Robust MRAC for a Wing Rock Phenomenon in Delta Wing Aircrafts

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ABSTRACT: Wing rock phenomenon is undesired vibration in rolling angle occurring in delta wing aircraft at large angles of attack. This paper presents three versions of model refernce adaptive controllers; namely classical MRAC, σ-modified MRAC and weighted σ-modified MRAC to control the rolling dynamics under wing rock phenomenon with the presence of unmatched disturbance. The stability analysis of new proposed strategy for weighted σ-modified MRAC has been presented. The effectiveness of the controllers is assessed in terms of their robusness under unmatched disturbance. The simulated results based on MATLAB/SIMULINK show that σ-modified adaptive controller gives better robustness characteristics than classical one; as it could confine different aircraft responses within smaller bounded limits. Moreover, as compared to σ-modified controller, weighted σ-modified controller gives better enhancement in terms of robustness, control effort and error charcteristics.

1- Introduction

Delta wing aircrafts development is highly funded aircrafts projects. Nowadays, many types of such aircrafts are still in military and commercial usage. Figure (1) shows different wing types of aircrafts. The delta wing aircraft appears as the third one in the figure.
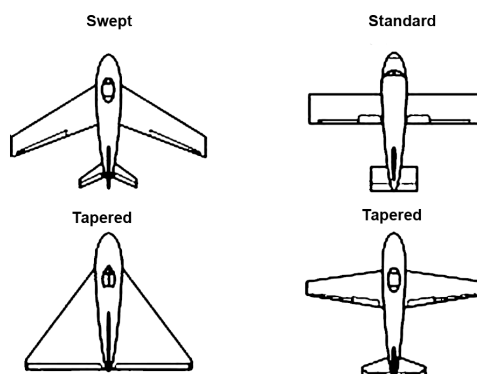

Fig. 1. Aircrafts wing types

The delta wing aircraft is characterized by high level of speed ranges and maneuverability. Its dynamic is complex and highly nonlinear [1]. The rolling motion of the delta wing aircraft are the rotation of the aircraft on the rolling axis. Rolling axis is the line passing through the aircraft's fuselage and the center of gravity as indicated in Figure (2). The ailerons at the end of the wing are the movable parts responsible for rolling motion control of the aircraft [2].

Wing rock phenomenon is an undesired motion appears in high angles of attack. This phenomenon comes out due to the presence of a limit cycle caused by wing rock dynamics. The wing rock phenomenon results in rolling the aircraft in both direction of roll angles with specified amplitude and frequency. The dynamics of wing rock phenomenon can be seen in many researches and studies [3]
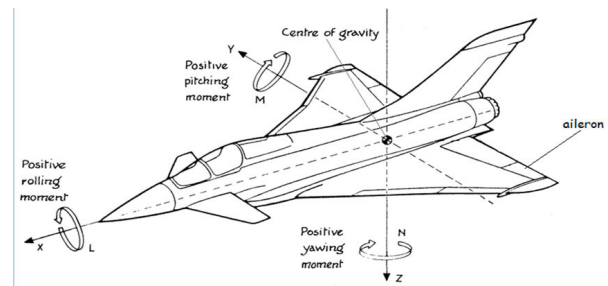

Fig. 2. Delta wing aircraft axis and ailerons

Different related studies are briefly reviewed here; Adaptive Feedback Linearization has been proposed by (Krstic, etal)[4]. Also, dynamic recurrent neural networks for stable adaptive control are presented by (Steven Boon Kooi)[5]. Nonlinear H_∞ method for control has been proposed by (Shyh-Pyng Shue and etal) as indicated in[6]. Simulation and Analysis of Wing Rock Physics is performed by (A. Saad, Ahmed) in [7]. Passino developed new techniques for nonlinear systems with a time-varying structure [8]. Comparative Analysis of various nonlinear active control has been proposed by (Józef Pietrucha) [9]. Linear and Nonlinear Robust Control strategies are presented by (Alon Kuperman and etal) [10]. Adaptive Backstepping type design proposed by (Ene Costin) can be found in [11]. It is clear from the above literature survey that most of the proposed techniques are interested in controller design and no mention of how well the robustness

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will be in presence of unmatched disturbance effect on the roll rate due to sensor inactivity.

Adaptive control theory are still seeking for robustness and robustness related problem are yet open and waiting to be solved [12],[13].

Recently, Model Reference Adaptive Control (MRAC) has acquainted significant attention due to its ability to adjust controller elements in real-time situation. This would enable the control system to operate in a highly uncertain environment in the presence of malfunctions or damages.

In theory, a Model Reference Adaptive Controller can drive the plant to asymptotically track a specified reference model if certain matching conditions on the uncertainties are met. However, practically, such matching conditions usually do not fit due to the presence of matched and unmatched unmodeled dynamics, disturbances and parameterization errors. This may lead to weakening of stability property of Uniform Ultimate Boundedness (UUB) and also a parameter drift would arise which in turn may lead to instability problems. In order to cope with such types of uncertainties and to have a robust MRAC, this requires a modification in the adaptive law of MRAC strategy. As such, several structures of modification have appeared like dead-zone modification, e-modification, σ-modification and others. The salient feature of σ-modification MRAC is that it does not need the upper bound of disturbance. Therefore, it would be our candidate for having a robust control of aircraft roll motion [14].

However, it is not difficult to see that the adaptive law in σ-modification MRAC add the same damping to all matrix elements of adaptive parameters. It is interesting to investigate the effect of adding different weight of damping to different matrix elements of adaptive parameters. This is equivalent to give different importance to different matrix elements of adaptive parameters. This is main key of the work contribution. This new version of modification is termed "weighted σ-modification" due to different weights of damping assigned to different matrix elements. Thus, in the present work, weighted σ-modification is introduced and the stability analysis is developed.

The objective of the present work is to assess the performance of three versions of model reference adaptive controllers under unmatched uncertainty for delta wing aircraft. The well performed controller is the one which gives the best robust characteristics under the considered uncertainty. The performance of σ-modified and weighted σ-modified controllers are compared to that of classical one. It is worthy to mention that the entire scenario occurs around small neighborhood of an operation point.

In order to make the case study under consideration more realistic, the work considered the limitation imposed on deflection ailerons angle in real application. This the other challenge which has been avoided by many researchers.

2- Mathematical Model

Wing rock phenomenon were studied and analyzed by many researchers in the literature. The derived model is formulated by fitting the experimental data gathered from wind tunnel simulation to a mathematical expression agreed to describe the wing rock phenomenon. The models got from the experimental fitting are a second order nonlinear models differs from each other's by nonlinearity terms. One of the first most cited works for modern control theory is the model

proposed by [3] . A further development on the model is suggested by [15]. The most recent wing rock phenomenon suppression controllers were designed considering the result of [16] depending on the original work in [17]. For the case of one degree of freedom which means that the considered movement in the aircraft inside the air tunnel is the roll motion only as shown in Figure (3).

The experimental procedure has done for an 80° delta wing aircraft with a number of aircrafts configuration and for different angle of attack simulation cycles for each configuration, as indicated in Figure (4).
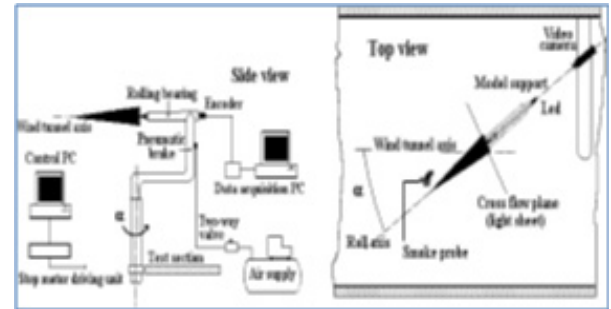

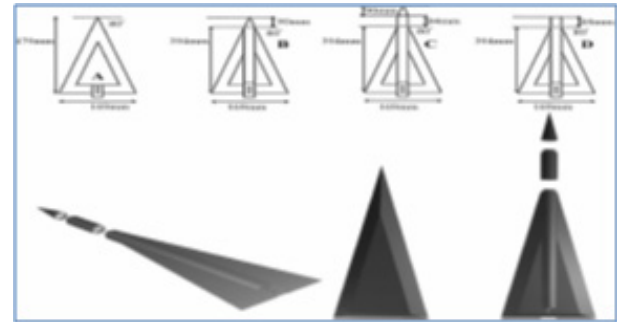**Fig. 3. experimental construction [16]**

Fig. 4. 80° delta wing aircraft configurations [16]

In this paper the C configuration shown in Figure (4) will be considered as a case study for the presnt work.

The C configuration is the closest configuration for the delta wings aircrafts in use either fighter or civilian aircrafts. The C configuration comprise the delta wing (80°swept wing), the forebody and the nose tip. The differential equation that describe the C configuration is constructed as:

For a case of one degree of freedom the considered model of wing rock dynamics is based on the following equation of motion [16], [3]:

$$\ddot{\phi} = \left(\rho\, U_\infty^2\, S_b\, / 2 I_{xx} \right) C_l + D\, u \tag{1}$$

where ϕ is the roll angle, ρ is the air density, U_∞ is the speed of the free stream of air, S is the area of the wing plane, b is the chord, I_{xx} is the moment of inertia of the wing around the axis of the roll span, u is the differential ailerons position and D is the effectiveness of the differential ailerons rolling.

Referring to [3] and [15], C_l can be given by;

$$C_l = a_1 \phi + a_2 \dot{\phi} + a_3 |\phi| \dot{\phi} + a_4 |\dot{\phi}| \dot{\phi} + a_5 \phi^3 \tag{2}$$

where a_1, a_2, a_3, a_4 and a_5 are unknown constants. Substituting Eq.(2) into Eq.(1), one can have the following equation:

$$\ddot{\phi} = b_1 \phi + b_2 \dot{\phi} + b_3 |\phi| \dot{\phi} + b_4 |\dot{\phi}| \dot{\phi} + b_5 \phi^3 + b_6 u \tag{3}$$

where b_1, b_2, b_3, b_4 and b_5 are still unknown constants.
Letting $\phi = x_1$ and $\dot{\phi} = \dot{x}_1 = x_2$, one can get the state equation

$$\dot{x}_1 = x_2 \tag{4}$$

$$\dot{x}_1 = b_1 x_1 + b_2 x_2 + (b_3 |x_1| + b_4 |x_2|) x_2 + b_5 x_1^3 + b_6 u \tag{5}$$

The first impression of the above system of equations is that it is a nonlinear system with unknown parameters. Gust and wind storms effects have not yet been included in the system. Since the gust affect directly to the ailerons, it will be treated later as a bounded disturbance. Then, the system the will be given as,

$$\dot{x}_1 = f_1 + \zeta_1 \tag{6}$$

$$\dot{x}_2 = f_2 + b_6 u \tag{7}$$

where,

$$f_1 = x_2 \tag{8}$$

$$f_2 = b_1 x_1 + b_2 x_2 + (b_3 |x_1| + b_4 |x_2|) x_2 + b_5 x_1^3 \tag{9}$$

and,

$$\| \zeta_1 \| \le \zeta_{max} \tag{10}$$

It is interesting to investigation the equilibrium points for the unforced and unperturbed system. It is simply found by equating f_1 and f_2 of Eq.(6) and (7) to zero, which results in, $x_2 = 0$ and $x_1 = 0$ or $x_1 = \pm\sqrt{-b_1/b_5}$ and the equilibrium points will be;

$$P_1 = (0,0), P_{2,3} = (\pm\sqrt{-b_1/b_5}, 0)$$

Three equilibrium points can be found and the origin can be classified as a saddle point and the two other are unstable nodes; a chance of appearance of limit cycle which is the cause of instability.

3- Controller Design

For a system with dynamics,

$$\dot{x} = Ax + B\Lambda(u + \theta^T \phi) + \zeta \tag{11}$$

$$y = C_m x \tag{12}$$

where,
$A \in R^{n \times n}$: is unknown matrix.
$B \in R^{n \times m}$: is unknown matrix.
$\Lambda \in R^{m \times m}$: is unknown matrix diagonal matrix.
$\theta \in R^{N \times m}$: is unknown matrix.
$\phi \in R^{N \times m}$: is known function matrix.
$y \in R^q$: is the output vector.
$C\_m \in R^{q \times n}$: is the output weight matrix (known)
$\zeta \in R^n$: is unmatched disturbance ((upper bound) known)
This system must behave as a reference system given by

$$\dot{x}_m = A_m x_m + B_m r \tag{13}$$

where,
$A_m \in R^{n \times n}$: is the reference model matrix.
$B_m \in R^{n \times m}$: is the reference input matrix.
$x_m \in R^n$: is reference state vector.
$r \in R^m$: is the desired input.
In order to design a controller u which able to make the system of Eqs.(11-12) track the desired input r based on reference model behavior, the controller has to perform the following tasks:
• Cancel out the nonlinearity of $\theta^T \phi$.
• Find the feedback and forward gains such that the system to be designed could coincide to model reference characteristics,
• Tackle the uncertainties represented by A, Λ and θ.
Choosing the control input as follows,

$$u = \widehat{K}^T \Psi \tag{14}$$

where:

$$\widehat{K}^T = [\widehat{K}_x^T \ \widehat{K}_r^T \ -\hat{\theta}^T] \tag{15}$$

$$\Psi = [x \ r \ \phi]^T \tag{16}$$

such that
$\hat{\theta} \in R^{N \times m}$: is a matrix of an estimated values of θ.
$\widehat{K}_x \in R^{n \times m}$: is a matrix of the adaptive feedback gains.
$\widehat{K}_r \in R^{m \times m}$: is a matrix of the adaptive feed forward gains.
Matching condition must exist for the system to mimic the characteristics of reference model. Existence of matching condition means that the following equalities must hold;

$$A + B\Lambda K_x^T = A_m \tag{17}$$

$$B\Lambda K_r^T = B_m \tag{18}$$

$$\Delta\theta^T = \widehat{\theta^T} - \theta^T = 0 \tag{19}$$

where K_x^T, K_x^T represent the ideal feedback and feed-forward gains, respectively, while $\widehat{\theta^T}$ stands for estimated parameters coefficients. The closed loop system becomes,

$$\dot{x} = Ax + B\Lambda(\widehat{K}^T \Psi + \theta^T \phi) + \zeta \tag{20}$$

The difference between the actual state and reference model state is given by the following error equation

$$e = x - x_m \tag{21}$$

The time derivative of the above error yields

$$\dot{e} = \dot{x} - \dot{x}_m$$

or

$$\dot{e} = A_m e + B\Lambda\Delta K^T \Psi + \zeta \tag{22}$$

In the sense of stability analysis, it is appropriate to choose the following Lyapunov candidate;

$$V(e, \Delta K^T) = e^T P e + tr(\Delta K^T \Gamma_k^{-1} \Delta K \Lambda) \tag{23}$$

where $V(e, \Delta K^T): R^{n \times m} \rightarrow R$ and $P \in R^{n \times n}$ |$P = P^T > 0$ is a positive definite symmetric matrix. The matrix P is calculated by solving the following equation:

$$PA_m + A_m^T P = -Q$$

where $Q \in R^{n \times n}$ is positive definite symmetric matrix. and $\Gamma_K \in R^{z \times z}$ |$\Gamma_K = \Gamma_K^T > 0$, $z = n + m + N$ is diagonal matrix known as adaptation or learning rate. The derivative of $V(e, \Delta K^T)$ will be:

$$\dot{V} = -e^T Q e + 2 \text{tr}(\Delta K^T (\Gamma_k^{-1} \widehat{K}$$
$$+ \Psi e^T PB)\Lambda + 2e^T P \zeta \tag{25}$$

For $\zeta = 0$ then the system can be asymptotically driven to the origin by choosing the adaptive law:

$$\widehat{K} = -\Gamma_k \Psi e^T PB \tag{26}$$

This leads to

$$\dot{V} = -e^T Q e \tag{27}$$

which is negative definite. If $\|\zeta\| \leq \zeta_{max}$, then

$$\dot{V} = -e^T Q e + d \tag{28}$$

where $d = 2e^T P \zeta$. Using the fact,

$$e^T Q e \leq \lambda_{max}(Q) \|e\|^2 \tag{29}$$

If one assumes that

$$d > \lambda_{max}(Q) \|e\|^2 \tag{30}$$

then $\dot{V}$ will no longer be negative definite and the stability will be threated. Instability begins with growth of $\widehat{K}^T$ elements which is called adaptive parameters drifts. Due to presence of disturbance the desired convergence to the desired equilibrium point will not be guaranteed any more. In case of none vanishing disturbance the best solution could reach to the uniform ultimate boundedness of the system trajectory. Internal stability is an important and basic concept in robust control; boundedness of all system signal is the condition of the internal stability [14]. Parameter drifts could degrade the internal stability by estimating high gains to the controller parameters.

3- 1- σ-modifications MRAC

The problem has just been argued above can be avoided by a modification in adaptive law of MRAC named as *Leakage* or sigma modification (σ-modification). This technique was presented recently by[18] and further discussed in [14]. Leakage or sigma modification has more than one form depending of choice of sigma and the form suggested in this paper is the fixed sigma modification. The modification is done to the adaptive law where it modified to:

$$\widehat{K} = -\Gamma_k (\Psi e^T PB - \sigma \widehat{K}) \tag{31}$$

where σ>0 is a constant chosen by the designer. Choosing the adaptive law indicated by Eq.(31), the *Lyapunov* time derivative function given by Eq.(25) becomes;

$$\dot{V} = -e^T Q e - 2\sigma \, \text{tr}(\Delta K^T \widehat{K} \Lambda)$$
$$+ 2e^T P \zeta \tag{32}$$

or,

$$\dot{V} \leq -\lambda_{min}(Q) \|e\|^2 + 2\|e\|\lambda_{max}(P)\zeta_{max} +$$
$$\sigma \|\Delta K\|_F^2 (2\Lambda_{min} + \|\Lambda\|_F) + \sigma \|K\|_F^2 \tag{33}$$

The inequality of Eq. (33) has two variables $\|e\|$ and $\|\Delta K\|_F^2$ and $\dot{V} < 0$ if:

$$\|\Delta K\|_F^2 > \frac{\|K\|_F^2 \|\Lambda\|_F}{(2\Lambda_{min} + \|\Lambda\|_F)} \tag{34}$$

and,

$$\|e\| > \frac{2\lambda_{max}(P)\zeta_{max}}{\lambda_{max}(Q)} \tag{35}$$

where inequalities (34) and (35) is guarantees the ultimate uniform boundedness of all adaptive gains, control input and the state variables and that's the aim of the robust adaptive control. However, combining both conditions of Eq.(34) and(35) to form the set defined as *α* outside which $\dot{V}$ is assured to be negative,

$$\alpha = \left\{ \|\Delta K_F\| \leq \sqrt{\frac{\|K\|_F^2 \|\Lambda\|_F}{(2\Lambda_{min} + \|\Lambda\|_F)}} \wedge \|e\| \leq \frac{2\lambda_{max}(P)\zeta_{max}}{\lambda_{min}(Q)} \right\}$$

where $e \in R^n$ and $\Delta K \in R^{N \times M}$.

3- 2- Weighted σ-modification MRAC

So far, the σ-modification technique add equally a damping constant, represented by constant σ, to all elements of $\widehat{K}$ The proposed weighted sigma modification is based on assigning different weight of sigma constant to each element of adaptive gain matrix $\widehat{K}$. Based on this notion, the adaptive law of Eq.(31) can be written as:

$$\widehat{K} = \Gamma_K (\Psi e^T PB - \sigma\Sigma\widehat{K}) \tag{36}$$

where σ>0 is a constant chosen by the designer and $\Sigma \in R^{N \times N}$ is diagonal positive definite matrix whose elements are the weights for the elements adaptive gain matrix such that $\Sigma_{ii} > 0$. Substituting the adaptive law of Eq.(36) into Eq.(25) we have;

$$\dot{V} = -e^T Q e - 2\sigma \, \text{tr}(\Delta K^T \Sigma \widehat{K} \Lambda) + 2e^T P \zeta \tag{37}$$

or

$$\dot{V} \leq -\lambda_{min}(Q) \|e\|^2 + 2\|e\|\lambda_{max}(P)\zeta_{max}$$
$$- \sigma \|\Delta K\|_F^2 (2\Sigma_{min}\Lambda_{min} + \|\Sigma\|_F \|\Lambda\|_F)$$
$$+ \sigma \|\Delta K\|_F^2 \|\Sigma\|_F \|\Lambda\|_F \tag{38}$$

The satisfication of the above equation depends on the bound of two variables $\|e\|$ and $\|\Delta K\|_F$. Then $\dot{V} < 0$, if

$$\|\Delta K\|_F^2 > \frac{\|K\|_F^2 \|\Sigma\|_F \|\Lambda\|_F}{\left(2\Sigma_{min}\Lambda_{min} + \|\Sigma\|_F \|\Lambda\|_F\right)} \qquad (39)$$

and

$$\|e\| > \frac{2\lambda_{max}(P)\zeta_{max}}{\lambda_{min}(Q)} \qquad (40)$$

It is easy to see that the there is no change of the error bound $\|e\|$ and this technique would scaling the bound $\|\Delta K\|$ only. Again combining both variables confined by Eq.(39) and (40) which gaurantees the negative definitness of $\dot{V}$ and guarantees the ultimate uniform boundedness of all adaptive gains, control input and the state variables and that's the aim of the robust adaptive control; one can find the unified set α_w associated with weighted σ-modifications,

$$\alpha_w = \{\|\Delta K\|_F \leq \sqrt{\frac{\|K\|_F^2 \|\Sigma\|_F \|\Lambda\|_F}{(2\Sigma_{min}\Lambda_{min} + \|\Sigma\|_F \|\Lambda\|_F)}}$$
$$\wedge \|e\| \leq \frac{2\lambda_{max}(P)\zeta_{max}}{\lambda_{min}(Q)}\} \qquad (41)$$

where $\|K\|_F^2 > 0$. One can argue that the region of stability for weighted modification α_w can be determined by Frobinus norm $\|\Sigma\|_F$. It works as a scaling factor of the region of stability; the region of stability may be tolarated to a larger bound for $\Sigma_{min} < 1$ or restricted to a smaller bounds for $\Sigma_{min} > 1$.

4- Simulated Results
The simulation of wing rock responses under supervision of suggested and proposed controller has been implemented using Matlab/Simulink. Before we proceed, the following conditions have to be kept in mind;
• The maximum allowable excursion of roll angle is limited to ±30 degree.
• An environmental gust conditions effect is embodied by applying an unmatched disturbance wind gust as a uniform random signal with the following maximum bounds:
$$-10 \leq \xi \leq 10 \quad \text{(Degree)}$$
• The system parameters used for simulation are listed below [3],[19]:
$b_1 = -0.018$, $b_2 = 0.015$, $b_3 = -0.062$,
$b_4 = 0.009$, $b_5 = 0.021$, $b_6 = 0.75$
The phase plane for the open loop system is shown in Figure (4). The figure shows the instability of the equilibrium points and the existence of limit cycle.
The reference model matrices are chosen to be:

$$A_m = \begin{bmatrix} 0 & 1 \\ -1 & -1.4 \end{bmatrix}, B_m = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Many tuning trials, the following appropriate diagonal values for the matrices Γ_K, Q, Λ and Σ are found;
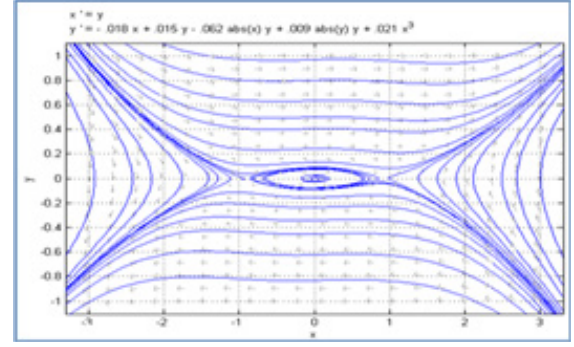

Fig. 4. Phase plane for the open loop

$$Q = \begin{bmatrix} 10 & 0 \\ 0 & 0.1 \end{bmatrix}, \Lambda = b_6$$

$$\Gamma_K = \begin{bmatrix} 500 & 0 & 0 & 0 & 0 & 0 \\ 0 & 500 & 0 & 0 & 0 & 0 \\ 0 & 0 & 500 & 0 & 0 & 0 \\ 0 & 0 & 0 & 100 & 0 & 0 \\ 0 & 0 & 0 & 0 & 10 & 0 \\ 0 & 0 & 0 & 0 & 0 & 10 \end{bmatrix}$$

$$\Sigma = \begin{bmatrix} 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.8 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.8 \end{bmatrix}$$

The desired trajectory in terms roll angle (model reference trajectory) and the disturbance behavior are depicted in Figure (5).
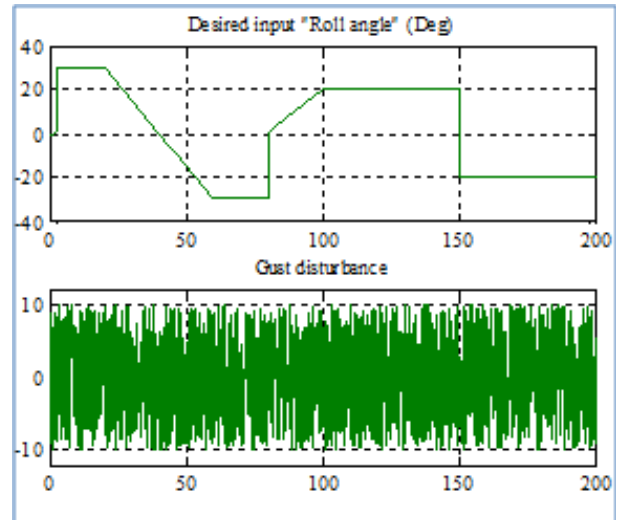

Fig. 5. Desired input and gust disturbance

The first scenario of simulation considers the case with unlimited aeleron deflection. Figure (6) describe the behaviors of roll angle and roll rate for the three suugested controllers. The value of sigma σ is set to value of 0.05. This value would satisfy both good performance and stability. The same value is assigned to the case of weighted sigma modification. Figure (7) shows the control action resulting from these controllers. It is seen that from the figures that all controllers perform well under exerted disturbance. It is evident from Fig.(7) that the maximum aileron excursion is the largest in the case of classical MRAC than that seen with other controllers.

Further comparison bewteen σ-modification controller and weighted σ-modification controller are illustrated in Fig. (8). The figure shows that the control action needed by weighted σ-modification controller is less than needed by σ-modification controller. Therefore, one may argue that the opputunity of reaching wind-up in weighted σ-modification controller is less than that in case of other controller.

Figure (9 shows the trajectory of errors between the states of model reference and plant.

One can see from Fig.(9) that the weighted σ-modification controller enable the states of aircraft to track the model reference states in least error than other controllers do.

However, the important feature of σ-modifiied and weighted σ-modifiied controllers is that they could keep the estimated forward and feedback gains and within a small bounded limits. Also, the controllers could successfully cancel the nonlinear gains. On the other hand, the classical MRAC fails to stop drifting of such gains. The drift which may lead to instability problems and it is undesirable. This is evident from Fig.s (10) to (15).

In what follows, the above scenarios have been repeated, but the presented controllers are applied to aircraft in which the movement of aircraft ailerons is restricted to the following limits [20];

$$-25 \leq \|u\| \leq 25 \quad \text{(Degree)}$$

The relevant simulations of such limited case are shown in Fig.s (16)-(25).

Figure (16) shows that there is high degradation in performance due to classical MRAC comapred to that resulting from modified controllers (sigma and weighted sigma). The latter controllers could sucessfully keep the performance of roll motion dynamics at high level.

Figure (17,18) shows the behavior of control actions due to the three control strategies. It is clear that the worst behavior is the one resulting from classical MRAC; as the control action shows undesirable chattering. Meanwhile, both modified controllers could confine the control action within the prescribed limits. This action is practically preferable, since it would save power and save the ailerons from damage. Furthermore, weighted σ-modifiied controller could give better and less control action than that obtained from σ-modifiied controller.

Figure (19) shows the error behaviors between the states of model refernce and the plant. It is clear from the figure that Both σ-modified controller and weighted σ-modified controller give less error variance as comapred to classical controller. Moreover, weighted σ-modified controller yields the least errors as compared to others.

However, the restriction of actuator led that classical MRAC could fix the gains at high bound level as indicated in Fig. (20). Meanwhile, both modified controllers could prevent the gain drift to exceed small bounds. This is clearly seen in Fig.s (21)-(25). Also, as illustrated from the figures that weighted σ-modified controller show better improvement in terms of gain drift characteristic.
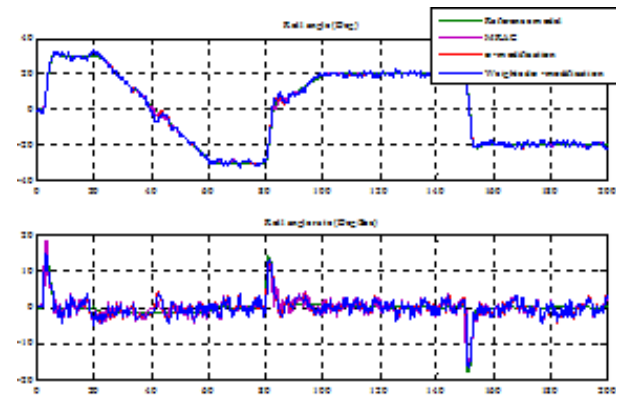

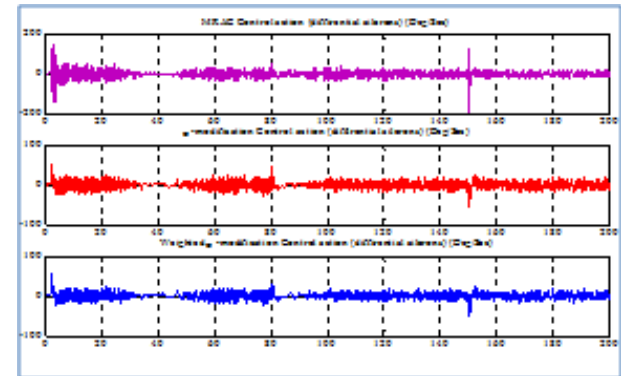**Fig. 6. Roll angle and roll angle rate (non-restricted actuator)**
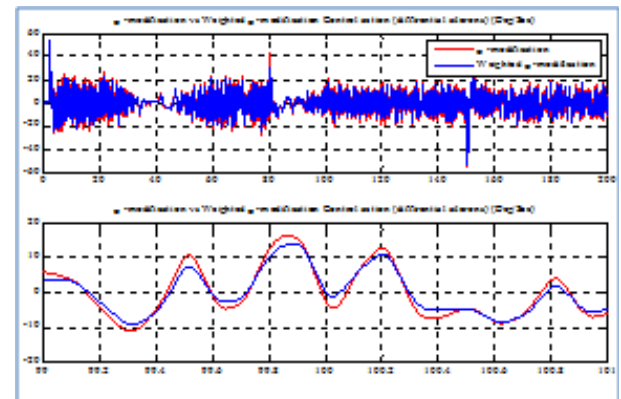

Figure 7. Control action (non-restricted actuator)

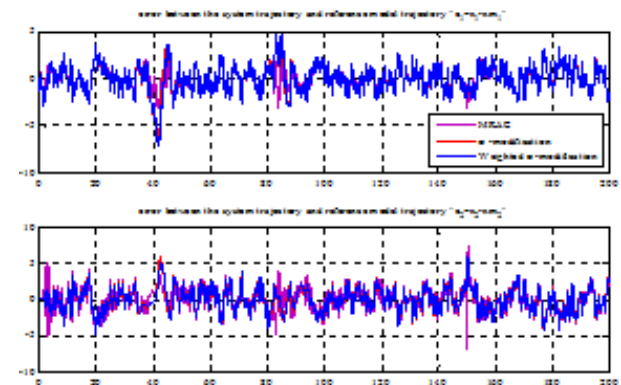**Fig. 8. Control action comparison (non-restricted actuator)**

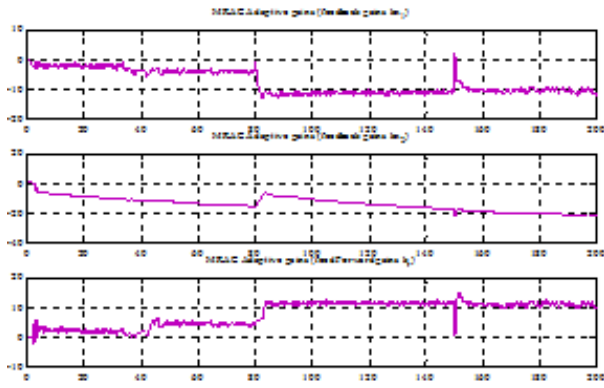
Fig. 9. Error trajectory (nonconstraint actuator)

Fig. 10. Feedback and feed forward gains MRAC (non-restricted actuator)
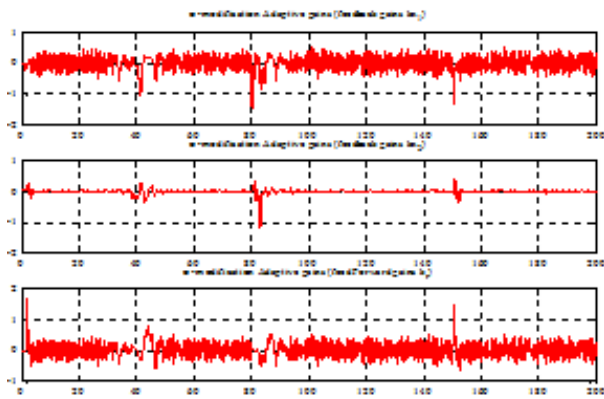

Fig. 11. Feedback and feed forward gains (σ-modification (non-restricteded actuator))
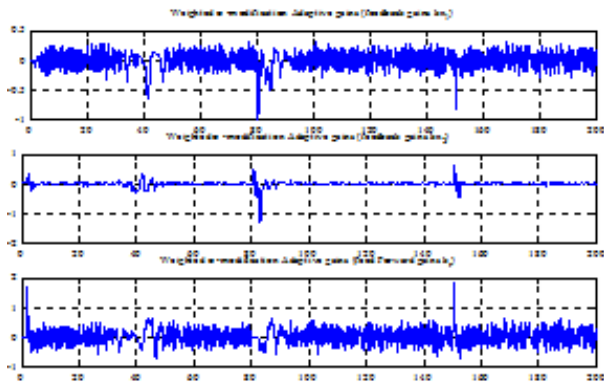

Fig. 12. Feedback and feed forward gains (Weighted σ-modification (non-restricted actuator))
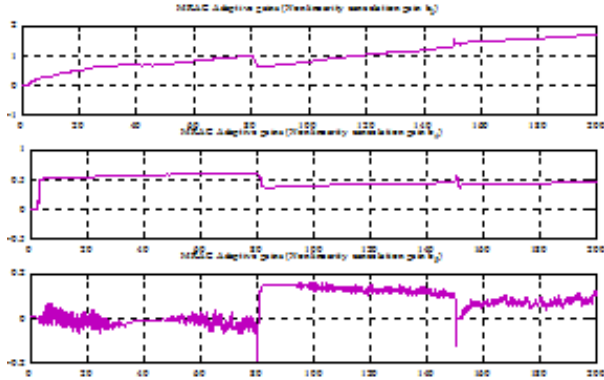

Fig. 13. Classical MRAC for cancelation of nonlinearity gains (non-restricted actuator)
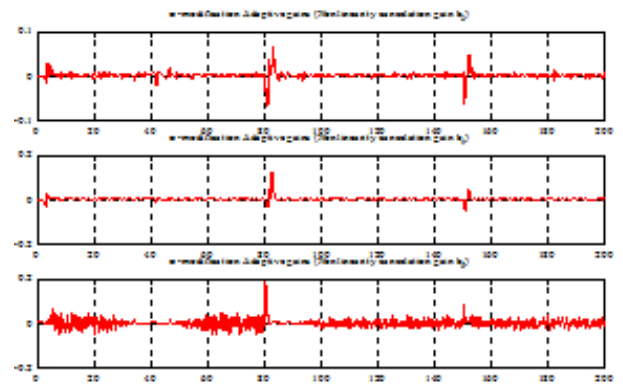

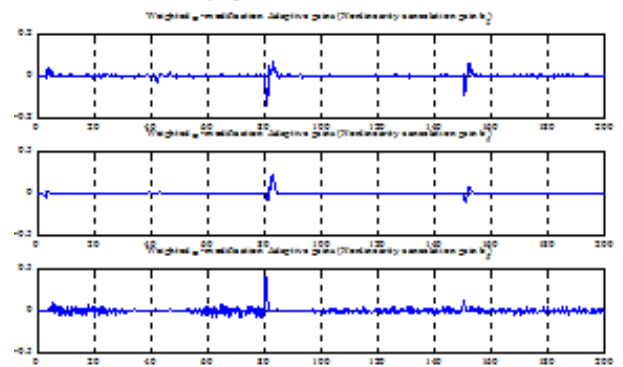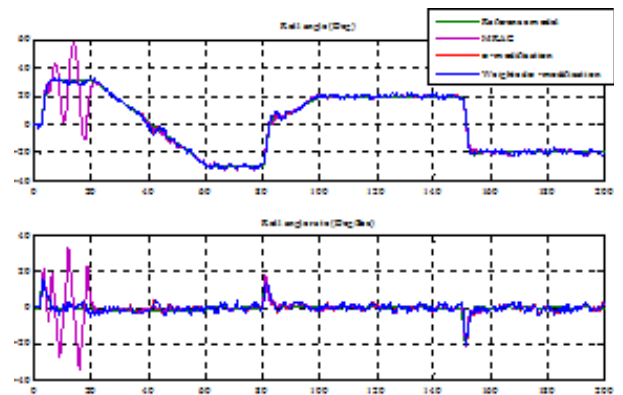**Fig. 14. Weighted σ-modified controller for cancellation for nonlinearity gains (non-restricted actuator)**

Fig. 15. σ-modified controller for cancellation of nonlinearity gains (non-restricted actuator)

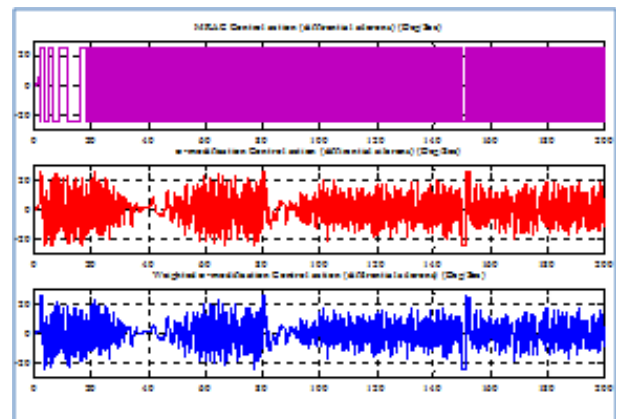**Fig. 16. Roll angle and roll angle rate (restricted actuator)**

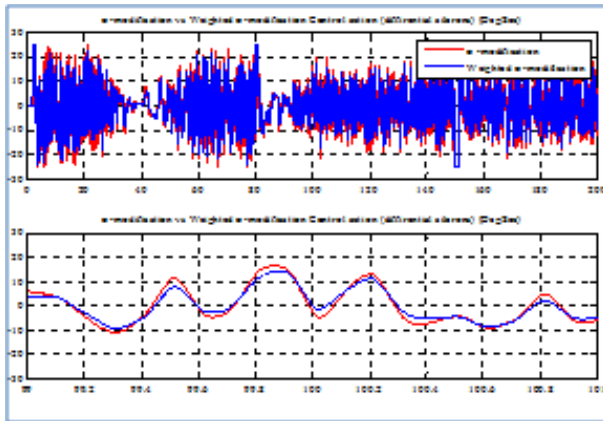
Fig. 17. Control input (restricted actuator)

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Fig. 18. Control input comparision (restricted actuator)
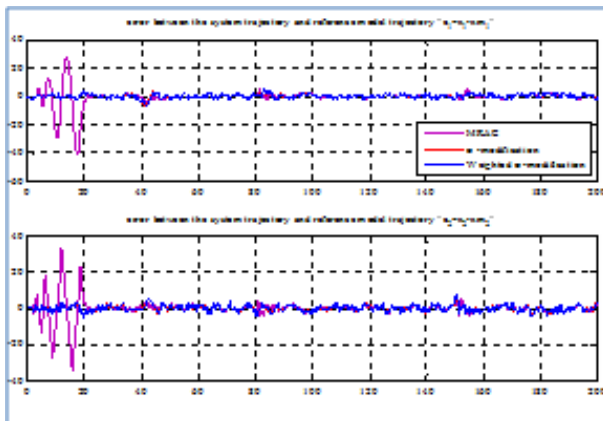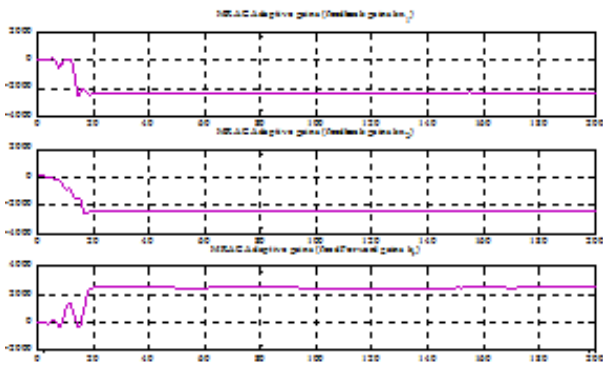

Fig. 19. Error trajectory (restricted actuator)

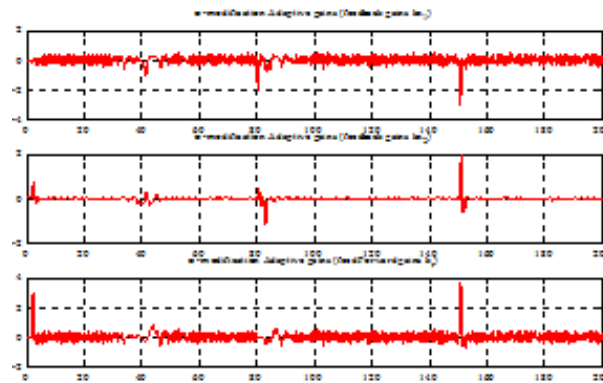**Fig. 20. Feedback and feed forward gains MRAC (restricted actuator)**

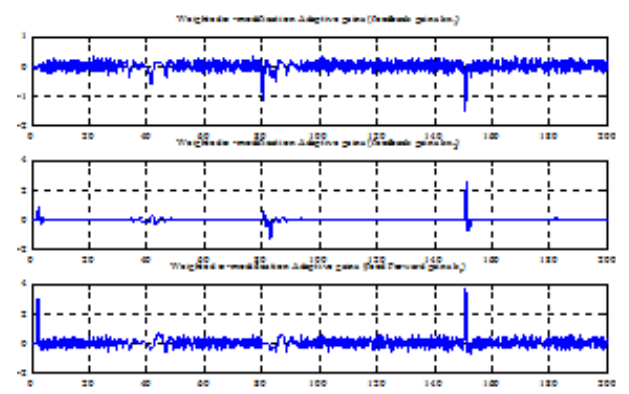**Fig. 21. Feedback and feed forward gains σ-modified (restricted actuator)**

Fig. 22. Feedback and feed forward adaptive gains Weighted σ-modified (restricted actuator)
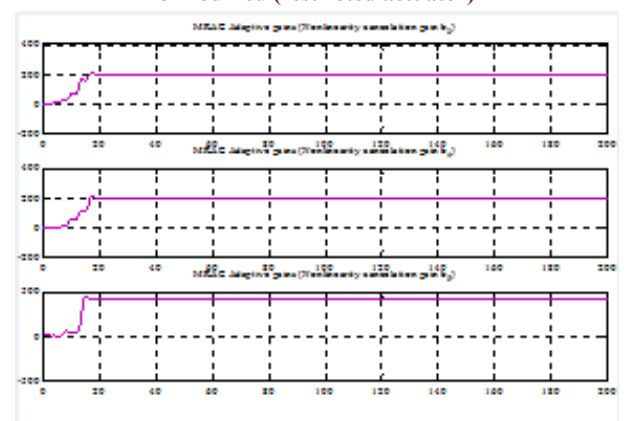

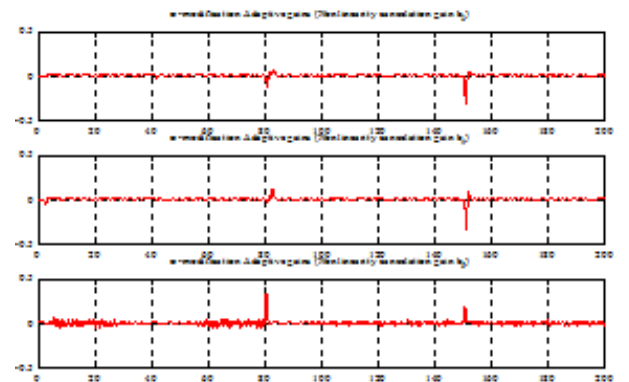**Fig. 23. MRAC Nonlinearity cancelation gains (restricted actuator)**

Fig. 24. σ-modified Nonlinearity cancelation gains (restricted actuator)
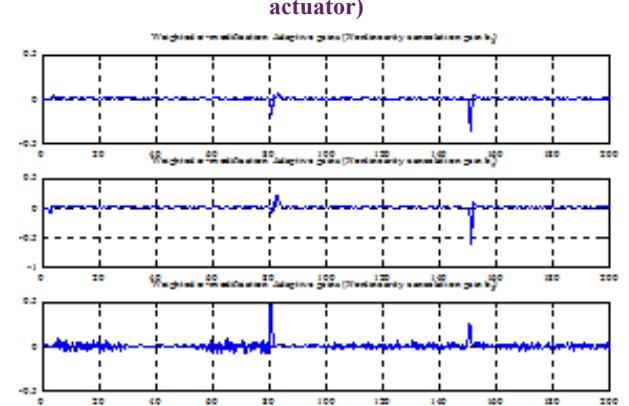

Fig. 25. Weighted σ-modified MRAC Nonlinearity cancelation gains (restricted actuator)

5- Conclusions

Three model reference adaptive controllers are presented here to control the roll motion of delta wing aircraft under unmatched uncertainty with consideration to the wing rock phenomenon. The weighted σ-modified controller is proposed in the present work and its performance is assessed and compared to fixed σ-modified controller and classical model refernce adative controller. Based on the observations of simulated results, the following points can be hilighted;

• Both σ-modified controller and weighted σ-modified controller prevents gain drifting and therefore grant the adaptive control system good robust charcteristics. On the contrary, classical MRAC could not confine the adative gains withing small limited bound and it is not robust against variation of paramters.

• Both σ-modified controller and weighted σ-modified controller give less error variance as comapred to classical controller. Generally, weighted σ-modified controller yields the least errors as compared to others.

• Both σ-modified controller and weighted σ-modified controller give less control action than that given by classical controller. However, control action behavior is better improved by weighted σ-modified controller.

6- Appendix

The modification is done to the adaptive law where it modified to:

$$\widehat{K} = \Gamma_K (\Psi e^T PB - \sigma\Sigma\widehat{K}) \tag{A.1}$$

where σ>0 is a constant chosen by the designer and $\Sigma \in R^{N \times N}$ is diagonal positive definite matrix represents the weighting added to each adaptive gain Σ_{ii}>o.

By choosing the Eq. (A.1) as adaptive law and by back to the *Lyapunov* candidate time derivative and substitute equation Eq. (A.1) in it *Lyapunov* candidate time derivative will be:

$$\dot{V} = -e^T Qe + 2tr(\Delta K^T (\Gamma_K^{-1}\Gamma_K (\Psi e^T PB - \sigma\Sigma\hat{K}) - \Psi e^T PB)\Lambda) + 2e^T P\zeta(t) \tag{A.2}$$

$$\Rightarrow \dot{V} = -e^T Qe - 2\sigma tr(\Delta K^T \Sigma\hat{K}\Lambda) + 2e^T P\zeta(t) \tag{A.3}$$

taking in consideration that:

$$\Delta K = \widehat{K} - K \Rightarrow \widehat{K} = \Delta K + K$$

and that led to:

$$\Rightarrow \dot{V} = -e^T Qe - 2\sigma tr(\Delta K^T \Sigma (\Delta K + K)\Lambda) + 2e^T P\zeta(t) \tag{A.4}$$

$$\Rightarrow \dot{V} = -e^T Qe - 2\sigma tr(\Delta K^T \Sigma\Delta K\Lambda) - 2\sigma tr(\Delta K^T \Sigma\Delta K\Lambda) + 2e^T P\zeta(t) \tag{A.5}$$

Considering the fact of Frobinus norm for matrices is:

$$\|A\|_F = \sqrt[2]{\sum_{i=1}^{N}\sum_{j=1}^{N} A_{ij}^2} = \sqrt[2]{tr(A^T A)}$$

and that's led to:

$$tr(\Delta K^T \Sigma\Delta K \Lambda) \geq \Delta K_F^2 \Sigma_{min}\Lambda_{min} \tag{A.6}$$

and by the famous Schwarz inequality of for any two vectors a and b the following inequality holds: |a^T b|≤‖a‖‖b‖ and also hold for more than two vectors which leds to:

$$2\sigma tr(\Delta K^T \Sigma K \Lambda)$$
$$\leq \Delta K_F \Sigma_F K_F \Lambda_F$$
$$\leq \Delta K_F K_F \Sigma_F \Lambda_F \tag{A.7}$$

Gathering the result of inequalities of (A.6) and (A.7) and substitute in Eq.(A.5) led to the inequality of:

$$\Rightarrow \dot{V} \leq -\lambda_{min}(Q)\|e\|^2$$
$$+2\|e\|\lambda_{max}(P)\zeta_{max}$$
$$-2\sigma\|\Delta K\|_F^2 \Sigma_{min}\Lambda_{min}$$
$$+2\sigma\|\Delta K\|_F \|K\|_F \|\Sigma\|_F \|\Lambda\|_F \tag{A.8}$$

For any two vectors a and b the fact of 2ab≤a^2+b^2 led to:

$$2\|\Delta K\|_F \|K\|_F \leq \|\Delta K\|_F^2 + \|K\|_F^2$$

And that led to:

$$\Rightarrow \dot{V} \leq -\lambda_{min}(Q)\|e\|^2$$
$$+2\|e\|\lambda_{max}(P)\zeta_{max}$$
$$-\sigma\|\Delta K\|_F^2 (2\Sigma_{min}\Lambda_{min}$$
$$+\|\Sigma\|_F \|\Lambda\|_F) \tag{A.9}$$

The inequality (A.9) has two variables ‖e‖ and ‖ΔK‖$_F$ and $\dot{V}$<0 if:

$$-\lambda_{min}(Q)e^2 + 2e\lambda_{max}(P)\zeta_{max} < 0 \tag{A.10}$$

And:

$$-\sigma\|\Delta K\|_F^2 (2\Sigma_{min}\Lambda_{min} + \|\Sigma\|_F \|\Lambda\|_F)$$
$$+\sigma\|K\|_F^2 \|\Sigma\|_F \|\Lambda\|_F < 0 \tag{A.11}$$

$$\Rightarrow \|e\|\left(-\lambda_{min}(Q)\|e\| + 2\lambda_{max}(P)\zeta_{max}\right) < 0 \qquad (A.12)$$

$$\|e\| < 0 \quad \text{or}$$
$$\left(-\lambda_{min}(Q)\|e\| + 2\lambda_{max}(P)\zeta_{max}\right) < 0 \qquad (A.13)$$

$$\Rightarrow \lambda_{min}(Q)\|e\| > 2\lambda_{max}(P)\zeta_{max} \qquad (A.14)$$

$$\Rightarrow \|e\| > \frac{2\lambda_{max}(P)\zeta_{max}}{\lambda_{min}(Q)} \qquad (A.15)$$

And

$$\Rightarrow \sigma\|\Delta K\|_F^2 \left(2\Sigma_{min}\Lambda_{min} + \|\Sigma\|_F\|\Lambda\|_F\right) \qquad (A.16)$$
$$> \sigma\|K\|_F^2\|\Sigma\|_F\|\Lambda\|_F$$

$$\Rightarrow \|\Delta K\|_F^2 > \frac{\|K\|_F^2\|\Sigma\|_F\|\Lambda\|_F}{\left(2\Sigma_{min}\Lambda_{min} + \|\Sigma\|_F\|\Lambda\|_F\right)} \qquad (A.17)$$

$$\Rightarrow \|\Delta K\|_F > \sqrt[2]{\frac{\|K\|_F^2\|\Sigma\|_F\|\Lambda\|_F}{\left(2\Sigma_{min}\Lambda_{min} + \|\Sigma\|_F\|\Lambda\|_F\right)}} \qquad (A.18)$$

Where inequalities (A.17) and (A.18) means that by using of weighted sigma modification the *Lyapunov* candidate time derivative negative definite outside the compact set:
It's clear that the set α_w where the *Lyapunov* candidate is

$$\alpha_w = \left\{ \begin{array}{l} \left(e, \Delta K^T\right): e \in \mathcal{R}^n, \Delta K \in \mathcal{R}^{N\times m} \\ e < \dfrac{2\lambda_{max}(P)\zeta_{max}}{\lambda_{min}(Q)} \\ \wedge \Delta K_F < \sqrt[2]{\dfrac{K_F^{\,2}\Sigma_F\Lambda_F}{\left(2\Sigma_{min}\Lambda_{min} + \Sigma_F\Lambda_F\right)}} \end{array} \right\} \qquad (A.19)$$

negative definite by the effect of the proposed weighted sigma modification; weighted sigma modification works as a scaling factor of the region of stability; the region of stability may be tolarated to a larger bound for $\Sigma_{min}<1$ or restricted to a smaller bounds for $\Sigma_{min}>1$.

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