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Generalized Aggregate Uncertainty Measure 2 for Uncertainty Evaluation of a Dezert-Smarandache Theory based Localization Problem

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ABSTRACT: In this paper, Generalized Aggregated Uncertainty measure 2 (GAU2), as a new uncertainty measure, is considered to evaluate uncertainty in a localization problem in which cameras' images are used. The theory that is applied to a hierarchical structure for a decision making to combine cameras' images is Dezert-Smarandache theory. To evaluate decisions, an analysis of uncertainty is executed at every level of the decision-making system. The second generalization of Aggregated Uncertainty measure (GAU2) which is applicable for DSmT results is used as a supervisor. The GAU2 measure in spite of the GAU1 measure can be applied to the problems with vague borders or continuous events. This measure may help to make decisions based on better preference combinations of sensors or methods of fusion. GAU2 is used to evaluate uncertainty after applying classic DSmT and hybrid DSmT with extra knowledge. Therefore by using the decision making system, results with less uncertainty are generated in spite of high conflict sensory data.

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1- Introduction

Information fusion generally deals with the integration of uncertain information from multiple sensors. Usually, physical constraints, detection algorithms, environmental noises, and transmitting channel of sensors cause uncertainties in information [1]. In 1986, the US Department of Defence constituted the Data Fusion Sub-Panel of the Joint Directors of Laboratories (JDL) to address key issues in data fusion and planned the new field in one work to unify the terminology and procedures [2]. Other models have been developed such that sensor fusion algorithms could be applied to variant applications . These models such as Multi-sensor integration fusion model, Thomopoulos architecture, Waterfall model, Behavioural knowledge-based data fusion model, Omnibus data fusion model, and Distributed blackboard data fusion architecture have been reviewed in [2].

There has been considerable researches on sensor fusion methods such as Bayesian approach and Dempster–Shafer theory in the recent years [3]. Other methods based on Shafer's model are also presented in [4-7]. The evidence theory, also known as Dempster–Shafer theory, is one of the most popular frameworks to deal with uncertain information. In the evidence theory, the singletons as in the probability theory have non-null confidence. Several applications of DST are stated in the literature to overcome its limitations. For example, in [8] alternative combination rules have been proposed to resolve the appeared conflicts of evidence.

The Dezert–Smarandache Theory (DSmT) has been suggested by Dezert and Smarandache in the recent years [9-12]. It can be considered as an extension of the classical Dempster–Shafer theory (DST) but with fundamental differences. DSmT formally allows a fusion of any kind of independent sources of information which are represented in terms of belief functions while it mainly focuses on the fusion of high conflict, uncertain and imprecise sources of evidence [10].

It is important to have an uncertainty assessment after sensor fusion for an improved decision making as uncertain information often exists at all levels of the process of information fusion [1]. Hartley [13] and Shannon [14] established the field of information theory and developed information entropy as a measure for redundancy, respectively. According to the approaches, information or preferably uncertainty-based information can be quantified by different common measures commonly called uncertainty measures [15]. In the evidence theory, a body of evidence or equally a belief function hides two types of uncertainty: conflict and non-specificity, which can be considered as ambiguity. In [16], a DS–AHP, Dempster–Shafer theory of evidence with the analytic hierarchy procedure is suggested. The method is proposed for the purpose of decision making using a multi-criteria system, MCDM, which permits extra analysis, including levels of uncertainty and conflict in the decisions made. Also, in [17], a DS–AHP method is suggested for multi-attribute decision making (MADM) problems with incomplete information, solving a problem directly based on its incomplete decision matrix. Also, a DSmT–AHP based multi-criteria decision making is offered in [18]. Several other applications of DSmT in classification problem are reported [19]. The applications and new advances of DSmT for information fusion are collected in [20].

Uncertainty measurement is a significant task to assess the results of the fusion. In previous researches such as in [20-26], various measures of ambiguity which are often called measures of total uncertainty, have been proposed. Among

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them, Maeda, Y. et al. [27] followed by [28] suggested a measure of aggregated uncertainty which is named AU. The measure is defined in the framework of the evidential theory that aggregates nonspecificity and conflict. It has been verified that the measure fulfills the five requirements defined in [15, 28, 29]. Jousselme, A.L. et al. [30] presented a new measure of aggregated uncertainty as another uncertainty measure, which is named AM for Ambiguity Measure which aims to remove the weaknesses of AU such as computing complexity. An alternative to measuring ambiguity in Dempster–Shafer theory is offered by AM. But the suggested measure is not sub-additive in general. By a particular counterexample, Klir and Lewis [31] showed that the used assumption in the last stage of the proof mentioned in [30] is improper and that AM violates sub-additivity indeed.

The AU uncertainty measure and its ass`ociated computing algorithm presented by Harmanec [29], in spite of their efficiencies, are devoted to DST framework and cannot be applied to DSmT. DSmT is a generalization of DST by removing the exhaustivity and the exclusivity conditions of events. Using a natural definition of defining entropy, this measure has already been introduced in specific by Dezert and Smarandache in their studies on the Dezert–Smarandache Theory (DSmT) which is called pignistic entropy [10, 11, 12, 25, 26]. DSmT is used in [32] to fuse fingerprint information. In [30] based on the pignistic probability (BetP) and likelihood ratio test, a decision is made to accept or reject attained results and a contextual unification framework is proposed to dynamically select the most suitable evidence-theoretic fusion algorithm for an assumed scenario. While measures similar to pignistic entropy and some uncertainty measures cited in [34] are generally used, they do not satisfy the needed axiomatic Klir and Wierman's requirements [15] for an uncertainty measure and sub-additivity. Consequently, it can be indicated that there is not any measure such as AU to evaluate uncertainty in the DSmT framework in the literature.

The AU measure has been generalized by the authors of this paper in two forms that are named GAU1 and GAU2 to deal with uncertainty measure in the DSmT framework in [36]. It has been shown that the new measures have a sufficient efficiency to assess the DSmT-based fused results. In this article, a study of uncertainty to localize an object on a plane, as a simple interpretation of more applicable examples, e.g. localization of cars in parking lots or highways, is considered. To this end, three cameras are used in different positions. Information of the occupied spaces on a plane by an object in views of the cameras is used within the ongoing sensor fusion frameworks. Dezert-Smarandache theory in classic and hybrid forms are applied as the fusion algorithm to deal with the problem using free and hybrid models, respectively. In [37], the results of the localization problem are assessed using Generalized Aggregated Uncertainty measure 1 (GAU1) that cannot be practical in the problems with unclear borders or continuous events. To assess the fused results from an uncertainty point of view, the second generalization of Aggregated Uncertainty (AU) measure named Generalized Aggregated Uncertainty 2 (GAU2) is used for the results. The uncertainty analysis by GAU2 measure is completed in a hierarchical decision-making system to compare the results of sensor fusion. The measure has formerly been studied by the authors for a different case study on a fusion of ultrasonic data in a target differentiation in [38] which two sensors have been used. In this paper, three sensors are used for the localization problem which yields a big conflict between sensors' data.

The remaining of this article is organized as follows. The Dezert-Smarandache theory is introduced in section 2 in both classic and hybrid forms. A brief discussion on the uncertainty measure AU and the present GAU2 measure is given in section 3. Section 4 is devoted to a brief overview of the experimental setup. In section 5, fusion results of cameras' images are offered and but this measures are discussed in a uncertainty point of view. Eventually, concluding remarks are presented in section 6.

2- Dezert-Smarandache Theory

2- 1- Foundation of DSmT

Dezert–Smarandache Theory is a theory of plausible and paradoxical reasoning [10-12]. The expansion of DSmT arises from the necessity to overcome the intrinsic limitations of Dempster–Shafer Theory [3] which are strictly related to the acceptance of Shafer's model for the under-consideration fusion problem.

This means that the frame of discernment $\Theta=\{\theta_1,\theta_2,\dots,\theta_n\}$ is implicitly defined as a finite set of exhaustive and exclusive hypotheses. The basis of DSmT is based on the definition of the Dedekind's lattice D^Θ which is additionally called hyper-power set of the frame Θ in the consequence. The hyper-power set D^Θ is defined as the set of all composite propositions made from elements of Θ with $\cup$ and $\cap$ operators such that:

1. $\emptyset, \theta_1, \theta_2,\dots,\theta_n \in D^\Theta$
2. if $A,B \in D^\Theta$ then $A\cup B\in D^\Theta$,$A\cap B\in D^\Theta$
3. No further elements belong to D^Θ except those that satisfy rule 1 or rule 2.

From a common frame Θ, a map associated with a given body of evidence is defined as:

$$m(.): D^\Theta \rightarrow [0,1] \quad m(\emptyset)=0, \quad \sum_{A\in D^\Theta} m(A)=1,\ 0\le m(A)\le 1 \tag{1}$$

The quantity m(A) is called the generalized basic belief assignment/mass (gbba) of A.

Shafer's model in DST is denoted by $M^0(\Theta)$. In DSmT, free DSm model is denoted by $M^f(\Theta)$, and hybrid DSm model is denoted by $M(\Theta)$ [10-12]. Almost in the same way as in the DST, the generalized belief and plausibility functions are defined, i.e.

$$Bel(A)=\sum_{B\in D^\Theta, B\subseteq A} m(B) \tag{2}$$

$$Pl(A)=\sum_{B\in D^\Theta, B\cap A\neq\emptyset} m(B) \tag{3}$$

2- 2- Classic DSmT Rule of Combination

The classic DSm rule of combination $m_{M^{f(\Theta)}}(.)=m(.)=[m_1\oplus m_2](.)$ of two independent sources of evidences over the same frame with belief functions $Bel_1(.)$, $Bel_2(.)$ associated with gbba $m_1(.)$, $m_2(.)$ corresponds to the conjunctive consensus of the sources. It is given by:

$$\forall C\in D^\Theta \quad m^f(C)=m(C)=\sum_{\substack{A,B\in D^\Theta \\ A\cap B=C}} m_1(A)m_2(B) \tag{4}$$

Since D^{Θ} is closed under ∪ and ∩ set operators, this new rule of combination guarantees that m(.) is a proper generalized belief assignment, i.e. $m(.):D^{\Theta}\longrightarrow[0,1]$.

2- 3- Hybrid DSmT Rule of Combination

To remove the degenerate vacuous fusion problem from the Classic DSmT rule of combination, the assumed hybrid DSm model M under consideration is always different from the vacuous model M; $I_t\neq\emptyset$ where $I_t\triangleq\theta_1\cup\theta_2\cup...\cup\theta_n$ is the total ignorance. The hybrid DSm rule of combination is defined in association with an assumed hybrid DSm model $M\neq M_{\emptyset}$; the hybrid rule of combination for two sources for all $A\in D\Theta$

$$m_{M(\Theta)}(A)=\phi(A)\left[S_1(A)+S_2(A)+S_3(A)\right] \quad (5)$$

$$S_1(A)=\sum_{\substack{X_1,X_2\in D^{\Theta}\\ X_1\cap X_2=A}} m_1(X_1).m_2(X_2) \quad (6)$$

$$S_2(A)=\sum_{\substack{X_1,X_2\in\emptyset\\ [u(X_1)\cup u(X_2)=A]\vee\\ [(u(X_1)\cup u(X_2))\in\emptyset\wedge(A=I_t)]}} m_1(X_1).m_2(X_2) \quad (7)$$

$$S_3(A)=\sum_{\substack{X_1,X_2\in D^{\Theta}\\ X_1\cup X_2=A\\ X_1\cap X_2=\emptyset}} m_1(X_1).m_2(X_2) \quad (8)$$

is as:

where $\phi(A)$ is the characteristic non-emptiness function of a set A i.e. $\phi(A)=1$ if $A\notin\{\emptyset_M,\emptyset\}$ and $\phi(A)=0$ otherwise. $\emptyset_M$ is the set of all elements of D^{Θ} which have been forced to be empty by the constraint of the model M and $\emptyset$ is the classical empty set. Also $U\triangleq u(X_1)\cup u(X_2)$. u(X) is the union of all singletons θ_i that compose X. For example, if $X=\theta_1\cap\theta_2$ or $X=\theta_1\cup\theta_2$ then $u(X)=\theta_1\cup\theta_2$.

3- Uncertainty Measurement

To evaluate the results of the fused data in uncertainty point of view, a suitable uncertainty measure is needed that can be applied to DSmT-based fused results. In this section, the aggregated uncertainty (AU) measure developed for DST framework by [29] is studied. Then, a generalization of AU is introduced in order to be practical to the results of DSmT-based fusion problem.

3- 1- Aggregated Uncertainty Measure

Although the objective of data fusion is to reduce the global uncertainties, in [29] the concept of comprehensive uncertainty measurement in the DST framework has been explored.

Definition 3.1. The measure of the Aggregated Uncertainty

$$AU(Bel)=max\left\{-\sum_{\theta\in\Theta}p_{\theta}\log_2 p_{\theta}\right\} \quad (9)$$

contained in Bel, indicated as AU(Bel), is defined by:

where the maximum is taken over all $\{p_{\theta}\}_{\theta\in\Theta}$ such that $p_{\theta}\in[0,1]$ for $\theta\in\Theta$, $\sum_{\theta\in\Theta}p_{\theta}=1$ and for all $A\subseteq\Theta$, $Bel(A)\leq\sum_{x\in A}p_x$.

While AU technique is not an efficient algorithm, it is proved in [29] that it satisfies all the properties for a reasonable uncertainty measurement, especially the sub-additivity and the additivity. The algorithm of computing Aggregated Uncertainty was originated in [29]. Under the suggested algorithm, the input is treated in the form of a frame of discernment Θ, with a belief function Bel on Θ. The algorithm is:

Input: a frame of discernment Θ, a belief function Bel on Θ

Output: AU(Bel), $\{p_{\theta}\}_{\theta\in\Theta}$ such that $AU(Bel)=-\sum_{\theta\in\Theta}p_{\theta}\log_2 p_{\theta}$, $0\leq p_{\theta}\leq1$, $\sum_{\theta\in\Theta}p_{\theta}=1$ and $Bel(A)\leq\sum_{x\in A}p_x$ for all $\emptyset\neq A\subseteq\Theta$

Line 1) begin

Line 2) Y = Θ, Bel` = Bel

Line 3) while $Y\neq\emptyset$ and Bel` (Y) > 0 do

Line 4) find a nonempty set $A\subseteq\Theta$ such that Bel(A) / |A| is maximal if there are more such sets A than one , take the one with maximal cardinality endif

Line 5) for each $x\in A$, do p_x=Bel`(A) / |A| end for

Line 6) for each $B\subseteq Y-A$, do Bel`(B)=Bel`(B∪A)-Bel`(A) endfor

Line 7) Y = Y−A

Line 8) endwhile

Line 9) if Bel`(Y)=0 and $Y\neq\emptyset$

Line 10) then for all $x\in Y$ do $p_x=0$ endfor

Line 11) endif

Line 12) $AU(Bel)=-\sum_{\theta\in\Theta}p_{\theta}\log_2 p_{\theta}$

Line 13) end

As it is obvious, the algorithm is applied for DST framework whereas it cannot be directly applied for DSmT. The reason is behind the line 6 of the algorithm of computing AU and mainly in the key difference of DST and DSmT. As mentioned previously, DSmT overcomes the limitation of DST in the Shafer's model. The frame of discernment of the fusion problem under consideration in the DST assumed to have exhaustive and exclusive elementary hypotheses but these conditions are violated in DSmT. Observing line 6 of the algorithm of computing AU measure concludes that at least one part of the information determined by A∩B will be missed if someone wants to use this algorithm to compute uncertainty in DSmT. Consequently, measuring of uncertainty would not be correct. Therefore, a generalized form of the AU measure, called Generalized Aggregated Uncertainty 2 (GAU2) measure and its associated algorithm are presented that can be used for DSmT-based fused results.

3- 2- Generalized Aggregated Uncertainty Measure 2 (GAU2)

In order to clarify the motivation of Generalized Aggregate Uncertainty 2, consider a set of exclusive events such as {A,B}. It is easy to see that A=A-B because of exclusivity of the events. Now consider the same set with nonexclusive events. In this case, it is concluded that A=A-B+(A∩B) because of intersection between events A and B. In the GAU2 measure, probability distribution assignments which are used in GAU2 are computed for nonexclusive events and all of their intersections. Consequently, a new set should be defined. Consider the set of n nonexclusive events $\Theta=\{\theta_1,\theta_2,\ldots,\theta_n\}$ or $\Theta=\{\theta|\theta=\theta_i,i=1,2,\ldots,n\}$. GAU2 is defined based on a class of probability distribution of events of a set such as Θ_P while the

$$\Theta_P=\left\{\theta_P\,|\,\theta_P=\theta_{Pj},\ j=1,2,\ldots,n_{\Theta_P}\right\}=$$

$$\left\{\theta_P\,|\,\theta_P=\theta_{Pj}=\bigcap_{\substack{i\in I\\ \emptyset\neq I\subseteq\{1,2,\ldots,n\}}}\theta_i\right\}$$

entropy of Shannon is maximized. Θ_P is equivalent to:
where I is any nonempty subset of the set {1,2,…,n}. Θ_P has the simple form:

$$\Theta_P = \Big\{ \{\theta_i\}_{i=1,2,\dots,n}, \{\theta_i \cap \theta_j\}_{\substack{i,j=1,2,\dots,n \\ i\neq j}}, \{\theta_i \cap \theta_j \cap \theta_k\}_{\substack{i,j,k=1,2,\dots,n \\ i\neq j\neq k}}, \dots \Big\}$$

This new set contains all the nonexclusive events in Θ and their intersections. The cardinality of Θ_P is defined as

$$|\Theta_P| = n_{\Theta_P} = \sum_{i=1}^{n} \binom{n}{i}$$

Definition 3.2. The measure of the Generalized Aggregated Uncertainty 2 contained in Bel, which is indicated as GAU2(Bel), is defined by:

$$GAU2(Bel) = max_{\{p_{\theta_P}\}} \left\{ -\sum_{\theta_P \in \Theta_P} p_{\theta_P} \log_2 p_{\theta_P} \right\} \quad (10)$$

p_{θ_P} is the associated probability distribution assignment of each event of Θ_P and the maximum is taken over all $\{p_{\theta_P}\}_{\theta_P \in \Theta_P}$ such that for all $\theta_P \in \Theta_P$, $0 \le p_{\theta_P} \le 1$,

$$\sum_{\theta_P \in \Theta_P} (-1)^{\alpha(\theta_P)+1} p_{\theta_P} = 1,$$

$$\alpha\left(\theta_P = \bigcap_{i\in I} \theta_i\right) = |I|, \ for\ any\ I \subseteq \{1,2,\dots,n\},\ I \neq \emptyset$$

$$Bel(A) \le \sum_{\theta_P \in A} p_{\theta_P} for\ all\ \emptyset \neq A \subseteq \Theta_P$$

The exceeding conditions endorse that p_{θ_P} is a probability distribution. For instance, consider the previous set with nonexclusive events {A,B}. The set of probability distribution assignments Θ_P which are defined for this problem is $\{p_A, p_B, p_{A\cap B}\}$ that must satisfy the probability condition $p_A + p_B - p_{A\cap B} = 1$.
The generalized algorithm to compute the GAU2 measure is:
Input: a frame of discernment Θ (with n nonexclusive events), a generalized belief function Bel on Θ
Output: GAU2(Bel), $\{p_{\theta_P}\}_{\theta_P \in \Theta_P}$ such that:

$$GAU2(Bel) = -\sum_{\theta_P \in \Theta_P} p_{\theta_P} \log_2 p_{\theta_P}$$

$$\Theta_P = \{\theta_P \mid \theta_P = \theta_{Pj},\ j = 1,2,\dots,n_{\Theta_P}\} = \{\theta_{P1}, \theta_{P2}, \dots, \theta_{Pn_{\Theta_P}}\}$$

$$p_{\theta_P} = p_{\theta_{Pj}} \Big|_{\theta_P = \theta_{Pj},\ j=1,2,\dots,n_{\Theta_P}},$$

$$\sum_{\theta_P \in \Theta_P} (-1)^{\alpha(\theta_P)+1} . p_{\theta_P} = 1 \quad , \quad 0 \le p_{\theta_P} \le 1$$

$$\alpha\left(\theta_P = \bigcap_{i\in I} \theta_i\right) = |I| \left(\emptyset \neq I \subseteq \{1,2,\dots,n\}\right)$$

$$Bel(A) \le \sum_{\theta_P \in A} p_{\theta_P} \quad for \quad all\ \emptyset \neq A \subseteq \Theta_P$$

Line 1) begin
Line 2) make

$$\Theta_P = \{\theta_P \mid \theta_P = \theta_{Pj},\ j = 1,2,\dots,n_{\Theta_P}\} = \left\{ \theta_P \mid \theta_P = \theta_{Pj} = \bigcap_{\substack{i\in I \\ \emptyset \neq I \subseteq \{1,2,\dots,n\}}} \theta_i \right\}$$

Line 3) Y=Θ_P, Bel\`=Bel
Line 4) while Y≠∅ and Bel\`(Y)>0 do
Line 5) find a nonempty set A⊆Θ_P such that Bel(A)∕|A| is maximal if there are more such sets A than one, take the one with maximal cardinality endif
Line 6) for each θ_P∈A do p_{θ_P}= Bel\`(A)∕|A| endfor
Line 7) for each B⊆(Y−A)∪(Y∩A) do
Bel\`(B)=Bel\`(B∪A) − Bel\`(A)+ Bel\`(B∩A) endfor
Line 8) Y=(Y−A)∪(Y∩A)
Line 9) end while
Line 10) if Bel\`(Y)=0 and Y≠∅ then
Line 11) for all θ_P∈Y do p_{θ_P} =0 endfor
Line 12) endif
Line 13) GAU2(Bel) = $-\sum_{\theta_P \in \Theta_P} p_{\theta_P} \log_2 p_{\theta_P}$
Line 14) end

It can be seen that the differences between the above algorithm and the main AU measure algorithm are:

- replacing the set of non-exclusive events Θ by the new set Θ_P
- the condition imposed to Θ_P in the Definition 3.2
- By comparison line 7 with line 6 of the AU measure algorithm, one may find that the term Y∩A is added (also Bel' (B∩A) is added in the next line). The term is added to compensate extra deletion of the term Y∩A in Y=Y-A
- By comparison line 8 with line 7 of the AU measure algorithm, one may find that the term Y∩A is added. This term is added to compensate extra deletion of the term Y∩A in Y=Y-A

In GAU2, clearness of the borders of events in the frame of discernment is not necessary. Hence GAU2 is an appropriate uncertainty measure for continuous frameworks.

4- Experimental Setup: Object Localization Using Cameras' Images

In this study, three cameras in three different positions pointing the plane are applied. The first camera is located at an angle of 60 degrees to the plane. The second camera is located on the other side with an angle of 30 degrees to the plane. The third camera is implemented on the top of the plane vertically. The architecture works at the decision level to find the location of an object on a plane using predefined regions.
Uncertainties are usually involved in the transform of cameras' information from 3-D space into 2-D space. It is necessary to find out the object's position in the projected plane and then estimate the position on the plane, in order to estimate the real object position. Decisions from cameras can be generated by applying a perspective-based basic belief assignment function. This basic belief assignment function represents uncertainty derived from cameras perspective locating object on the plane. It is possible to apply projective transform in order to estimate objects positions on the ground plane for surveillance tasks where objects positions have to

be given according to the ground plane, however, this process might carry errors from perspective [39].

The calculation of the area of the projected image of the object in the predefined zone in 2-D space is an alternative technique to assign basic belief functions. This value will be an accurate basic belief assignment (bba) function or a generalized bba with respect to the area of the total projected image of that object. Some pre-processes, e.g. edge detection, noise elimination and partitioning of the plane to distinct areas, must be performed on the captured images before determining the bba. Images are transformed to 2-D space at first and the projected area of the object for each zone is calculated after edge detection and deciding on margins of zones. Then, the ratio of the occupied space of an area to the overall occupied space by the object can be assigned as gbba. This process is accomplished for all cameras' images.

A 2-D Gaussian distribution for the localization of the object in each area is considered because of the existence of the noise and uncertainty in cameras' images. The mean values of these Gaussian distributions are equal to the center of each area. Accordingly, the maximum of the distributions is assigned to the center of each area. Lastly, decision fusion is used to combine the results, i.e. these distributions, to make the final decision.

Consider Figure 1. Assume that the object is located in area 2. The object's position is determined in areas 1 and 2 according to the projected image of Camera 1 in 2-D plane, and it is determined in areas 2 and 3 according to the projected image of Camera 2 in 2-D plane. In this simple position, cameras make conflicting decisions. Hence, a suitable choice is to combine the results by a theory of fusion that can deal with such high conflict problems. DSmT in classic and hybrid forms to carry out this task is applied in the succeeding section.

5- Data Fusion Results

5- 1- Assignment of mass function to areas

At first, a value is assigned to the occupied space of each area by the object as gbba in a Gaussian distribution form. Figures 1 to 3 represent these assigned distributions.

Associated distribution of Camera 1 in areas 1 and 2 shows that this camera has located the object in areas 1 and 2 as it is clear in Figure 1. Figure 2 illustrates localization results by Camera 2. Areas 2 and 3 are decided as locations of the object in this case. The result of Camera 3 is shown in Figure 3. Camera 3 yields the results with less uncertainty. Numerical results of the assigned mass functions to each area by Cameras 1, 2 and 3 are listed in Table 1. Clearly, there are values for a total ignorance in each camera's results in the last row of Table 1 because of noise and cameras' uncertainty and some calculation approximations.

5- 2- Results of data fusion; classic DSmT

DSmT in classic form is used to fuse all possible pairs of sensors according to (4). Figures 4 to 7 illustrate the results of fusion results of Cameras 1&2, Cameras 1&3, Cameras 2&3 and Cameras 1&2&3, respectively. Table 2 presents the fusion results for the hyper-power set D^{Θ} with 19 possible events. In the tables: $m_{ij}^{f}(A)=m_i \oplus m_j$;i,j=1,2,3. The final decision in locating the object is area 2 in all four cases. Furthermore, total ignorance (i.e. $\theta_1 \cup \theta_2 \cup \theta_3$)

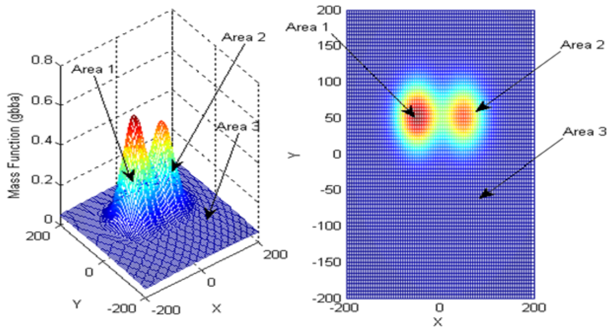

Fig. 1. Results of localization of the object by camera1

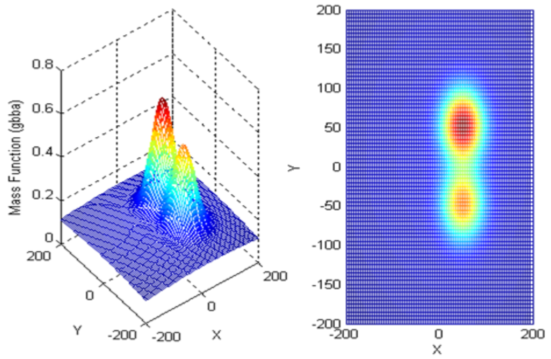

Fig. 2. Results of localization of the object by camera2

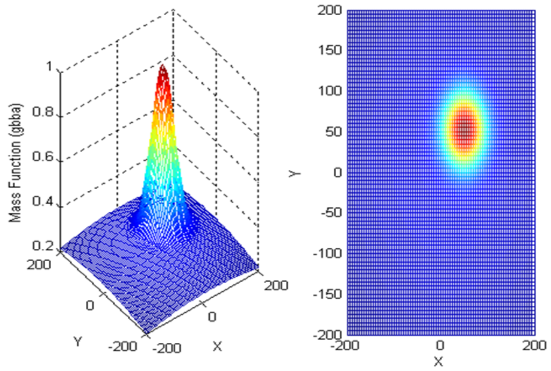

Fig. 3. Results of localization of the object by camera3

Table 1. Associated mass functions of cameras 1, 2, 3

Events (A)	m_1 (A)	m_2 (A)	m_3 (A)
$\emptyset$	0	0	0
θ_1	0.4841	0	0
θ_2	0.3963	0.4656	0.7166
θ_3	0	0.3408	0
$\theta_1 \cap \theta_2$	0.0541	0	0
$\theta_1 \cap \theta_3$	0	0	0
$\theta_2 \cap \theta_3$	0	0.0479	0
$\theta_1 \cap \theta_2 \cap \theta_3$	0	0	0
$\theta_1 \cup \theta_2 \cup \theta_3$	0.0654	0.1457	0.2834

is reduced in the fusion processes. These results confirm the effectiveness of DSmT to deal with problems with a high conflict. For well decision making, the assessment of the DSmT-based results in uncertainty point of view that could be done by a suitable uncertainty measure like GAU2 measure.

5- 3- Results of data fusion; hybrid DSmT

Supplementary knowledge may help with making meaningful decisions. As can be assumed that area 1 and area 3 have no shared area, DSmT can be used in the hybrid form. In this case: $\theta_1 \cap \theta_3 \equiv \emptyset$ and therefore $\theta_1 \cap \theta_2 \cap \theta_3 \equiv \emptyset$. Thus, hybrid DSmT can be used for this problem according to (5) to (8). Table 3 presents results of hybrid DSmT-based fusion of cameras' images for Cameras 1&2. The condition $\theta_1 \cap \theta_3 \equiv \emptyset$ has converted some events to simpler events such as $(\theta_1 \cap \theta_3) \cup \theta_2 \equiv \theta_2$ which are illustrated in Table 4. The number of events is reduced accordingly. If the associated values to these events are nonzero, they should be added to their equivalent events.

In the problem, all these simplified events have zero mass functions. Hence, $m_{12}(A)$ has not changed but the number of events is reduced. Hybrid DSmT transfers the sum of relatively empty sets to the nonempty sets which cause more precise decisions.

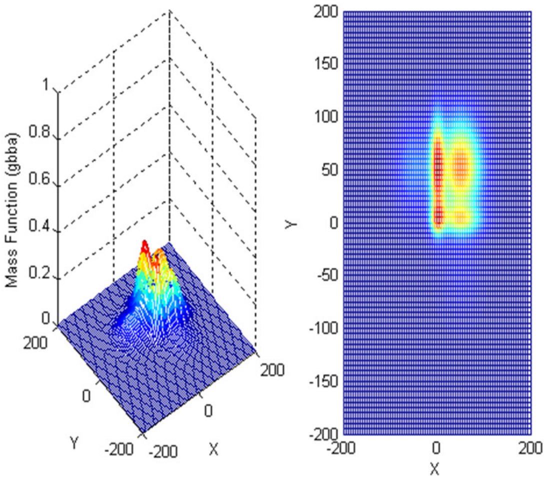

Fig. 4. Results of localization after fusing cameras 1&2

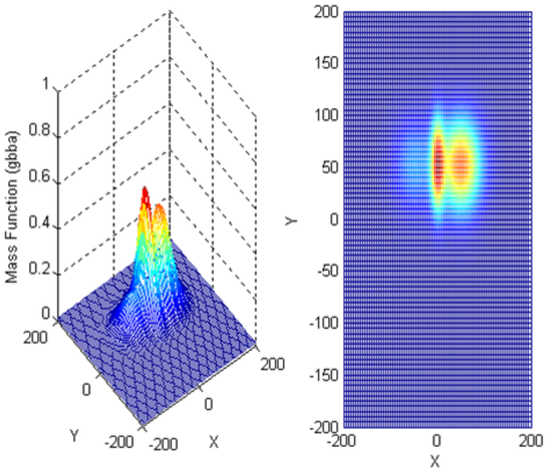

Fig. 5. Results of localization after fusing cameras 1&3

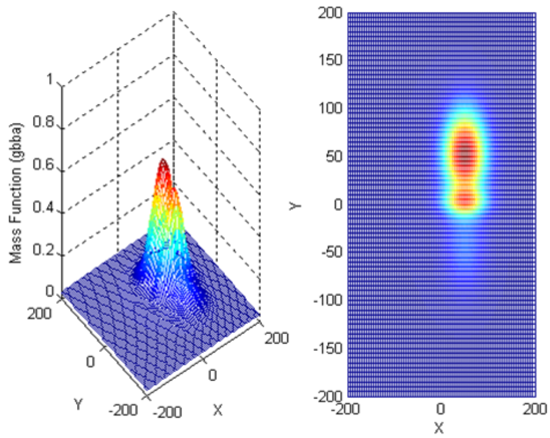

Fig. 6. Results of localization after fusing cameras 2&3

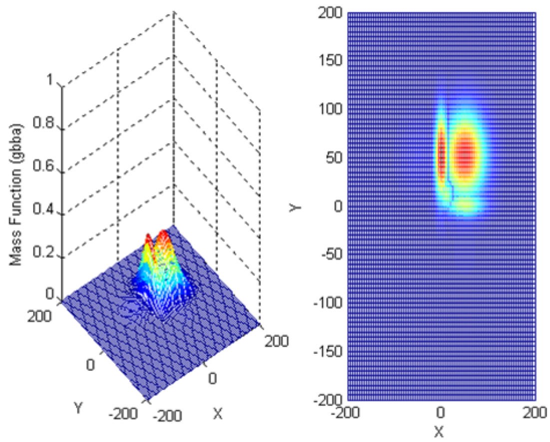

Fig. 7. Results of localization after fusing cameras 1&2&3

Table 2. Results of data fusion; Classic DSmT

Events (A)	$m_{12}^f(A)$	$m_{13}^f(A)$	$m_{23}^f(A)$	$m_{123}^f(A)$
$\emptyset$	0	0	0	0
θ_1	0.0705	0.1372	0	0.0200
θ_2	0.2727	0.4432	0.5700	0.2796
θ_3	0.0223	0	0.0966	0.0063
$\theta_1 \cap \theta_2$	0.2585	0.4010	0	0.3090
$\theta_1 \cap \theta_3$	0.1650	0	0	0.0467
$\theta_2 \cap \theta_3$	0.1572	0	0.2922	0.1732
$\theta_1 \cap \theta_2 \cap \theta_3$	0.0442	0	0	0.1625
$\theta_1 \cup \theta_2$	0	0	0	0
$\theta_1 \cup \theta_3$	0	0	0	0
$\theta_2 \cup \theta_3$	0	0	0	0
$\theta_1 \cup \theta_2 \cup \theta_3$	0.0095	0.0185	0.0413	0.0027
$(\theta_1 \cap \theta_2) \cup \theta_3$	0	0	0	0
$(\theta_1 \cap \theta_3) \cup \theta_2$	0	0	0	0
$(\theta_2 \cap \theta_3) \cup \theta_1$	0	0	0	0
$(\theta_1 \cup \theta_2) \cap \theta_3$	0	0	0	0
$(\theta_1 \cup \theta_3) \cap \theta_2$	0	0	0	0
$(\theta_2 \cup \theta_3) \cap \theta_1$	0	0	0	0
$(\theta_1 \cap \theta_2) \cup (\theta_1 \cap \theta_3) \cup (\theta_2 \cap \theta_3)$	0	0	0	0

Table 3. Results of data fusion; Hybrid DSmT $\theta_1\cap\theta_3\equiv\emptyset$

Events (A)	ϕ(A)	S_1 (A)	S_2 (A)	S_3 (A)	m_{12} (A)
$\emptyset$	0	0	0	0	0
θ_1	1	0.0705	0	0	0.0705
θ_2	1	0.2727	0	0	0.2727
θ_3	1	0.0223	0	0	0.0223
$\theta_1\cap\theta_2$	1	0.2585	0	0	0.2585
$\theta_1\cap\theta_3$	0	0.1650	0	0	0
$\theta_2\cap\theta_3$	1	0.1572	0	0	0.1572
$\theta_1\cap\theta_2\cap\theta_3$	0	0.0442	0	0	0
$\theta_1\cup\theta_2$	1	0	0	0	0
$\theta_1\cup\theta_3$	1	0	0	0.1650	0.1650
$\theta_2\cup\theta_3$	1	0	0	0	0
$\theta_1\cup\theta_2\cup\theta_3$	1	0.0095	0	0	0.0095
$(\theta_1\cap\theta_2)\cup\theta_3$	1	0	0	0.0185	0.0185
$(\theta_1\cap\theta_3)\cup\theta_2$	1	0	0	0	0
$(\theta_2\cap\theta_3)\cup\theta_1$	1	0	0	0.0232	0.0232
$(\theta_1\cup\theta_2)\cap\theta_3$	1	0	0	0	0
$(\theta_1\cup\theta_3)\cap\theta_2$	1	0	0	0.0026	0.0026
$(\theta_2\cup\theta_3)\cap\theta_1$	1	0	0	0	0
$(\theta_1\cap\theta_2)\cup(\theta_1\cap\theta_3)\cup(\theta_2\cap\theta_3)$	1	0	0	0	0

Table 4. Result of data fusion; Hybrid DSmT- reduced events

Events (A)	m_{12} (A)	m_{13} (A)	m_{23} (A)	m_{123} (A)
$\emptyset$	0	0	0	0
θ_1	0.0705	0.1372	0	0.0200
θ_2	0.2727	0.4432	0.5700	0.2815
θ_3	0.0223	0	0.0966	0.0063
$\theta_1\cap\theta_2$	0.2585	0.4010	0	0.3090
$\theta_1\cap\theta_3\equiv\emptyset$	-	-	-	-
$\theta_2\cap\theta_3$	0.1804	0	0.2922	0.1732
$\theta_1\cap\theta_2\cap\theta_3\equiv\emptyset$	-	-	-	-
$\theta_1\cup\theta_2$	0.0026	0	0	0.0166
$\theta_1\cup\theta_3$	0.1650	0	0	0
$\theta_2\cup\theta_3$	0	0	0	0.0132
$\theta_1\cup\theta_2\cup\theta_3$	0.0095	0.0185	0.0413	0.1802
$(\theta_1\cap\theta_2)\cup\theta_3$	0.0185	0	0	0
$(\theta_1\cap\theta_3)\cup\theta_2\equiv\theta_2$	-	-	-	-
$(\theta_2\cap\theta_3)\cup\theta_1$	0.0232	0	0	0
$(\theta_1\cup\theta_2)\cap\theta_3\equiv\theta_2\cap\theta_3$	-	-	-	-
$(\theta_1\cup\theta_3)\cap\theta_2$	0.0026	0	0	0
$(\theta_2\cup\theta_3)\cap\theta_1\equiv\theta_1\cup\theta_2$	-	-	-	-
$(\theta_1\cap\theta_2)\cup(\theta_1\cap\theta_3)\cup(\theta_2\cap\theta_3)\equiv\theta_2\cap(\theta_1\cup\theta_3)$	-	-	-	-

Table 5. 1st step of computing GAU2 measure for results of m_{12}^f(A); Classic DSmT

D^Θ	m_{12}^f (A)	Bel(A)	Bel(A)/|A|
θ_1	0.0705	0.5382	0.5382
θ_2	0.2727	0.7326	0.7326
θ_3	0.0223	0.3887	0.3887
$\theta_1\cap\theta_2$	0.2585	0.3027	0.3027
$\theta_1\cap\theta_3$	0.1650	0.2092	0.2092
$\theta_2\cap\theta_3$	0.1572	0.2014	0.2014
$\theta_1\cap\theta_2\cap\theta_3$	0.0442	0.0442	0.0442
$\theta_1\cup\theta_2$	0	0.9681	0.4841
$\theta_1\cup\theta_3$	0	0.7177	0.3589
$\theta_2\cup\theta_3$	0	0.9199	0.4600
$\theta_1\cup\theta_2\cup\theta_3$	0.0095	1	0.3333
$(\theta_1\cap\theta_2)\cup\theta_3$	0	0.6472	0.3236
$(\theta_1\cap\theta_3)\cup\theta_2$	0	0.8976	0.4488
$(\theta_2\cap\theta_3)\cup\theta_1$	0	0.5382	0.2691
$(\theta_1\cup\theta_2)\cap\theta_3$	0	0.3664	0.1832
$(\theta_1\cup\theta_3)\cap\theta_2$	0	0.4599	0.2299
$(\theta_2\cup\theta_3)\cap\theta_1$	0	0.4677	0.2339
$(\theta_1\cap\theta_2)\cup(\theta_1\cap\theta_3)\cup(\theta_2\cap\theta_3)$	0	0.6249	0.2083

Table 6. 1st step of computing GAU2 measure for results of m_{12}(A); Classic DSmT

D^Θ	m_{12} (A)	Bel(A)	Bel(A)/|A|
θ_1	0.0705	0.3290	0.3290
θ_2	0.2727	0.7142	0.7142
θ_3	0.0223	0.2027	0.2027
$\theta_1\cap\theta_2$	0.2585	0.2585	0.2585
$\theta_1\cap\theta_3\equiv\emptyset$	-	-	-
$\theta_2\cap\theta_3$	0.1804	0.1804	0.1804
$\theta_1\cap\theta_2\cap\theta_3\equiv\emptyset$	-	-	-
$\theta_1\cup\theta_2$	0.0026	0.8105	0.4053
$\theta_1\cup\theta_3$	0.1650	0.7410	0.3705
$\theta_2\cup\theta_3$	0	0.7550	0.3775
$\theta_1\cup\theta_2\cup\theta_3$	0.0095	1	0.3333
$(\theta_1\cap\theta_2)\cup\theta_3$	0.0185	0.4823	0.2412
$(\theta_1\cap\theta_3)\cup\theta_2\equiv\theta_2$	-	-	-
$(\theta_2\cap\theta_3)\cup\theta_1$	0.0232	0.5352	0.2676
$(\theta_0\cup\theta_2)\cap\theta_3\equiv\theta_2\cap\theta_3$	-	-	-
$(\theta_1\cup\theta_3)\cap\theta_2$	0.0026	0.4415	0.2208
$(\theta_2\cup\theta_3)\cap\theta_1\equiv\theta_1\cup\theta_2$	-	-	-
$(\theta_1\cap\theta_2)\cup(\theta_1\cap\theta_3)\cup(\theta_2\cap\theta_3)\equiv\theta_2\cap(\theta_1\cup\theta_3)$	-	-	-

Table 7. 2nd step of computing GAU2 measure for results of $m_{12}(A)$; Hybrid DSmT

$D^{\Theta'}$	Bel`(A)	Bel`(A)/|A|
θ_1	0.3548	0.3548
θ_3	0.2212	0.2212
$\theta_1\cap\theta_2$	0.2585	0.2585
$\theta_2\cap\theta_3$	0.1804	0.1804
$\theta_1\cup\theta_3$	0.7273	0.3637
$(\theta_1\cap\theta_2)\cup\theta_3$	0.4823	0.2412
$(\theta_2\cap\theta_3)\cup\theta_1$	0.5378	0.2689
$(\theta_1\cup\theta_3)\cap\theta_2$	0.4415	0.2208

Table 8. 3rd step of computing GAU2 measure for the results of $m_{12}(A)$; Hybrid DSmT

$D^{\Theta'}$	Bel`(A)	Bel(A)/|A|
$\theta_1\cap\theta_2$	0.2585	0.2585
$\theta_2\cap\theta_3$	0.1804	0.1804
$(\theta_1\cup\theta_3)\cap\theta_2$	0.4415	0.2208

Table 9. Uncertainty values for DSmT-based fusion results measured by GAU2 measure

Uncertainty Value	Camera 1	Camera 2	Camera 3	m_{12}	m_{13}	m_{23}	m_{123}
Classic DSmT	1.4846	1.6385	1.1434	2.9974	1.3229	1.4225	3.0805
Hybrid DSmT	1.4846	1.6385	1.1434	2.3611	1.3225	1.4225	2.0572

5- 4- Uncertainty measurement for the DSmT-based fusion results

5- 4- 1- Uncertainty measurement of classic DSmT-based fusion results

Apply GAU2 measure to the set $\Theta_P=\{\theta_{P_1}, \theta_{P_2}, \theta_{P_3}, \theta_{P_4}, \theta_{P_5}, \theta_{P_6}, \theta_{P_7}\}$ where $\theta_{P_1}=\theta_1$, $\theta_{P_2}=\theta_2$, $\theta_{P_3}=\theta_3$, $\theta_{P_4}=\theta_1\cap\theta_2$, $\theta_{P_5}=\theta_1\cap\theta_3$, $\theta_{P_6}=\theta_2\cap\theta_3$, $\theta_{P_7}=\theta_1\cap\theta_2\cap\theta_3$. GAU2 measures uncertainty in DSmT-based results by probability assignment to the events of the set Θ_P which keeps the continuity of the main events. In Table 5, the maximum value of Bel(A)/|A| , which is underlined, is obtained for the event θ_2 according to the computing algorithm of GAU2 measure. Consequently, the probability distribution assignment for $\theta_{P_2}=\theta_2$ i.e. $p_{\theta_{P_2}}$ is equal to 0.7326. The algorithm is followed from Line 7 by eliminating the event $\theta_{P_2}=\theta_2$ to compute the other probability distribution assignments and $p_{\theta_{P_1}}=0.5382$, $p_{\theta_{P_3}}=0.3983$, $p_{\theta_{P_4}}=0.3027$, $p_{\theta_{P_5}}=0.2092$, $p_{\theta_{P_6}}=0.2014$, $p_{\theta_{P_7}}=0.0442$. Lastly, the value of uncertainty involved in DSmT fusion results calculated by GAU2 measure (Eq. (10)) is 2.9974. These computing steps can be followed for individual cameras or other fusion results. Table 9 summarizes the uncertainty values of cameras and the uncertainty values of fusion of all possible pairs of cameras by classic DSmT.

The uncertainty values in Table 9, measured by GAU2, illustrates that the uncertainty involved in camera 3 is less than the other cameras. Consequently, decisions made by camera 3 are more precise than those of camera 1 and camera 2 in uncertainty point of view. Furthermore, the uncertainty in DSmT fusion results is less than the sum of uncertainties in the associated cameras and even less than the uncertainty of each camera. Lastly, it can be concluded that DSmT has improved the results in uncertainty point of view using the generalized aggregated uncertainty measure, GAU2. In this case, the fusion of the cameras' data with less sum of uncertainty (e.g. camera 2, camera 3) leads to final fusion results with less uncertainty (i.e. m_{23}^f).

5- 4- 2- Uncertainty measurement of the hybrid DSmT-based fusion results

Supplementary knowledge affects the decisions made by cameras using hybrid DSmT. The effects of this knowledge which may be given by an expert are inspected in uncertainty point of view. As mentioned before, it is supposed that $\theta_1\cap\theta_3\equiv\emptyset$. Thus, there are three nonexclusive events while there is no community between θ_1 and θ_3. Tables 6 to 8 illustrate the computing steps of GAU2 measure for the fusion results by hybrid DSmT for m_{12} (A) as an example.
The maximum value is underlined in Table 6. The maximum value of Bel(A)/|A| is obtained for the event θ_2. Hence probability distribution assignment for ϑ_2 is $p_{\vartheta_2}=0.7142$.
Table 7 is attained by discarding ϑ_2 to compute the other probability distribution assignments. In Table 7, the maximum value is for $\theta_1\cup\theta_3$. Consequently, $p_{\theta_1}=p_{\theta_3}=0.3637$. Therefore, $p_{\theta_1}\cap\theta_2=0.2585$ and at last $p_{\theta_2}\cap\theta_3=0.1830$. Finally, the value of uncertainty involved in m_{12} (A) according to Eq. (10) is 2.3611. The uncertainties involved in cameras and fusion of other pairs of cameras by hybrid DSmT are summarized in Table 9. It is supposed that $\theta_1\cap\theta_3\equiv\emptyset$ in hybrid DSmT based results. Therefore, the values of uncertainties in cameras' data are changed in comparison to classic DSmT-based results.
The following conclusions are drawn from Table 9. The uncertainty values in the results using hybrid model are less than the uncertainty values in the results using free model. This is because of supplementary knowledge considered in hybrid model. Besides, extra knowledge reduces the uncertainty in measurements and final decisions as it is expected.
The uncertainty in hybrid DSmT fusion results is less than the sum of uncertainties in the associated cameras. Therefore, it can be concluded that DSmT has improved the results from uncertainty point of view.
Even though fusion of the cameras' data with fewer sums of uncertainty (e.g. Camera1, Camera3) leads to final fusion results with less uncertainty, i.e. m_{13}, it seems that the conclusion is not generally true. As an instance, the sum of uncertainties in Camera 1 and Camera 2 is less than that of Camera 2 and Camera3 but uncertainty in m_{12} is greater than m_{23}. When supplementary knowledge is considered, combination of sensors' data that support each other leads to the results with less uncertainty in comparison to combination of conflicting sensory data.
In this case, as can be understood from the result of m_{123}, increasing the number of cameras essentially does not give the final results with the least uncertainty.

6- Conclusion

Dezert-Smarandache theory was used in this paper to fuse the attained information of cameras for the localization of an object on a plane. Three cameras in three different positions were used in the experimental setup. The cameras' images lead to decisions with conflict to locate the object. Therefore, DSmT was selected to deal with this the high conflict problem. Proficiency of DSmT to deal with high conflict problems and generality of the model used in DSmT rather than other fusion algorithms such as Dempster-Shafer theory are the motives to select DSmT as data fusion algorithm. To achieve fusion task, an associated mass function for each camera was calculated by some pre-processes on collected data, including noise removal, edge detection, and assignment of Gaussian distribution to the decisions of each camera. Then DSmT was applied to fuse data of sensors in free and hybrid models. There are some conditions in modeling the events that convert some events to empty sets. Hence, Hybrid DSmT was applied regarding to nature of the problem. The fusion results showed the ability of DSmT in problems with conflict. Uncertainty measurement was carried out at every level of fusion to select the best choice of sensors or to select the best results of sensor fusion from uncertainty point of view. An appropriate uncertainty measure is required like AU that is developed for DST. A generalized AU measure, i.e. GAU2, was introduced by suitable extension of events to overcome the limitation of AU measure in problems with non-exclusive events such as the model used in DSmT. More reliable results in measurements and fusion are available using GAU2 in the framework, and also final decisions were made with less uncertainty.

Given the present results, the subsequent conclusions may be drawn.

- The uncertainty in the results of classic and hybrid DSmT-based fusion problem was less than the sum of uncertainties in the associated cameras. Consequently, DSmT improves the results from uncertainty point of view which is assessed by GAU2 measure.
- Due to additional knowledge which is considered in the hybrid model in hybrid DSmT in comparison to classic model, the uncertainty values were less than the uncertainty values in classic DSmT-based fusion results.
- Even though fusion of the cameras' data with less sum of uncertainty leads to final fusion results with less uncertainty, it appears that this result should be considered as a theoretical one.
- It should be noted that when there is an extra knowledge, a combination of sensors' data that are in agreement leads to the results with less uncertainty in comparison to the combination of conflicting sensory data.
- Increasing the number of cameras to fuse data by classic DSmT gave the final results with the least uncertainty but it did not yield the least uncertain final results when hybrid DSmT was applied. This issue should be examined theoretically as well.

GAU2 is an appropriate uncertainty measure for DSmT. However, when the number of events increases, its application is complicated.

For future works, other fusion methods like Proportional Conflict Redistribution rules can be analysed. Besides, the effects of an improved experimental setup and assignments of mass function to each area can be examined.

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