=-(a_0(t)y(t)+a_1(t)\dot{y}(t)\\
&+\hat{c}(t))+\ddot{y}_r(t)-4\dot{e}(t)-4e(t)\\
&\hat{c}(t)=c(t)-\gamma(t,T)
\end{aligned} \tag{18}$$

The initial conditions for output and input signals are chosen: $u(0)=1$, $y(0)=1$ and $\dot{y}(0)=0$.

Since the chosen T=0.001 is so small, it is expected $\gamma(t,T)$ is insignificant. Fig. 2 shows the plots of the output signal $y(t)$, input signal $u(t)$ and the reference signal $y_r(t)$. As can be seen, the $y(t)$ is almost asymptotically converged to the $y_r(t)$.

Case 2

The simulation in Case1 is repeated for the Case 2 where T=0.1; Fig. 3 shows the output signal $y(t)$, input signal $u(t)$ and the reference signal $y_r(t)$. As it is seen, since the effect of the $\gamma(t,T)$ is considerable, there is a difference between $y(t)$ and $y_r(t)$.

Case 3

In this case, we have an evolving instant at t=5. Actually, for $0\leq t<5$, the represented system in Case 1 is used, where n_t=2 and m_t=1; however, for $t\geq 5$ the armature current is considered fixed (x_1=1) and dynamic order of input and output of the system changes to n_t=1 and m_t=0. The differential equation for the system can be computed as follows:

$$\begin{cases}\ddot{y}(t)=F_{(2,1)}(\dot{y},y,\dot{u},u):Like\,Case\,1 & 0\leq t<5\\ \dot{y}=F_{(1,0)}(y,u)=-\theta_4 y+\theta_5 u & t\geq 5\end{cases} \tag{19}$$

We have determined the control strategy for $0\leq t<5$ in Case 1; here, we determine the control strategy for $t\geq 5$.

$$\begin{aligned}
&\dot{y}(t)=a_0(t)y(t)+b_0(t)u(t)+c(t)\\
&a_0(t)=\frac{\partial F_{(2,1)}}{\partial y}=-\theta_4=-1
\end{aligned} \tag{20}$$

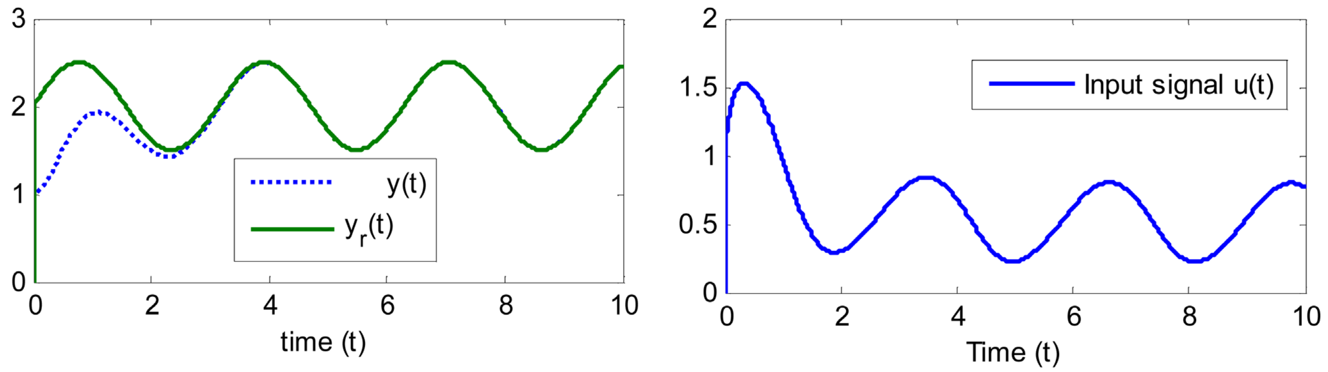


Fig. 2. The output signal $y(t)$, the reference signal $y_r(t)$ and input signal $u(t)$ when the sampling period T=0.001 second

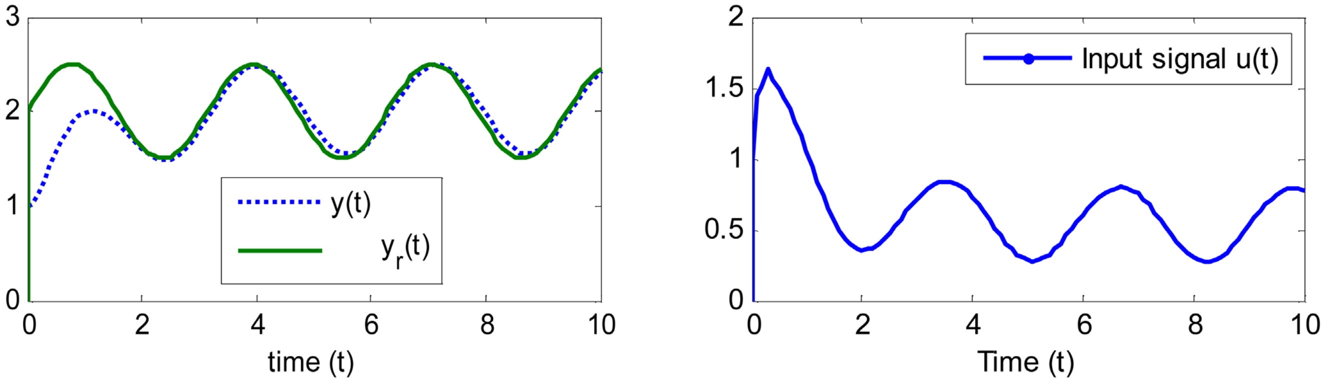


Fig. 3. The output signal $y(t)$, the reference signal $y_r(t)$ and input signal $u(t)$ when the sampling period T=0.1 second

$$b_0(t)=\frac{\partial F_{(2,1)}}{\partial u}=\theta_5=1$$
$$c(t)=\dot{y}(t-T)-a_0(t)y(t-T)-b_0(t)u(t-T)$$
$$+\gamma(t,T),\ \ T=0.001$$

The required initial condition for the signal is chosen: $y(5)=1$. Signal $e(t)=y_r(t)-y(t)$, the proposed signal control is computed by solving following equation:

$$b_0(t)u(t)=\mu(t)-\left(a_0(t)y(t)+\hat{c}(t)\right)+\dot{y}_r(t)$$
$$-4e(t)-\int_{\tau=5}^{t}4e(\tau)d\tau \qquad (21)$$
$$\hat{c}(t)=c(t)-\gamma(t,T)$$

where by considering n_{max}=2, n_t=1 and σ_t=5:

$$\mu(t)=\sum_{k=1}^{n_{max}-n_t}\frac{1}{(k-1)!}e^{[k+n_t-1]}(t_0)(t-t_0)^{k-1}=\dot{e}(5) \qquad (22)$$

One can compute $\dot{e}(5)$ by solving following equation and compute $\dot{e}(t)|_{t=5}$.

$\ddot{e}-4\dot{e}(t)-4e(t)\approx 0$, $\dot{e}(0)=\dot{y}(0)-\dot{y}_r(0)=-1$, $e(0)=y(0)-y_r(0)=-1$.

Fig. 4 shows the plot of the output signal $y(t)$, input signal $u(t)$ and the reference signal $y_r(t)$. As it is observed, the $y(t)$ is asymptotically converged to the $y_r(t)$ even with evolving in the structure at t=5.

As seen in Fig. 4, in the plots of the input signal control and output control, there is an abrupt change at evolving instant but the output signal changes continuously through the time.

B. Robust Control Strategy for an ELM with Some Uncertainties

Definition 1: For the ELM defined in (7), the partially known ELM is defined as follow:

$$y^{[n_0]}(t)=\sum_{i=0}^{n_0-1}\hat{a}_i(t)y^{[i]}(t)+\sum_{j=0}^{m_0}\hat{b}_j(t)u^{[j]}(t)+\hat{c}(t) \qquad (23)$$

where n_0 and m_0 denote the dynamic orders of the output and input signals and uncertainties are defined as follows:

$\forall t$:$|\hat{a}_i(t)-a_i(t)|<\bar{a}_i$ (i=0,1,2...,n_0) ,$|\hat{c}(t)-c(t)|<\bar{c}$, $|\hat{b}_j(t)-b_j(t)|<\bar{b}_j$ (j=1,2,...,m_0-1), $\beta^{-1}\le b_{m_0}(t)/\hat{b}_{m_0}(t)\le\beta$, $\beta\ge 1$, $|a_i(t)|\le\tilde{a}_i$, $|b_j(t)|\le\tilde{b}_j$, $|c(t)|\le\tilde{c}$.

Theorem 2: For the partially known ELM in Definition 1 and considering Assumptions 5 and 6,

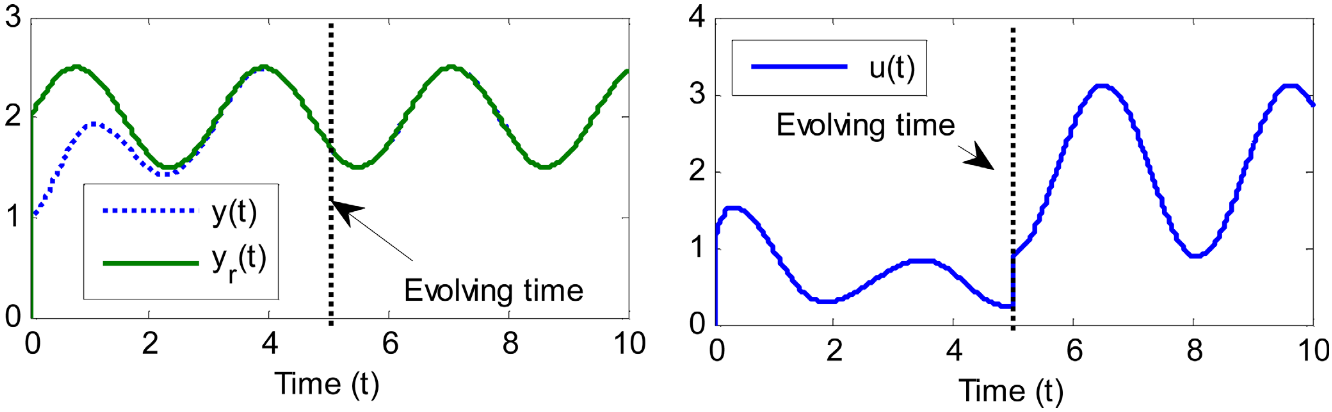


Fig. 4 The output signal $y(t)$, input signal $u(t)$ and the reference signal $y_r(t)$ for the Case 3 when the sampling period T=0.001 second

the control signal computed from (24) guarantees that $y(t)$ will converginge to $y_r(t)$, if there is $\delta\geq 0$ such that the condition (26) is satisfied.

$$u^{[m_0]}(t)+\sum_{j=0}^{m_0-1}\frac{\hat{b}_j(t)}{\hat{b}_{m_0}(t)}u^{[j]}(t)=$$

$$\frac{1}{\hat{b}_{m_0}(t)}\sum_{j=0}^{m_0-1}[(-\sum_{i=0}^{n_0-1}\hat{a}_i(t)y^{[i]}(t)-\hat{c}(t)+y_r^{[n_0]}(t) \quad (24)$$

$$-\sum_{l=1}^{n0-1}d_l e^{[n_0-l]}(t))-\zeta sign(S(t))]$$

$$e(t)=y(t)-y_r(t)$$

$$S(t)=e^{[n_0-1]}(t)+d_1e^{[n_0-2]}(t)+d_2e^{[n_0-3]}(t)$$

$$+\ldots+d_{n_0-1}e(t)$$

$$\zeta=\delta+\beta\Delta$$

$$\Delta=\sum_{i=0}^{n_0-1}[\beta\quad \tilde{a}_i]\begin{bmatrix}\bar{a}_i\\(\beta-1)\end{bmatrix}\bar{y}_i+$$

$$\sum_{j=0}^{m_0}[\beta\quad \tilde{b}_j]\begin{bmatrix}\bar{b}_j\\(\beta-1)\end{bmatrix}\bar{u}_j+[\beta\quad \tilde{c}]\begin{bmatrix}\bar{c}\\(\beta-1)\end{bmatrix} \quad (25)$$

$$+(\beta-1)\nabla$$

$$\nabla=\bar{y}_{r,n_0}+\sum_{l=1}^{n_0-1}d_l\bar{e}_{n_0-l}$$

$(\bar{e}_{n_0-1}\ (l=1,2,\ldots,n_0-1))$ are defined later)

where $\zeta\epsilon R^+$ and the roots of the characteristic function $s^{n_0-1}+d_1s^{n_0-2}+d_2s^{n_0-3}+\ldots+d_{n_0-1}=0$ are in the left part of the complex plane $s=\sigma+j\omega$.

$$\zeta=\delta+\beta\Delta$$

$$\Delta=\sum_{i=0}^{n_0-1}[\beta\quad \tilde{a}_i]\begin{bmatrix}\bar{a}_i\\(\beta-1)\end{bmatrix}\bar{y}_i+\sum_{j=0}^{m_0}[\beta\quad \tilde{b}_j]\begin{bmatrix}\bar{b}_j\\(\beta-1)\end{bmatrix}\bar{u}_j$$

$$+[\beta\quad \tilde{c}]\begin{bmatrix}\bar{c}\\(\beta-1)\end{bmatrix}+(\beta-1)\nabla \quad (26)$$

$$\nabla=\bar{y}_{r,n_0}+\sum_{l=1}^{n_0-1}d_l\bar{e}_{n_0-l}\ ,\ \mathrm{R}\begin{bmatrix}\bar{u}_0\\\vdots\\\bar{u}_{m_0-2}\\\bar{u}_{m_0-1}\end{bmatrix}\geq \mathrm{H}$$

$$\mathrm{R}_{m_0\times m_0}=(\underbrace{I_{m_0}}_{Idenitity\ matix}\quad -\beta\bar{\sigma}^{-1}\begin{bmatrix}0\\\vdots\\0\\1\end{bmatrix}$$

$$[(\beta\bar{b}_0+(\beta-1)\tilde{b}_0)\quad\ldots\quad(\beta\bar{b}_{m_0-1}+(\beta-1)\tilde{b}_{m_0-1})])$$

$$H=(\begin{bmatrix}|u(t_0)|\\\vdots\\|u^{[m_0-2]}(t_0)|\\|u^{[m_0-1]}(t_0)|\end{bmatrix}+[\bar{z}+\beta\sum_{i=0}^{n_0-1}[\beta\quad \tilde{a}_i][\frac{\bar{a}_i}{(\beta-1)}]\bar{y}_i+$$

$$\beta[\beta\quad \tilde{c}][\frac{\bar{c}}{(\beta-1)}]+\beta(\beta-1)\nabla+\delta]\tilde{b}_{m_0}^{-1}\bar{\sigma}^{-1}\begin{bmatrix}0\\\vdots\\0\\1\end{bmatrix})$$

$$\forall\tau\geq t_0:\bar{\sigma}=max\left(\left|eig\left(\mathrm{A}_u(\tau)\right)\right|\right)$$

$$\mathrm{A}_u(\tau)=\begin{bmatrix}0&1&\cdots&0&0\\\vdots&\vdots&&\vdots&\vdots\\0&0&\ddots&0&1\\\frac{-\hat{b}_0(t)}{\hat{b}_{m_0}(t)}&\frac{-\hat{b}_1(t)}{\hat{b}_{m_0}(t)}&\cdots&\frac{-\hat{b}_{m_0-2}(t)}{\hat{b}_{m_0}(t)}&\frac{-\hat{b}_{m_0-1}(t)}{\hat{b}_{m_0}(t)}\end{bmatrix}$$

$$\bar{z}=\left(\sum_{i=0}^{n_0-1}\tilde{a}_i\bar{y}_i+\tilde{c}+\bar{y}_{r,n_0}(t)+\sum_{l=1}^{n_0-1}d_l\bar{e}_{n_0-1}\right)$$

$$\bar{e}_{n_0-1}=\left(\mathrm{S}_0+d_1\bar{e}_{n_0-2}+d_2\bar{e}_{n_0-3}+..+d_{n_0-1}\bar{e}\right)$$

$$\mathrm{S}_0=e^{[n_0-1]}(t_0)+d_1e^{[n_0-2]}(t_0)+d_2e^{[n_0-3]}(t_0)$$

$$+\cdots+d_{n_0-2}e(t_0)$$

$$\begin{bmatrix}\bar{y}\\\vdots\\\bar{y}_{n_0-2}\\\bar{y}_{n_0-1}\end{bmatrix}=\begin{bmatrix}\bar{e}+\bar{y}_r\\\vdots\\\bar{e}_{n_0-2}+\bar{y}_{r,n_0-2}\\\bar{e}_{n_0-1}+\bar{y}_{r,n_0-1}\end{bmatrix}$$

Proof: the proof is given in Appendix A.

Following results can be stated from Theorem 2:

- The suggested robust control strategy is similar to the sliding mode control because the error dynamic equation $S(t)$ (sliding surface) too is utilized in the sliding mode. However, in sliding mode control, the affine of the input signal in output differential equation must be affine but here, this constraint is not necessary; however, here, some constraints about the bounds of the signal control and its derivatives are posed.

- One can identify the unknown parameters of the ELM recursively through some estimation methods, such as Least Square or Kalman filter. It is expected by converging the parameters to their original values, the uncertainties vanish gradually.

- If the uncertainties starts to vanish through the time, it is expected that after some while $\beta\to 1$, $\bar{a}_i\to 0$, $\bar{b}_j\to 0$ and $c\to 0$. In such state, it is seen that $\zeta\to\delta\to 0^+$ and consequently upper bound for the signal control and its derivatives can be chosen easily as follows:

$$\begin{bmatrix}\bar{u}_0\\ \vdots\\ \bar{u}_{m_0-2}\\ \bar{u}_{m_0-1}\end{bmatrix}\geq\begin{bmatrix}\left|u(t_0)\right|\\ \vdots\\ \left|u^{[m_0-2]}(t_0)\right|\\ \left|u^{[m_0-1]}(t_0)\right|\end{bmatrix}+\bar{z}\tilde{b}_{m_0}^{-1}\bar{\sigma}^{-1}\begin{bmatrix}0\\ \vdots\\ 0\\ 1\end{bmatrix} \tag{27}$$

1) Illustrative Examples

Example 1: In this example, the same example stated in section 4.1.1 (Case 1) is considered. The original ELM and its partially known ELM are introduced as follows:

The original ELM: $\ddot{y}(t)=a_0(t)y(t)+a_1(t)\dot{y}(t)+b_0(t)u(t)+b_1(t)\dot{u}(t)+c(t)$.

The Partially known ELM: $\ddot{y}(t)=\hat{a}_0(t)y(t)+\hat{a}_1(t)\dot{y}(t)+\hat{b}_0(t)u(t)+\hat{b}_1(t)\dot{u}(t)+\hat{c}(t)$.

where the parameters are known as follows:

$a_0(t)=-u^2+\dot{u}u^{-1}-1$; $\hat{a}_0(t)=a_0(t)+\varepsilon$; $\bar{a}_0=\varepsilon$

$a_1(t)=\dot{u}u^{-1}-2$; $\hat{a}_1(t)=a_1(t)+\varepsilon$; $\bar{a}_1=\varepsilon$

$b_0(t)=-2yu+5-(\dot{y}+y)\dot{u}u^{-2}$; $\hat{b}_0(t)=\beta b_0(t)$

$b_1(t)=(\dot{y}+y)u^{-1}$; $\hat{b}_1(t)=b_1(t)+\varepsilon$; $\bar{b}_1=\varepsilon$

$c(t)=\ddot{y}(t-T)-a_0(t)y(t-T)-a_1(t)y(t-T)-b_0(t)u(t-T)-b_1(t)\dot{u}(t-T))+\gamma(t,T)$; $\hat{c}(t)=c(t)+\varepsilon$; $S(t)=\dot{e}(t)+2e(t)$

As it is explained in Theorem 2, the signal control is computed as follows:

$$\begin{aligned}&\dot{u}(t)+\frac{\hat{b}_0(t)}{\hat{b}_1(t)}u(t)=\frac{1}{\hat{b}_1(t)}[(-\hat{a}_0(t)y(t)-\hat{a}_1(t)\dot{y}(t)\\&-\hat{c}(t)+y_r^{[2]}(t)-d_I\dot{e}(t))-\zeta sign(S(t))]\end{aligned} \tag{28}$$

The resulting output (input) signals and reference signal are shown in four cases in Fig. 5. In (a) the uncertainty of the ELM is assumed ε=-0.2 and β=1.1. The control signal is computed for ζ=1. As it is seen in (a), the output signal converges to reference signal but in (c) when ζ=0 is chosen, the system is not robust against considered uncertainty. In (b), the uncertainty of the ELM is assumed more than (a): ε=0.5 and β=1.5. The control signal is computed for ζ=5. As it is seen in (c), the output signal converges to reference signal but in (d) when the uncertainty in ε becomes bigger, the system is not robust. Also, the plots of signal controls in (a) and (b) are shown in Fig. 5.

Example 2: In this example, a flexible joint robot is considered. The system dynamic is represented as follows:

$$\begin{aligned}&J\ddot{q}_1+MgLsin(q_1)+k(q_1-q_2)=0\\&J\ddot{q}_2-k(q_1-q_2)=u\end{aligned} \tag{29}$$

where q_1 and q_2 denote respectively, link position and motor position and, u denotes the voltage motor. Fig. 6 shows a scheme of the flexible joint robot.

Considering $y=q_1$ the following input-output differential equation is obtained.

$$\begin{aligned}&y^{[4]}=F_{(4,0)}(y^{[3]},y^{[2]},\dot{y},y,u)=-(kI^{-1}+k^2J^{-1}\\&+I^{-1}MgL\cos(y))y^{[2]}+I^{-1}MgL\sin(y)\dot{y}^2\\&-J^{-1}I^{-1}k^2MgLsin(y)+J^{-1}I^{-1}ku\end{aligned} \tag{30}$$

Now, we can compute its ELM as follows:

$$\begin{aligned}&y^{[4]}(t)=a_0(t)y(t)+a_1(t)\dot{y}(t)+a_2(t)y^{[2]}(t)\\&+a_3(t)y^{[3]}(t)+b_0(t)u(t)+c(t)\end{aligned} \tag{31}$$

$a_0(t)=\partial F_{(4,0)}/\partial y=MgL[-I^{-1}\sin(y)y^{[2]}+I^{-1}\cos(y)y^2]-J^{-1}I^{-1}k^2MgL\cos(y)$

$a_1(t)=\partial F_{(4,0)}/\partial\dot{y}=2I^{-1}MgL\sin(y)\dot{y}$

$a_2(t)=\partial F_{(4,0)}/\partial y^{[2]}=-(kI^{-1}+k^2J^{-1}+I^{-1}MgL\cos(y))$

$a_3(t)=\partial F_{(4,0)}/\partial y^{[3]}=0$

$b_0(t)=\partial F_{(4,0)}/\partial u=J^{-1}I^{-1}k$

$c(t)=y_r^{[4]}(t-T)-\sum_{i=1}^{3}a_i(t)y^{[i]}(t-T)-b_0(t)u(t-T)+\gamma(t,T)$

$\hat{a}_i(t)=a_i(t)+\varepsilon$, i=0,1,2,3

$\hat{b}_0(t)=\beta b_0(t)$

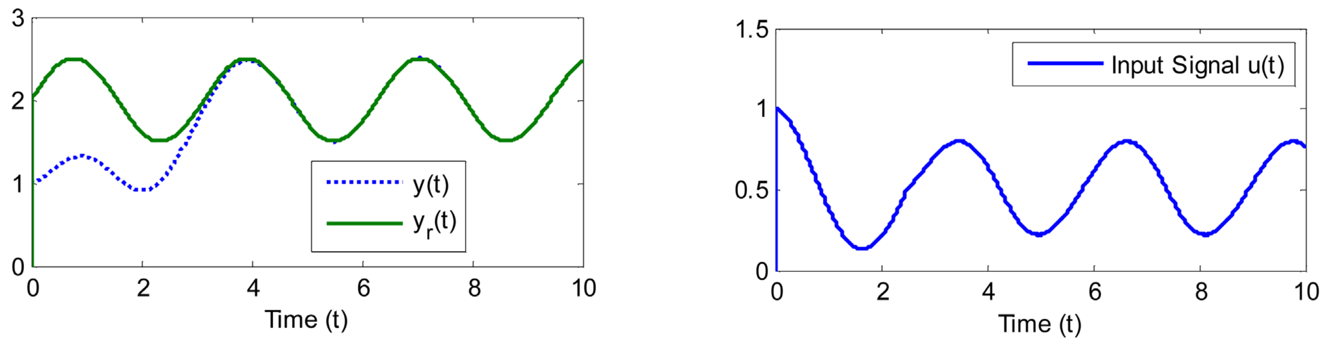
For the considred case, we assume M=1, L=0.1, g=10, k=1, J=1, I=1. Now, the error eqution is considred as follows:

$$\begin{aligned}&S(t)=e^{[3]}(t)+6e^{[2]}(t)+12\dot{e}(t)+8e(t)\\&e(t)=y(t)-y_r(t)\\&y_r(t)=0.1+0.5\cos(2t)+\cos(0.3t)\end{aligned} \tag{32}$$

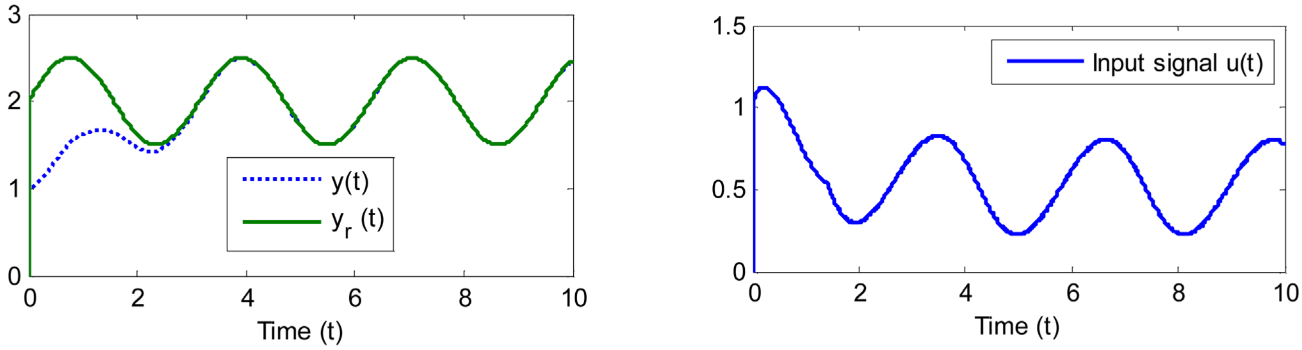
Respecting Theorem 2, the following differential equation of input signal is achieved.

$$\begin{aligned}&u(t)=\frac{1}{\hat{b}_0(t)}[(-\sum_{i=0}^{3}\hat{a}_i(t)y^{[i]}(t)-\hat{c}(t)+y_r^{[4]}(t)\\&-6e^{[3]}(t)-12e^{[2]}(t)-8\dot{e}(t))-\zeta sign(S(t))]\end{aligned} \tag{33}$$

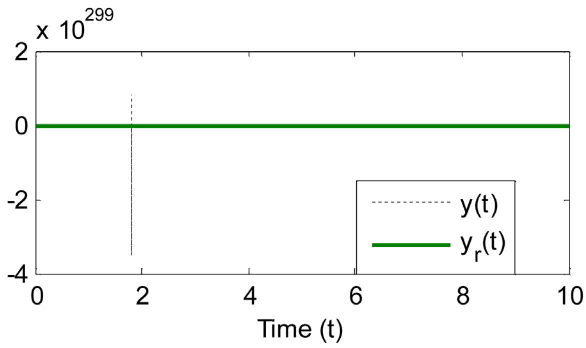
For different modes, we simulate the system with suggested control strategy: Fig. 7 shows the .resulting simulation in different cases. In (a), the uncertainty of the ELM is assumed ε=-0.2 and β=1.1. The control signal is computed for ζ=1. As it is seen in (a), the output signal converges to reference signal but in (c) (which is similar to (a)) when ζ=0 is chosen, the system is not robust against considered uncertainty. In (b), the uncertainty of the ELM is assumed more than (a): ε=0.5 and β=1.5. The control signal is computed



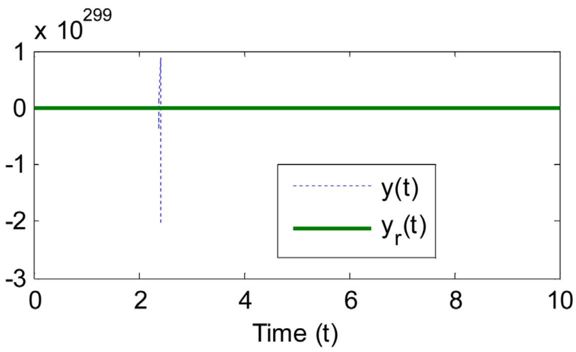
(a) $\varepsilon = -0.2, \beta = 1.1, \zeta = 1$



(b) $\varepsilon = 0.5, \beta = 1.5, \zeta = 5$



(c) $\varepsilon = -0.2, \beta = 1.1, \zeta = 0$



(d) $\varepsilon = 1, \beta = 1.5, \zeta = 5$

Fig. 5. The output signal $y(t)$, input signal $u(t)$ and reference signal $y_r(t)$ in (a) and (b) and $y(t)$ and $y_r(t)$ in (c) and (d) for Example 1

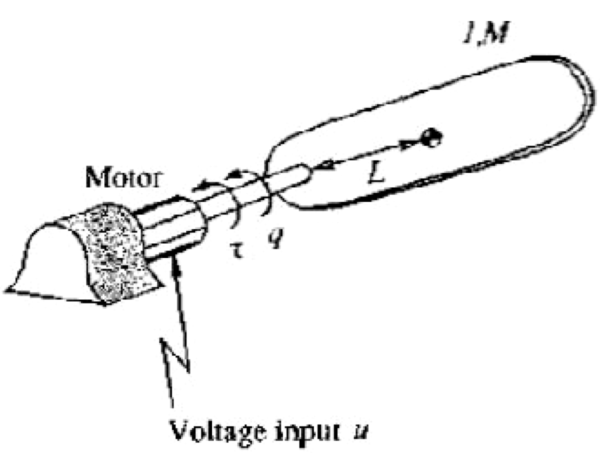
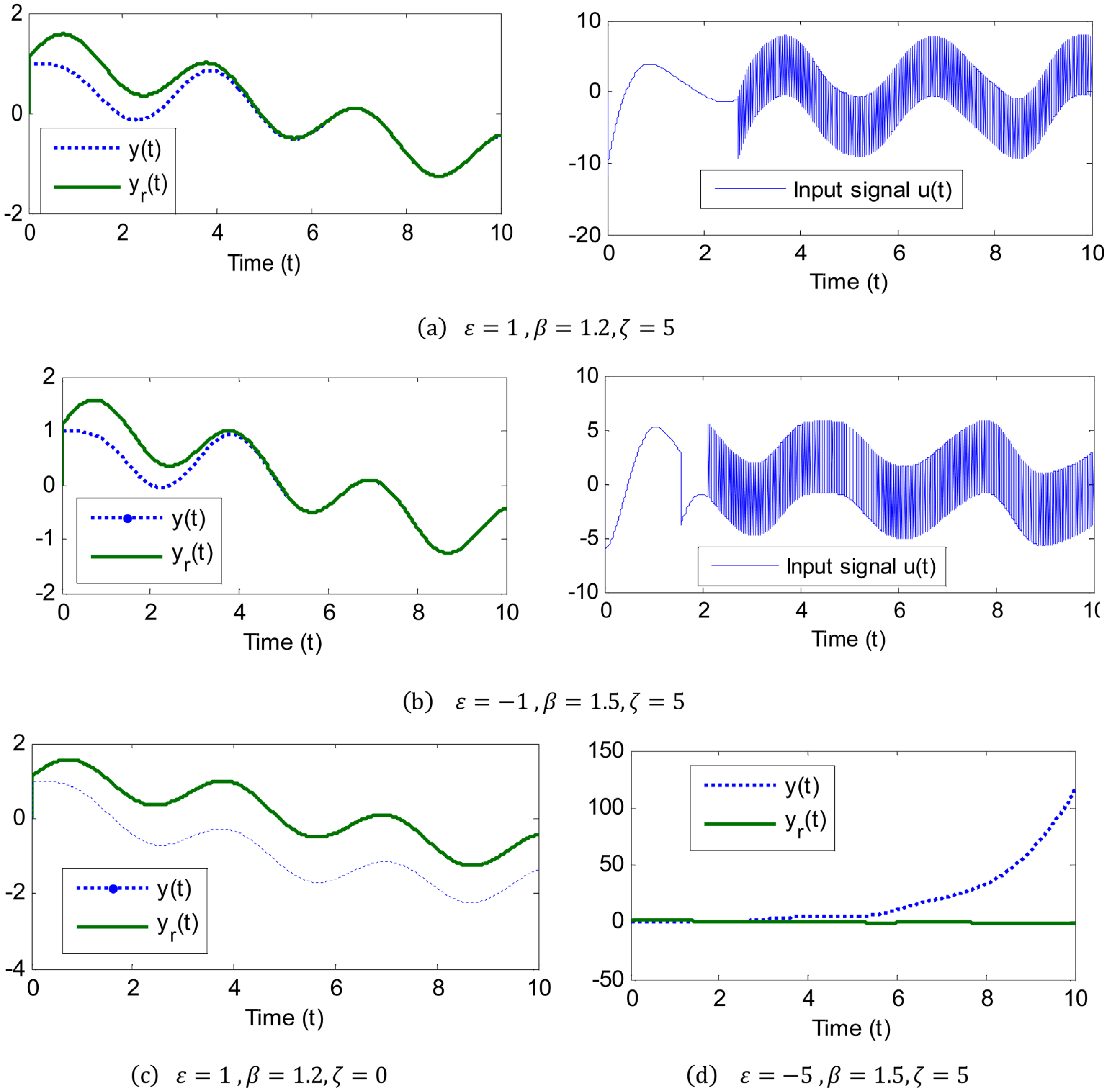


Fig. 6. A link driven by a low motor

for ζ=5. As it is seen in (c), the output signal converges to reference signal but in (d) (which is similar to (b)) when the uncertainty in ε become bigger, the system is not robust. The input signals shown in (a) and (b) are oscillating in some intervals. This is due to using *sign* function in the suggested signal control and the effect of the quantization noise. From the suggested solutions in sliding mode controllers, we know by using some smoothed variants of *sign* functions, such as bipolar sigmoid functions, more smoothed behavior in both output and control signals will be resulted, [8].



(a) $\varepsilon = 1, \beta = 1.2, \zeta = 5$

(b) $\varepsilon = -1, \beta = 1.5, \zeta = 5$

(c) $\varepsilon = 1, \beta = 1.2, \zeta = 0$

(d) $\varepsilon = -5, \beta = 1.5, \zeta = 5$

Fig. 7. Output signal $y(t)$, input signal $u(t)$ and reference signal $y_r(t)$ in (a) and (b) and $y(t)$ and $y_r(t)$ in (c) and (d) for Example 2

5- Conclusion

In this paper, potentials of Evolving Linear Models (ELMs) in modeling and tracking control system of nonlinear time-varying SISO systems were introduced. We explained a nonlinear time varying system, which is continuously differentiable except some finite instants, can be presented as an ELM. Then, based on two theorems, two tracking control strategies were proposed, respectively for two different states: (1) when the parameters of the ELM is known perfectly and (2) when the parameters of the ELM have certain uncertainties but the dynamic orders of input and output signals are fixed. In both states, the system was assumed minimum phase and in the second state it was stated the upper bounds of the signal control must satisfy an inequality. The proposed control strategies are used in speed control of a DC motor and position control of a flexible robot link. The results show that ELMs are capable of being used in tracking control systems of those cases in which the nonlinearity, time-varying and uncertainty are existed. However, some other challenges of using

ELMs in control systems such as: tracking control systems for non minimum phase systems and multi input and multi output ELMs are focused in our future works.

APPENDIX A

Proof of Theorem 2: Consider the energy function as follows:

$$V(t) = 0.5S^2(t) \tag{A.1}$$

Its time derivative is computed as follows:

$$\dot{V}(t) = S(t)\dot{S}(t) \tag{A.2}$$

$$\begin{aligned}\dot{S}(t) &= e^{[n_0]}(t) + d_1 e^{[n_0-1]}(t) + d_2 e^{[n_0-2]}(t) + \cdots \\ &+ d_{n_0-1}\dot{e}(t) = y^{[n_0]}(t) - y_r^{[n_0]}(t) + d_1 e^{[n_0-1]}(t) \\ &+ d_2 e^{[n_0-2]}(t) + \cdots + d_{n_0-1}\dot{e}(t)\end{aligned} \tag{A.3}$$

Putting the suggested control signal in $y^{[n_0]}(t)$, we will have:

$$\begin{aligned}\dot{S}(t) &= [\sum_{i=0}^{n_0-1}(\mathrm{a_i}(t) - \frac{b_{m_0}(t)}{\hat{b}_{m_0}(t)}\hat{a}_{\mathrm{i}}(t))y^{[i]}(t) \\ &+ \sum_{j=0}^{m_0}(\mathrm{b_j}(t) - \frac{b_{m_0}(t)}{\hat{b}_{m_0}(t)}\hat{b}_{\mathrm{j}}(t)) \\ & u^{[j]}(t) + (\mathrm{c}(t) - \frac{b_{m_0}(t)}{\hat{b}_{m_0}(t)}\hat{c}(t)) + (1 - \frac{b_{m_0}(t)}{\hat{b}_{m_0}(t)}) \\ & \sum_{l=1}^{n_0-1} d_l e^{[n_0-l]}(t) + (-1 + \frac{b_{m_0}(t)}{\hat{b}_{m_0}(t)}) \\ & y_r^{[n_0]}(t) - \frac{b_{m_0}(t)}{\hat{b}_{m_0}(t)}\zeta\, sign(S(t))]\end{aligned} \tag{A.4}$$

$$\begin{aligned}S(t)\dot{S}(t) &= [\sum_{i=0}^{n_0-1}(a_i(t) - \frac{b_{m_0}(t)}{\hat{b}_{m_0}(t)}\hat{a}_i(t))y^{[i]}(t) \\ &+ \sum_{j=0}^{m_0}(b_j(t) - \frac{b_{m_0}(t)}{\hat{b}_{m_0}(t)}\hat{b}_j(t))u^{[j]}(t) + (c(t) \\ &- \frac{b_{m_0}(t)}{\hat{b}_{m_0}(t)}\hat{c}(t)) + (1 - \frac{b_{m_0}(t)}{\hat{b}_{m_0}(t)})\sum_{l=1}^{n0-1} d_l e^{[n_0-l]}(t) \\ &+ (-1 + \frac{b_{m_0}(t)}{\hat{b}_{m_0}(t)})y_r^{[n_0]}(t)]S(t) - \frac{b_{m_0}(t)}{\hat{b}_{m_0}(t)}\zeta\left|S(t)\right|\end{aligned} \tag{A.5}$$

$$\begin{aligned}S(t)\dot{S}(t) &= [\sum_{i=0}^{n_0-1}(a_i(t) - \frac{b_{m_0}(t)}{\hat{b}_{m_0}(t)}\hat{a}_{\mathrm{i}}(t) \pm \frac{b_{m_0}(t)}{\hat{b}_{m_0}(t)}a_i(t)) \\ & y^{[i]}(t) + \sum_{j=0}^{m_0}(b_j(t) - \frac{b_{m_0}(t)}{\hat{b}_{m_0}(t)}\hat{b}_j(t) \pm \frac{b_{m_0}(t)}{\hat{b}_{m_0}(t)}b_j(t)) \\ & u^{[j]}(t) + (\mathrm{c}(t) - \frac{b_{m_0}(t)}{\hat{b}_{m_0}(t)}\hat{c}(t) \pm \frac{b_{m_0}(t)}{\hat{b}_{m_0}(t)}\mathrm{c}(t)) \\ &+ (1 - \frac{b_{m_0}(t)}{\hat{b}_{m_0}(t)})\sum_{l=1}^{n0-1} d_l e^{[n_0-l]}(t) + (-1 + \frac{b_{m_0}(t)}{\hat{b}_{m_0}(t)}) \\ & y_r^{[n_0]}(t)]S(t) - \frac{b_{m_0}(t)}{\hat{b}_{m_0}(t)}\zeta\left|S(t)\right|\end{aligned} \tag{A.6}$$

$$\begin{aligned}S(t)\dot{S}(t) &\le [\sum_{i=0}^{n_0-1}(\frac{b_{m_0}(t)}{\hat{b}_{m_0}(t)}\left|a_i(t) - \hat{a}_i(t)\right| + \left|1 - \frac{b_{m_0}(t)}{\hat{b}_{m_0}(t)}\right| \\ & \left|a_i(t)\right|)\left|y^{[i]}(t)\right| + \sum_{j=0}^{m_0-1}(\frac{b_{m_0}(t)}{\hat{b}_{m_0}(t)}\left|b_j(t) - \hat{b}_j(t)\right| \\ &+ \left|1 - \frac{b_{m_0}(t)}{\hat{b}_{m_0}(t)}\right|\left|b_j(t)\right|)\left|u^{[j]}(t)\right|\frac{b_{m_0}(t)}{\hat{b}_{m_0}(t)}\left|c(t) - \hat{c}(t)\right| \\ &+ \left|1 - \frac{b_{m_0}(t)}{\hat{b}_{m_0}(t)}\right|\left|c(t)\right| + \left|1 - \frac{b_{m_0}(t)}{\hat{b}_{m_0}(t)}\right|\sum_{l=1}^{n_0-1} d_l e^{[n_0-l]}(t) \\ &+ \left|1 - \frac{b_{m_0}(t)}{\hat{b}_{m_0}(t)}\right| y_r^{[n_0]}(t)]\left|\mathrm{S}(t)\right| - \frac{b_{m_0}(t)}{\hat{b}_{m_0}(t)}\zeta\left|S(t)\right|\end{aligned} \tag{A.7}$$

Considering the given constraints in the theorem the following inequality is achieved.

$$\begin{aligned}S(t)\dot{S}(t) &\le \sum_{i=0}^{n_0-1}\begin{bmatrix}\beta & \tilde{a}_i\end{bmatrix}\begin{bmatrix}\bar{a}_i \\ (\beta-1)\end{bmatrix}\bar{\mathrm{y}}_i + \sum_{j=0}^{m_0}\begin{bmatrix}\beta & \tilde{b}_j\end{bmatrix} \\ & \begin{bmatrix}\bar{b}_j \\ (\beta-1)\end{bmatrix}\bar{\mathrm{u}}_j + \begin{bmatrix}\beta & \tilde{c}\end{bmatrix}\begin{bmatrix}\bar{c} \\ (\beta-1)\end{bmatrix} + \beta\bar{y}_{r,n_0} + (\beta-1)\nabla] \\ & \left|S(t)\right| - \beta^{-1}\zeta\left|S(t)\right|\end{aligned} \tag{A.8}$$

$$\begin{aligned}\Delta &= \sum_{i=0}^{n_0-1}\begin{bmatrix}\beta & \tilde{a}_i\end{bmatrix}\begin{bmatrix}\bar{a}_i \\ (\beta-1)\end{bmatrix}\bar{\mathrm{y}}_i + \sum_{j=0}^{m_0}\begin{bmatrix}\beta & \tilde{b}_j\end{bmatrix}\begin{bmatrix}\bar{b}_j \\ (\beta-1)\end{bmatrix} \\ & \bar{\mathrm{u}}_j + \begin{bmatrix}\beta & \tilde{c}\end{bmatrix}\begin{bmatrix}\bar{c} \\ (\beta-1)\end{bmatrix} + \beta\bar{y}_{r,n_0} + (\beta-1)\nabla]\end{aligned}$$

$$\nabla = \bar{y}_{r,n_0} + \sum_{l=1}^{n_0-1} d_l \bar{e}_{n_0-l} \tag{A.9}$$

where $\bar{e}_{n_0-1}$ (l=1,2,...,n_0-1) as upper bounds of $e^{[n_0-l]}(t)$ are defined later. Now, we can rewrite $\dot{V}(t)$ as follows:

$$\dot{V}(t) = \dot{S}(t)S(t) \le \left[\Delta - \beta^{-1}\zeta\right]\left|S(t)\right| \le 0 \tag{A.10}$$

For $\zeta > \beta\Delta$, $\dot{V}(t)<0$ and then $S(t)\to 0$ or $y(t)\to y_r(t)$; however, we should consider that the signal control is affected by ζ too. It means we must check that for the chosen ζ if $|u(t)|\le \bar{u}_j$. For purpose, we compute the supremum values for the error signal, output signal and their derivatives.

Assume there is ζ that the resulting signal control guarantee $\dot{V}(t)<0$ or $|S(t)|\to 0$. Considering $|S(t)|\big|_{t=t_0}=S_0$ and $\dot{V}(t)<0$ for $t<t_0$: $|S(t)|<S_0$.

$$e^{[n_0-1]}(t)\ +d_1e^{[n_0-2]}(t)+d_2e^{[n_0-3]}(t)+\cdots +d_{n_0-2}e(t)=S(t) \tag{A.11}$$

We can rewrite the above linear time invariant differential equation as following ordinal differential equation (ODE) form:

$$\begin{bmatrix}\dot{e}(t)\\ \vdots\\ e^{[n_0-2]}\\ e^{[n_0-1]}\end{bmatrix}\underbrace{\begin{bmatrix}0 & 1 & \cdots & 0 & 0\\ \vdots & \vdots & & \vdots & \vdots\\ 0 & 0 & \ddots & 0 & 1\\ -d_{n_0-2} & -d_{n_0-3} & \ldots & -d_2 & -d_1\end{bmatrix}}_{A_e} \begin{bmatrix}e(t)\\ \vdots\\ e^{[n_0-3]}(t)\\ e^{[n_0-2]}(t)\end{bmatrix}+\underbrace{\begin{bmatrix}0\\ \vdots\\ 0\\ 1\end{bmatrix}}_{B_e}S(t) \tag{A.12}$$

Solving the above ODE, we have:

$$\begin{bmatrix}e(t)\\ \vdots\\ e^{[n_0-3]}(t)\\ e^{[n_0-2]}(t)\end{bmatrix}=\exp\left(\mathrm{A}_e\left(t-t_0\right)\right)\begin{bmatrix}e(t_0)\\ \vdots\\ e^{[n_0-3]}(t_0)\\ e^{[n_0-2]}(t_0)\end{bmatrix} +\int_{\tau=t_0}^{t}\exp\left(\mathrm{A}_e\left(t-\tau\right)\right)B_eS(\tau)d\tau \tag{A.13}$$

We can show easily that:

$$\begin{bmatrix}|e(t)|\\ \vdots\\ |e^{[n_0-3]}(t)|\\ |e^{[n_0-2]}(t)|\end{bmatrix}\le \max(eig\left(\left(\exp\left(\mathrm{A}_e\left(t-t_0\right)\right)\right)\right) \begin{bmatrix}|e(t_0)|\\ \vdots\\ |e^{[n_0-2]}(t_0)|\\ |e^{[n_0-1]}(t_0)|\end{bmatrix}+\int_{\tau=t_0}^{t}\exp\left(\mathrm{A}_e\left(t-\tau\right)\right)\mathrm{B}_e\left|S(\tau)\right|d\tau \tag{A.14}$$

Since the matrix A_e is negative definite (ND), we have:

$$\begin{bmatrix}|e(t)|\\ \vdots\\ |e^{[n_0-3]}(t)|\\ |e^{[n_0-2]}(t)|\end{bmatrix}\le \exp\left(\max\left(eig\left(A_e\right)\right)\left(t-t_0\right)\right) \begin{bmatrix}|e(t_0)|\\ \vdots\\ |e^{[n_0-3]}(t_0)|\\ |e^{[n_0-2]}(t_0)|\end{bmatrix}+S_0\int_{\tau=t_0}^{t}\exp\left(A_e\left(t-\tau\right)\right)B_ed\tau \tag{A.15}$$

We know for the ND matrix A_e, max($eig(A_e)$)→0, hence we have:

$$\begin{bmatrix}|e(t)|\\ \vdots\\ |e^{[n_0-3]}(t)|\\ |e^{[n_0-2]}(t)|\end{bmatrix}\le\begin{bmatrix}|e(t_0)|\\ \vdots\\ |e^{[n_0-3]}(t_0)|\\ |e^{[n_0-2]}(t_0)|\end{bmatrix} +S_0\left[A_e^{-1}\exp\left(A_e\left(t-t_0\right)\right)-A_e^{-1}\right]B_e \tag{A.16}$$

$$\begin{bmatrix}|e(t)|\\ \vdots\\ |e^{[n_0-3]}(t)|\\ |e^{[n_0-2]}(t)|\end{bmatrix}\le\begin{bmatrix}|e(t_0)|\\ \vdots\\ |e^{[n_0-3]}(t_0)|\\ |e^{[n_0-2]}(t_0)|\end{bmatrix}-\mathrm{S}_0\mathrm{A}_e^{-1}\mathrm{B}_e=\begin{bmatrix}\bar{e}\\ \vdots\\ \bar{e}_{n_0-3}\\ \bar{e}_{n_0-2}\end{bmatrix} \tag{A.17}$$

On the other hand, we can compute a supremum for $|e^{[n_0-1]}(t)|$:

$$S(t)=e^{[n_0-1]}(t)\ +d_1e^{[n_0-2]}(t)+d_2e^{[n_0-3]}(t) +\cdots+d_{n_0-1}e(t) \tag{A.18}$$

Since all roots of the characteristic function are in the left part of the complex plane, from Ruth-Horvitz principle we know $d_i>0$. Thus, we have:

$$\left|e^{[n_0-1]}(t_0)\right|<(S_0+d_1\bar{e}_{n_0-2}+d_2\bar{e}_{n_0-3}+..+d_{n_0-1}\bar{e}) =\bar{e}_{n_0-1} \tag{A.19}$$

Since the reference signal is defined by designer, we can compute supremum values for it and its derivatives as follows:

$$\begin{bmatrix}|y_r(t)|\\ \vdots\\ |y_r^{[n_0-1]}(t)|\\ |y_r^{[n_0]}(t)|\end{bmatrix}\le\begin{bmatrix}\bar{y}_r\\ \vdots\\ \bar{y}_{r,n_0-1}\\ \bar{y}_{r,n_0}\end{bmatrix} \tag{A.20}$$

Then, we can compute supremum values for the output signal and its derivatives as follows:

$$\begin{bmatrix} |y(t)| \\ \vdots \\ |y^{[n_0-2]}(t)| \\ |y^{[n_0-1]}(t)| \end{bmatrix} \le \begin{bmatrix} |e(t)|+|y_r(t)| \\ \vdots \\ |e^{[n_0-2]}(t)|+|y_r^{[n_0-2]}(t)| \\ |e^{[n_0-1]}(t)|+|y_r^{[n_0-1]}(t)| \end{bmatrix} \le \begin{bmatrix} |\bar{e}|+|\bar{y}_r| \\ \vdots \\ \bar{e}_{n_0-2}+\bar{y}_{r,n_0-1} \\ \bar{e}_{n_0-1}+\bar{y}_{r,n_0-1} \end{bmatrix} = \begin{bmatrix} \bar{y} \\ \vdots \\ \bar{y}_{n_0-2} \\ \bar{y}_{n_0-1} \end{bmatrix} \quad \text{(A.21)}$$

Now, we can compute the supremum values for the input signal and its derivatives by solving the following linear time-varying differential equation:

$$\sum_{j=0}^{m_0} \hat{b}_j(t) u^{[j]}(t) = z(t) \quad \text{(A.22)}$$

$$z(t) = (-\sum_{i=0}^{n_0-1} \hat{a}_i(t) y^{[i]}(t) - \hat{c}(t) + y_r^{[n_t]}(t) + \sum_{l=1}^{n_0-1} d_l e^{[n_0-l]}(t)) - \zeta sign(S(t)) \quad \text{(A.23)}$$

A time-varying linear ODE representation for the above equation is stated as follows:

$$\begin{bmatrix} \dot{u}(t) \\ \vdots \\ u^{[m_0-1]}(t) \\ u^{[m_0]}(t) \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & & 0 & 1 \\ -\mu_0(t) & -\mu_1(t) & \cdots & -\mu_{m_0-2}(t) & -\mu_{m_0-1}(t) \end{bmatrix}}_{\mathrm{A}_u(t)} \begin{bmatrix} u(t) \\ \vdots \\ u^{[m_0-2]}(t) \\ u^{[m_0-1]}(t) \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}}_{\mathrm{B}_u} \frac{z(t)}{\hat{b}_{m_0}(t)} \quad \text{(A.24)}$$

$$\mu_k(t) = \frac{\hat{b}_k(t)}{\hat{b}_{m_0}(t)};\ k = 0,1,2..,m_0-1 \quad \text{(A.25)}$$

By solving the above equation through state transient matrix $\varphi_{A_u}(t)(t,t_0)$, we have:

$$\begin{bmatrix} u(t) \\ \vdots \\ u^{[m_0-2]}(t) \\ u^{[m_0-1]}(t) \end{bmatrix} = \varphi_{A_u(t)}(t,t_0) \begin{bmatrix} u(t_0) \\ \vdots \\ u^{[m_0-2]}(t_0) \\ u^{[m_0-1]}(t_0) \end{bmatrix} + \int_{\tau=t_0}^{t} \varphi_{A_u(t)}(t,\tau) B_u \frac{z(\tau)}{\hat{b}_{m_0}(\tau)} d\tau \quad \text{(A.26)}$$

$$\varphi_{A_u(t)}(t,t_0) = I + \int_{\tau=t_0}^{t} A_u(\tau) d\tau + \int_{\tau_1=t_0}^{t} A_u(\tau_1) \left[\int_{\tau=t_0}^{\tau_1} A_u(\tau_2) d\tau_2 \right] d\tau_1 + \cdots \quad \text{(A.27)}$$

For the signal $z(t)$ considering $|\hat{a}_i(t)| \le \tilde{a}_i(t)$, $|\hat{b}_j(t)| \le \tilde{b}_j(t)$, $c(t) \le \tilde{c}$, $y_r^{[n_t]}(t) \le \bar{y}_{r,n_0}(t)$ and $e^{[n_0-1]}(t) \le \bar{e}_{n_0-1}$ we have:

$$|z(t)| \le \underbrace{\sum_{i=0}^{n_0-1} \tilde{a}_i \bar{y}_i + \tilde{c} + \bar{y}_{r,n_0}(t) + \sum_{l=1}^{n_0-1} d_l \bar{e}_{n_0-1}}_{\bar{z}} + \zeta = \bar{z} + \zeta \quad \text{(A.28)}$$

Also, the ELM is the minimum phase, the matrix $A_u(t)$ is ND for $t>t_0$. Accordingly, we have:

$$max\left(\left|eig(\varphi_{\mathrm{A}_u(t)}(t,t_0))\right|\right) \le [max\left(eig(\mathrm{I})\right) + \int_{\tau=t_0}^{t} max\left(\left|eig(\mathrm{A}_u(\tau))\right|\right) d\tau + \int_{\tau_1=t_0}^{t} max\left(\left|eig(\mathrm{A}_u(\tau))\right|\right) [\int_{\tau=t_0}^{\tau_1} max\left(\left|eig(\mathrm{A}_u(\tau))\right|\right) d\tau_2] d\tau_1 + \cdots] \quad \text{(A.29)}$$

For $\hat{b}_{m_0}(\tau)$, we assume that $|\hat{b}_{m_0}(\tau)| \ge \breve{b}_{m_0}$. Considering for all time τ max$(|eig(A_u(\tau)|) \le \bar{\sigma}$ and $\bar{\sigma} \le 0$.

$$max\left(\left|eig(\varphi_{\mathrm{A}_u(t)}(t,t_0))\right|\right) \le \exp\left(\int_{\tau=t_0}^{\tau=t} \bar{\sigma} d\tau\right) = exp\left(\bar{\sigma}(t-t_0)\right) \quad \text{(A.30)}$$

$$\begin{bmatrix} |u(t)| \\ \vdots \\ |u^{[m_0-2]}(t)| \\ |u^{[m_0-1]}(t)| \end{bmatrix} \le \hat{b}_{m_0}(\tau) max\left(\left|eig(\varphi_{A_u(t)}(t,t_0))\right|\right) \begin{bmatrix} |u(t_0)| \\ \vdots \\ |u^{[m_0-2]}(t_0)| \\ |u^{[m_0-1]}(t_0)| \end{bmatrix} + \breve{b}_{m_0}^{-1}(\bar{z}+\zeta) \quad \text{(A.31)}$$

$$\int_{\tau=t_0}^{t} max\left(\left|eig\left(\varphi_{A_u(t)}(t,t_0)\right)\right|\right)B_u d\tau$$

$$\begin{bmatrix} |u(t)| \\ \vdots \\ |u^{[m_0-2]}(t)| \\ |u^{[m_0-1]}(t)| \end{bmatrix} \le \exp\left(\bar{\sigma}(t-t_0)\right)\begin{bmatrix} |u(t_0)| \\ \vdots \\ |u^{[m_0-2]}(t_0)| \\ |u^{[m_0-1]}(t_0)| \end{bmatrix}$$

$$+\tilde{b}_{m_0}^{-1}\left(\bar{z}+\zeta\right)\int_{\tau=t_0}^{t}\exp\left(\bar{\sigma}(\tau-t_0)\right)d\tau B_u d\tau \quad \text{(A.32)}$$

$$\begin{bmatrix} |u(t)| \\ \vdots \\ |u^{[m_0-2]}(t)| \\ |u^{[m_0-1]}(t)| \end{bmatrix} \le \begin{bmatrix} |u(t_0)| \\ \vdots \\ |u^{[m_0-2]}(t_0)| \\ |u^{[m_0-1]}(t_0)| \end{bmatrix}\tilde{b}_{m_0}^{-1}\left(\bar{z}+\zeta\right)\bar{\sigma}^{-1}$$

$$\left(1-\exp\left(\bar{\sigma}(t-t_0)\right)\right)B_u$$

$$\begin{bmatrix} |u(t)| \\ \vdots \\ |u^{[m_0-2]}(t)| \\ |u^{[m_0-1]}(t)| \end{bmatrix} \le \begin{bmatrix} |u(t_0)| \\ \vdots \\ |u^{[m_0-2]}(t_0)| \\ |u^{[m_0-1]}(t_0)| \end{bmatrix} + \tilde{b}_{m_0}^{-1}\left(\bar{z}+\zeta\right)\bar{\sigma}^{-1}B_u \quad \text{(A.33)}$$

Since we assume $\zeta \ge \beta\Delta$, there is $\delta \ge 0$ that $\zeta = \beta\Delta + \delta$.

$$\begin{bmatrix} |u(t_0)| \\ \vdots \\ |u^{[m_0-2]}(t_0)| \\ |u^{[m_0-1]}(t_0)| \end{bmatrix} + [\bar{z} + \beta\sum_{i=0}^{n_0-1}(\beta\bar{a}_i + (\beta-1)\tilde{a}_i)\bar{y}_i$$

$$+\beta\sum_{j=0}^{m_0-1}(\beta\bar{b}_j + (\beta-1)\tilde{b}_j)\bar{u}_j + \beta\left(\beta\bar{c} + (\beta-1)\tilde{c}\right) \quad \text{(A.34)}$$

$$+\beta(\beta-1)\nabla + \delta]\tilde{b}_{m_0}^{-1}\bar{\sigma}^{-1}B_u \le \begin{bmatrix} \bar{u} \\ \vdots \\ \bar{u}_{m_{0-2}} \\ \bar{u}_{m_{0-1}} \end{bmatrix}; \ \delta \ge 0$$

Considering:

$$R = (I_{m_0} - \beta\tilde{b}_{m_0}^{-1}\bar{\sigma}^{-1}B_u$$
$$[((\beta\bar{b}_0 + (\beta-1)\tilde{b}_0) \ \cdots \ (\beta\bar{b}_{m_0-1} + (\beta-1)\tilde{b}_{m_0-1}))]) \quad \text{(A.35)}$$

$$H = \begin{bmatrix} |u(t_0)| \\ \vdots \\ |u^{[m_0-2]}(t_0)| \\ |u^{[m_0-1]}(t_0)| \end{bmatrix} + [\bar{z} + \beta\sum_{i=0}^{n_0-1}(\beta\bar{a}_i + (\beta-1)\tilde{a}_i)\bar{y}_i \quad \text{(A.36)}$$
$$+\beta\left(\beta\bar{c} + (\beta-1)\tilde{c}\right) + \beta(\beta-1)\nabla + \delta]\tilde{b}_{m_0}^{-1}\bar{\sigma}^{-1}B_u$$

The condition for the upper bounds of signal control and its derivatives is achieved as follows:

$$R\begin{bmatrix} \bar{u} \\ \vdots \\ \bar{u}_{m_0-2} \\ \bar{u}_{m_0-1} \end{bmatrix} \ge H \quad \text{(A.37)}$$

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