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Mathematics, Federal University of Agriculture, Abeokuta, Nigeria

Lateef O Sogbetun*, Bakai Ishola Olajuwon, Olalekan Fagbemiro

Department of Mathematics, Federal University of Agriculture, Abeokuta, Nigeria.

ABSTRACT: This paper examines the flow and heat transfer characteristics of an Eyring-Powell fluid passing over a stretched sheet surface that is being heated by hot fluid from beneath. The thermal mechanism of the model is analyzed on the considerations that the thermal conductivity is a linear function of temperature, the fluid viscosity obeys the Reynolds model, and that the Cattaneo–Christov heat flux model is incorporated into the energy equation. The governing nonlinear partial differential equations were transformed into a system of nonlinear ordinary differential equations using suitable similarity variables. The resulting self-similar problems were then solved using the spectral quasi-linearization method (SQLM). The effectiveness and accuracy of this method were demonstrated through error analysis and comparative studies with relevant existing results. Graphical outcomes illustrating the impact of pertinent fluid parameters in the model equations are presented as velocity and temperature profiles. It is noteworthy that both fluid temperature and velocity decline when the thermal relaxation parameter and slip velocity parameter are increased. The results also reveal that the fluid variables, such as the thermal relaxation time parameter , Eyring-Powell parameter , slip velocity parameter , surface-convection parameter , or radiation parameter boost the rate of heat transfer when any of these parameters is increased.

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1- Introduction

Rheological fluids are generally recognized by scientists and engineers as being more suitable than Newtonian fluids for various real-world engineering and industrial applications. Notable uses include the extrusion of polymers, the molding of metals, and petroleum drilling. Researchers have explored a variety of non-Newtonian fluids that exhibit distinct rheological behaviors [1–5]. Among these, the Eyring–Powell model has gained considerable importance. This model's relevance is not merely derived from empirical observations; it can also be understood through the kinetic theory of liquids. One significant advantage of the Erying-Powell model is its ability to accurately describe fluid behavior across a range of shear rates. Specifically, it aligns with Newtonian fluid characteristics at both low and high shear rates, making it versatile for different operational conditions. The insights gained from this model have been instrumental in understanding flows associated with modern industrial materials such as powdered graphite and ethylene glycol.

The Eyring–Powell model finds extensive application across various natural, geophysical, and industrial processes. These applications include temperature and moisture distribution in agricultural fields, environmental pollution dynamics, underground energy transfer mechanisms, crop damage due to freezing temperatures, thermal insulation considerations, and more. Recent studies have delved into different aspects of the Erying-Powell fluid under various conditions. Patel and Timol [6] conducted a numerical investigation on the flow behavior of an Eyring-Powell fluid under asymptotic boundary conditions. The influence of radiation on magnetohydrodynamic (MHD) flows of Eyring-Powell fluids over stretching surfaces was examined by Hayat et al. [7]. Rosca and Pop [8] examined the Powell-Eyring fluid flow and heat transmission over a decreasing surface in a parallel free stream. The flow of Powell-Eyring fluid over a nonlinear stretching sheet was studied analytically by Panigrahi et al. [9] Hayat et al. [10] looked at how convective boundary conditions affected the constant flow of Powell-Eyring fluid over a moving surface. Hayat et al. [11] looked at the impact of heat flux and chemical reactions on the flow of Eyring-Powell fluid past an exponentially extending sheet. Moreover, quantitative analyses involving thermal radiation and magnetic fields affecting Erying-Powell fluid flows have been approached using spectral collocation methods [12]. The MHD dissipative Powell-Eyring fluid flow caused by a stretching sheet with a convective boundary condition and slip was solved via the shooting technique and the Runge-

*Corresponding author's email: sogbetunlateef@gmail.com

Kutta Method [13].

The study of boundary layer flow over stretching sheets has gained significant attention recently due to its wide-ranging applications in various manufacturing processes. Understanding fluid behavior on different surfaces is essential to optimizing industrial operations. Abbas et al. [14] emphasized the practical benefits derived from analyzing fluid flow over stretching surfaces, particularly in the production of glass, polymer, aerodynamic plastic sheet extrusion, liquid coating of photographic films, condensation of metallic plate in cooling baths, and lubricant systems. The boundary layer flow of a viscous fluid across a sheet moving with a velocity proportional to the aperture distance was studied by Crane et al. [15]. Kumari and Nath [16] reported the exact solution of two-dimensional flow across a stretching surface in the presence of a magnetic field. Mukhopadhyay [17] investigated the MHD boundary layer flows past exponentially stretched sheets while incorporating heat radiation and slip conditions alongside the suction/blowing factor. The role of chemical reactions and unstable free convection on heat transmission via an expandable sheet in a permeable medium was investigated by Chamkha and Mansour [18]. Turkyilmazoglu ([19]) discussed solutions to two-dimensional laminar flow in a closed shape that crosses a continuously stretched sheet. Tawade et al. [20] presented a numerical method designed for solving unsteady, two-dimensional, laminar nanofluid flows using fourth-order Runge-Kutta schemes complimented by the shooting technique. Finally, the stagnation point flows involving MHD nanofluids toward expandable sheets were studied by Ibrahim et al. [21].

The heat transfer process, driven by temperature differences within or between bodies, plays a critical role in various applications such as power generation, nuclear fusion, and so on. The foundational understanding of heat conduction was established by Fourier [22], whose law describes the instantaneous rate of heat flow and leads to the parabolic heat equation. However, recognizing that different materials exhibit varied thermal relaxation times, Cattaneo [23] proposed in 1948 an integration of thermal relaxation time into Fourier's model. This adjustment enhances the accuracy of modeling effective heat transfer rates. Building on this idea, Christov [24] introduced a time derivative model called the Cattaneo-Christov heat flux model. A comparative study conducted by Hayat et al. [25] utilized the Cattaneo-Christov double diffusion model to analyze viscoelastic nanofluid flow. Furthermore, advancement was made by Liu et al. [26], who developed an improved model for heat conduction using Riesz fractional calculus combined with the Cattaneo-Christov framework. Meraj et al. [27] studied the role Cattaneo-Christov heat flux model on the Jeffrey fluid through the Darcy-Forchheimer flow with variable conductivity. Reddy et al. [28] used the Cattaneo-Christov heat flux to analyze the energy equation while examining the cross diffusion effects. Hadad [29] investigated thermal instability in porous media under the influence of a heat flux model. Hayat et al. [30] investigated how variations in surface thickness affect flows governed by the Cattaneo-Christov model framework. The impact of the Cattaneo-Christov heat flux model on the thermal behavior of a three-dimensional Maxwell fluid with varying thermal conductivity was demonstrated by Abbasi and Shehzad [31].

In this current study, we investigate the flow and heat transfer characteristics of an Eyring-Powell fluid over a convectively heated stretched sheet. This paper intends to achieve the following objectives:

To explore the thermal mechanisms associated with a convectively heated Erying-Powell fluid, particularly in the context of nonlinear thermal conductivity, temperature-dependent viscosity, and Cattaneo–Christov heat flux model

To provide critical insights on the roles of convective conditions and slip effects in modifying the flow dynamics under practical application scenarios.

To analyze the combined effects of non-linear thermal conductivity, temperature-dependent viscosity, viscous dissipation, joule heating, and slip velocity on the flow characteristics over a stretching surface.

The novelty of this research lies in its focus on the aforementioned objectives. This study extends existing literature, particularly the work of Sogbetun et al. [32], by integrating complex factors such as non-linear thermal conductivity and variable viscosity within the framework outlined by Cattaneo-Christov for heat flux modeling. To the best of our knowledge, this is one of the first studies that comprehensively addresses these aspects concerning convectively heated Erying-Powell fluids.

2- Model Formulation

An incompressible, electrically conducting, steady transport of an Eyring-Powell fluid passing over a convectively heated two-dimensional linearly stretching sheet with Cattaneo-Christov heat flux is studied. (x, y) are taking as the coordinate system of the flow and (u, v) are the respective velocity components. The x–axis is considered as the flow direction, while y axis is perpendicular to it, and hot fluid exists beneath the expandable sheet's bottom surface, as shown in Fig. 1. A non-varying magnetic field of strength B is imposed along the positive y-axis. The magnetic Reynolds number is considered to be small, hence the induced magnetic field is neglected. The continuous sheet moves in its own plane with the velocity $U_w = ax$ where a the stretching parameter is. The stretched sheet surface is heated by hot fluid from beneath through convection with the temperature $T_f = T_\infty + Ax^m$, $T_\infty < T_f$ where T_∞ denotes a stable cold fluid temperature away from the sheet, A and m are constants. The flow and heat transfer is being influenced by thermo-physical terms like viscous dissipation, radiation term, heat source, thermal relaxation parameter, and Joule heating.

Under the aforementioned assumptions, the governing boundary layer equations for an Eyring-Powell hydromagnetic fluid in the presence of Cattaneo-Christov heat flux take the form

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=$$

$$\frac{1}{\rho_\infty}\frac{\partial}{\partial y}\left[\mu^*\frac{\partial u}{\partial y}+\frac{1}{\tilde{\beta}C}\frac{\partial u}{\partial y}-\frac{1}{6\tilde{\beta}C^3}\left(\frac{\partial u}{\partial y}\right)^3\right] \tag{2}$$

$$+g\beta^*(T-T_\infty)-\frac{\sigma B^2}{\rho_\infty}u$$

$$\rho_\infty c_p\left[u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right]=$$

$$-\nabla.\vec{q}+\left[\mu^*\left(\frac{\partial u}{\partial y}\right)^2+\frac{1}{\tilde{\beta}C}\left(\frac{\partial u}{\partial y}\right)^2-\frac{1}{6\tilde{\beta}C^3}\left(\frac{\partial u}{\partial y}\right)^4\right] \tag{3}$$

$$+\sigma B^2u^2+Q_0(T-T_\infty)-\frac{\partial q_r}{\partial y}$$

Here, u and v denote the velocity components along the *x* and *y* axes, and the symbols $T,\rho_\infty,\sigma,c_p,k(T)$ and $\vec{q}$ denote the fluid temperature, ambient fluid density, electricity conductivity parameter, specific heat term, nonlinear thermal conductivity, dynamic viscosity and heat flux, respectively. The Powell-Eyring fluid parameters are symbolized by $\tilde{\beta}$ and *C*. These Eyring-Powell parameters, $\tilde{\beta}$ and *C*, are very important because they determine the unique rheological properties and behavior of the fluid. They have a direct impact on how the fluid behaves and interacts in different circumstances, providing vital information about how it reacts to external factors.

The Cattaneo-Christov heat flux model for the fluid takes the form

$$\vec{q}+\lambda_e\left[\frac{\partial q}{\partial t}+V.\nabla q-q.\nabla V+(\nabla.V)q\right]=-k(T)\nabla T \tag{4}$$

The liquid thermal conductivity and thermal relaxation time are presented by $k(T)$ and λ_e respectively. Replacing by $\lambda_e=0$ Eq. (4) reduced to the Fouriar's law. For incompressible, Eq. (4) gives

$$\vec{q}+\lambda_e[V.\nabla q-q.\nabla V]=-k(T)\nabla T \tag{5}$$

Now eliminate $\vec{q}$ from Eq. (3), we have

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}+\lambda_e\begin{bmatrix}\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)\frac{\partial T}{\partial x}\\+\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)\frac{\partial T}{\partial y}+u^2\frac{\partial^2T}{\partial x^2}\\+v^2\frac{\partial^2T}{\partial y^2}+2uv\frac{\partial^2T}{\partial x\,\partial y}\end{bmatrix}=$$

$$\frac{1}{\rho_\infty c_p}\frac{\partial}{\partial y}\left[k(T)\frac{\partial T}{\partial y}\right] \tag{6}$$

$$+\frac{1}{\rho_\infty c_p}\left[\mu^*\left(\frac{\partial u}{\partial y}\right)^2+\frac{1}{\tilde{\beta}C}\left(\frac{\partial u}{\partial y}\right)^2-\frac{1}{6\tilde{\beta}C^3}\left(\frac{\partial u}{\partial y}\right)^4\right]$$

$$+\frac{\sigma B^2u^2}{\rho_\infty c_p}+\frac{Q_0(T-T_\infty)}{\rho_\infty c_p}-\frac{1}{\rho_\infty c_p}\frac{\partial q_r}{\partial y}$$

With the boundary conditions:

$$u=ax+\frac{\lambda_1}{\mu_\infty}\left[\mu^*\frac{\partial u}{\partial y}+\frac{1}{\tilde{\beta}C}\frac{\partial u}{\partial y}-\frac{1}{6\tilde{\beta}C^3}\left(\frac{\partial u}{\partial y}\right)^3\right], \tag{7}$$

$$v=0,\quad -k\left(\frac{\partial T}{\partial y}\right)=h_f\left(T_f-T_\infty\right)\text{ at } y=0$$

$$u\to 0,\qquad T\to T_\infty\qquad \text{as } y\to\infty \tag{8}$$

We introduce the following stream function and similarity variables in Eq. (9)

$$\psi=\sqrt{a\nu_\infty}\,xf(\eta),$$

$$\eta=\sqrt{\frac{a}{\nu_\infty}}\,y, \tag{9}$$

$$\theta(\eta)=\frac{T-T_\infty}{T_f-T_\infty}$$

The fluid viscosity and thermal conductivity are expressed as:

$$\mu^*=\mu_\infty e^{-\gamma\theta}\quad\text{and}$$

$$k(T)=k_\infty\left(1+\delta^*(T-T_\infty)\right) \tag{10}$$

where γ is a viscosity variation parameter and μ_∞ is the fluid dynamic viscosity at the ambient temperature, k_∞ is the constant value of the coefficient of thermal conductivity far from the plate, and δ^* is a small parameter.

Applying Eqs. (9)-(10) to Eqs. (1), (2) and (6)-(8) give:

$$f'''\left(e^{-\gamma\theta}+\alpha\left(1-\delta f''^{2}\right)\right)-\gamma f''e^{-\gamma\theta}\theta' \\ -f'^{2}+ff''-Mf'+G_{rx}\theta=0 \tag{11}$$

$$\frac{1}{\Pr}\left(\left(1+\varepsilon\theta+R_d\right)\theta''+\varepsilon\theta'^{2}\right) \\ -\xi\left[4f'^{2}\theta-2ff''\theta-3ff'\theta'+f^{2}\theta''\right] \\ +f\theta'-2f'\theta+Ecf''^{2}\left(e^{-\gamma\theta}+\alpha-\frac{\alpha\delta}{3}f''^{2}\right) \\ +MEcf'^{2}+Q\theta=0 \tag{12}$$

$$f(0)=0 \\ f'(0)=1+\lambda e^{-\gamma\theta}f''(0)+\lambda\alpha f''(0)-\frac{1}{3}\lambda\alpha\delta f''^{3}(0) \\ \theta'(0)=-\Delta\frac{(1-\theta(0))}{(1+\varepsilon\theta(0))} \qquad f'(\infty)\to 0 \qquad \theta(\infty)\to 0 \tag{13}$$

Here $\alpha,\delta,\lambda,M,G_{rt},\Delta,Q,Ec,\Pr,\varepsilon,R_d$ and ξ represent the Eyring-Powell fluid parameters, slip parameter, magnetic parameter, thermal Grashof number, surface-convection parameter, heat source, Eckert number, Prandtl number, thermal conductivity parameter, radiation parameter and thermal relaxation term, respectively. Their mathematical expressions are hereby represented as follows:

$$\alpha=\frac{1}{\tilde{\beta}C\mu_\infty},\quad \delta=\frac{aU_w^2}{2\nu_\infty C^2},$$

$$\lambda=\lambda_1\sqrt{\frac{a}{\nu_\infty}},\quad M=\frac{\sigma B^2}{\rho_\infty a},$$

$$G_{rx}=\frac{g\beta^*\left(T_f-T_\infty\right)}{a^2x},\quad \Delta=\frac{h_f}{k_\infty}\sqrt{\frac{\nu_\infty}{a}},$$

$$Q=\frac{Q_0}{\rho_\infty c_p a},\qquad Ec=\frac{U_w^2}{c_p\left(T_f-T_\infty\right)},$$

$$\Pr=\frac{c_p\mu_\infty}{k_\infty},\qquad \varepsilon=\delta^*\left(T_f-T_\infty\right),$$

$$R_d=\frac{16\sigma^*T_\infty^3}{3k^*k_\infty},\quad \xi=\lambda_e a,\quad \mathrm{Re}_x^{\frac{1}{2}}=x\sqrt{\frac{a}{\nu_\infty}}$$

3- Skin Friction and Nusselt Number

The dimensional surface drag force and Nusselt number associated with the model are expressed in Eq. (14) respectively

$$Cf=\frac{\tau_w}{\rho\left(U_w\right)^2},\qquad Nu_x=\frac{x^2q_w}{k_\infty\left(T_f-T_\infty\right)} \tag{14}$$

Where

$$\tau_{ij}=-\left[\mu^*\frac{\partial u}{\partial y}+\frac{1}{\tilde{\beta}}\sinh^{-1}\left(\frac{1}{C}\frac{\partial u}{\partial y}\right)\right]_{y=0}= \\ -\left[\mu^*\frac{\partial u}{\partial y}+\frac{1}{\tilde{\beta}C}\frac{\partial u}{\partial y}-\frac{1}{6\tilde{\beta}C^3}\left(\frac{\partial u}{\partial y}\right)^3\right]_{y=0} \tag{15}$$

$$q_w=\left(-k(T)\frac{\partial T}{\partial y}+q_z\right)_{y=0} \tag{16}$$

Using Eqs. (14)-(16), the dimensionless skin friction and Nusselt number is obtained as

$$Cf\,\mathrm{Re}_x^{\frac{1}{2}}=-\left[\left(e^{-\gamma\theta(0)}+\alpha\right)f''(0)-\frac{\alpha\delta}{3}f''^{3}(0)\right] \tag{17}$$

$$Nu_x\,\mathrm{Re}_x^{-\frac{1}{2}}=-\left(\left(1+\theta(0)\varepsilon\right)\theta'(0)+R_d\theta'(0)\right) \tag{18}$$

The solution to the Eqs. (11)-(12) and their boundary conditions Eq. (13), are sought after using the Spectral Quasi-Linearisation Method. Applying the Taylor series of multiple variables Eqs. (11)-(13) yield

$$a_{0r}f_{r+1}'''+a_{1r}f_{r+1}''+a_{2r}f_{r+1}' \\ +a_{3r}f_{r+1}+a_{4r}\theta_{r+1}'+a_{5r}\theta_{r+1}=R_1 \tag{19}$$

$$b_{0r}\theta_{r+1}''+b_{1r}\theta_{r+1}'+b_{2r}\theta_{r+1} \\ +b_{3r}f_{r+1}''+b_{4r}f_{r+1}'+b_{5r}f_{r+1}=R_2 \tag{20}$$

Subject to

$$
\begin{aligned}
&f_{r+1}(0)=0 \\
&f_{r+1}{}'(0)=1+\lambda e^{-\gamma\theta_{r+1}} f_{r+1}{}''(0)+\lambda\alpha f_{r+1}{}''(0)-\frac{1}{3}\lambda\alpha\delta f_{r+1}{}''^{3}(0) \\
&\theta_{r+1}{}'(0)=-\Delta\frac{(1-\theta_{r+1}(0))}{(1+\varepsilon\theta_{r+1}(0))} \\
&f_{r+1}{}'(\infty)\to 0 \qquad \theta_{r+1}(\infty)\to 0
\end{aligned}
\tag{21}
$$

Where

$$
\begin{aligned}
R_1 &= a_{0r} f_r''' + a_{1r} f_r'' + a_{2r} f_r' \\
&\quad + a_{3r} f_r + a_{4r}\theta_r' + a_{5r}\theta_r - F_r
\end{aligned}
\tag{22}
$$

$$
\begin{aligned}
R_2 &= b_{0r}\theta_r'' + b_{1r}\theta_r' + b_{2r}\theta_r \\
&\quad + b_{3r} f_r'' + b_{4r} f_r' + b_{5r} f_r - G_r
\end{aligned}
\tag{23}
$$

$$
\begin{aligned}
F_r &= f_r'''\left(e^{-\gamma\theta_r}+\alpha\left(1-\delta f_r''^{2}\right)\right)-\gamma f_r'' e^{-\gamma\theta_r}\theta_r' \\
&- f_r'^{2} + f_r f_r'' - M f_r' + G_{rx}\theta_r
\end{aligned}
\tag{24}
$$

$$
\begin{aligned}
G_r &= \frac{1}{\Pr}\left(\left(1+\varepsilon\theta+R_d\right)\theta''+\varepsilon\theta'^{2}\right) \\
&-\xi\left[4f'^{2}\theta-2ff''\theta-3ff'\theta'+f^{2}\theta''\right] \\
&+f\theta'-2f'\theta+Ecf''^{2}\left(e^{-\gamma\theta}+\alpha-\frac{\alpha\delta}{3}f''^{2}\right) \\
&+MEcf'^{2}+Q\theta
\end{aligned}
\tag{25}
$$

$$
\begin{aligned}
a_{0r} &= \left(e^{-\gamma\theta_r}+\alpha\left(1-\delta f_r''^{2}\right)\right) \\
a_{1r} &= -2\alpha\delta f_r''' f_r'' - \gamma e^{-\gamma\theta_r}\theta_r' + f_r \\
a_{2r} &= -2f_r' - M
\end{aligned}
\tag{26}
$$

$$
\begin{aligned}
&a_{3r} = f_r'' \qquad a_{4r} = -\gamma f_r'' e^{-\gamma\theta} \\
&a_{5r} = -\gamma f_r''' e^{-\gamma\theta} + \gamma^2 f_r'' e^{-\gamma\theta}\theta' + G_{rt}
\end{aligned}
\tag{27}
$$

$$
\begin{aligned}
b_{0r} &= \frac{1}{P_r}\left(1+\varepsilon\theta_r+R_d\right)-\xi f_r^{2} \\
b_{1r} &= \frac{2\varepsilon\theta_r'}{P_r}+f_r+3\xi f_r f_r'
\end{aligned}
\tag{28}
$$

$$
\begin{aligned}
b_{2r} &= \frac{\varepsilon\theta_r''}{P_r}-4\xi f_r'^{2}+2\xi f_r f_r'' \\
&-2f_r'-\gamma Ecf_r''^{2}e^{-\gamma\theta_r}+Q
\end{aligned}
\tag{29}
$$

$$
b_{3r} = 2\xi f_r\theta_r + 2Ec\left(e^{-\gamma\theta}+\alpha\right)f_r'' - \frac{4\alpha\delta Ec}{3} f_r''^{3} \tag{30}
$$

$$
\begin{aligned}
b_{4r} &= -8\xi f_r'\theta_r + 3\xi f_r\theta_r' - 2\theta_r + 2MEcf_r' \\
b_{5r} &= 2\xi f_r''\theta_r + 3\xi f_r'\theta_r' - 2\xi f_r\theta_r'' + \theta_r'
\end{aligned}
\tag{31}
$$

Eqs. (19)-(20) are integrated using Chebyshev Spectral Collocation method which is based on the Lagrange interpolation polynomial defines on the interval $[-1,1]$. By employing algebraic mapping in Eq. (32)

$$
\varepsilon = \frac{2\eta}{L}-1, \qquad \varepsilon\in[-1,1] \tag{32}
$$

the semi-infinite domain $[0,\infty)$ in Eq. (21) is transformed into the computational domain $[-1,1]$ and thereafter discretize into N collocation points by Gauss-Lobatto collocation points using Eq. (33)

$$
\begin{aligned}
&\varepsilon_j = \cos\frac{\pi j}{N}; \qquad \varepsilon\in[-1,1]; \\
&j = 0,1,2,3,\ldots,N
\end{aligned}
\tag{33}
$$

Using the relation $D=\frac{2}{L}\mathrm{D}$ where D is the Chebyshev spectral differentiation matrix, applying the spectral transformation in Eqs. (32)-(33) on the coupled Eqs. (19)-(20) and their boundary condition Eq. (21), we have

$$
AY = B \tag{34}
$$

Subject to

$$f_{r+1}\left(\varepsilon_{N_\infty}\right)=0$$

$$f'_{r+1}\left(\varepsilon_{N_\infty}\right)=1+\lambda e^{-\gamma\theta_{r+1}}f''_{r+1}\left(\varepsilon_{N_\infty}\right)+\lambda\alpha f''_{r+1}\left(\varepsilon_{N_\infty}\right)-\frac{1}{3}\lambda\alpha\delta f''^{3}_{r+1}\left(\varepsilon_{N_\infty}\right) \tag{35}$$

$$\theta'_{r+1}\left(\varepsilon_{N_\infty}\right)=-\Delta\frac{\left(1-\theta\left(\varepsilon_{N_\infty}\right)\right)}{\left(1+\varepsilon\theta\left(\varepsilon_{N_\infty}\right)\right)} \tag{36}$$

$$f'_{r+1}\left(\varepsilon_{N_0}\right)=0 \qquad \theta_{r+1}\left(\varepsilon_{N_0}\right)=0 \tag{37}$$

In Eq. (34), A represents $2\left(N+1\right)\times 2\left(N+1\right)$ matrix, Y and B connotes $4\left(N+1\right)\times 1$ matrices as defined below

$$A=\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \tag{38}$$

$$A_{11}=a_{0r}D^3 f_{r+1}\left(\varepsilon_j\right)+a_{1r}D^2 f_{r+1}\left(\varepsilon_j\right)+a_{2r}Df_{r+1}\left(\varepsilon_j\right)+a_{3r}f_{r+1}\left(\varepsilon_j\right)I \tag{39}$$

$$A_{12}=a_{4r}D\theta_{r+1}\left(\varepsilon_j\right)+a_{5r}\theta_{r+1}\left(\varepsilon_j\right)I \tag{40}$$

$$A_{21}=b_{3r}D^2 f_{r+1}\left(\varepsilon_j\right)+b_{4r}Df_{r+1}\left(\varepsilon_j\right)+b_{5r}f_{r+1}\left(\varepsilon_j\right)I \tag{41}$$

$$A_{22}=b_{0r}D^2\theta_{r+1}\left(\varepsilon_j\right)+b_{1r}D\theta_{r+1}\left(\varepsilon_j\right)+b_{2r}\theta_{r+1}\left(\varepsilon_j\right)I \tag{42}$$

$$Y=\begin{bmatrix} f_{r+1}\left(\varepsilon_0\right), f_{r+1}\left(\varepsilon_1\right),\ldots,f_{r+1}\left(\varepsilon_N\right), \\ \theta_{r+1}\left(\varepsilon_0\right),\theta_{r+1}\left(\varepsilon_1\right),\ldots,\theta_{r+1}\left(\varepsilon_N\right) \end{bmatrix}^T \tag{43}$$

$$B=\begin{bmatrix} R_1\left(\xi_0\right),R_1\left(\xi_1\right),\ldots,R_1\left(\xi_N\right), \\ R_2\left(\xi_0\right),R_2\left(\xi_1\right),\ldots,R_2\left(\xi_N\right) \end{bmatrix}^T \tag{44}$$

In Eqs. (38)-(44), T signifies the transpose of a matrix, I denotes an $\left(N+1\right)\times\left(N+1\right)$ identity matrix and a_{kr} and b_{kr}, $\left(k=0,1\ldots,n\right),\left(r=0,1,\ldots,N\right)$ are diagonal matrices of size $\left(N+1\right)\times\left(N+1\right)$. Applying the invertible method, the solution to Eqs. (34)-(37) is expressed as

$$Y=A^{-1}B \tag{45}$$

The appropriate initial solutions for Eqs. (11)-(12) and their boundary conditions in Eq. (13) are given by Eqs. (46)-(47)

$$f_r=\left(\frac{1}{3}\lambda\alpha\delta-\lambda e^{-\gamma\theta}-\lambda\alpha-2\right)\eta e^{-\eta}+\left(\frac{2}{3}\lambda\alpha\delta-2\lambda e^{-\gamma\theta}-2\lambda\alpha-3\right)e^{-\eta}+2\lambda e^{-\gamma\theta}+2\lambda\alpha-\frac{2}{3}\lambda\alpha\delta+3 \tag{46}$$

$$\text{Thus, } \theta=\frac{\Delta}{1+\Delta}e^{-\left(\frac{1+\Delta}{1+\Delta+\varepsilon\Delta}\right)\eta} \tag{47}$$

4- Result and Discussion

Here, numerical and graphical representations of the influence of salient fluid variables in the model Eqs. (11) – (13) are presented for discussion for better understanding. For the numerical computation, the fluid parameters were assigned the following values: $\Delta=Q=\gamma=0.1$, $\alpha=\xi=\varepsilon=\delta=\lambda=Ec=M=0.2$, $Rd=Grx=0.5$ and $\Pr=1.2$ except otherwise stated.

The validation of our chosen solution methodology is clearly established through the comparative numerical results presented in Table 1. By setting $\Pr=1$ and all other fluid parameters to zero while maintaining non-zero values for α and δ, we created conditions directly comparable to previous studies. Our computational results demonstrate excellent agreement with three independent benchmark solutions: Abbas et al. [13] obtained using the fourth-order Runge-Kutta method combined with shooting technique, Hayat et al. [33] derived through homotopy analysis method (HAM), and Sogbetun et al. [32] solved via the spectral quasi-linearization method (SQLM). This strong correlation across different numerical approaches not only verifies the accuracy of our implementation but also robustly confirms the effectiveness and precision of SQLM in solving boundary value problems. The consistent alignment of our results with these established methods underscores the reliability of both our current findings and the broader applicability of SQLM for similar computational challenges.

Fig. 2(a, b) show the effect of the thermal relaxation time parameter (ξ) on the velocity distribution $(f'(\eta))$ and temperature distribution $(\theta(\eta))$ respectively. The relaxation time parameter shows the same effect on both the velocity and temperature distributions for the Eyring-Powell fluid. For a greater estimation of thermal relaxation parameter, the thickness of both the fluid velocity and temperature distribution wane. An increase in thermal relaxation time indicates that it takes longer for particles to exchange heat with nearby particles, resulting in a drop in system temperature and thus leads to a decrease in the dispersion of temperatures and fluid velocity.

Fig. 3(a, b) illustrate the impact of the slip velocity parameter (λ) on the heat transfer rate $\theta(\eta)$ and the velocity distribution $f'(\eta)$. A similar downward trend is observed for both the velocity profile and thermal profile for an increasing slip velocity parameter (λ) in both Fig. 13(a) and 3(b). This observable fact can be explained physically by the slip velocity effect, which signifies the existence of roughness or irregularities on the sheet's surface. Therefore, the fluid velocity in the boundary layer area is slowed down by this specific characteristic.

The consequence of the Powell-Eyring parameter (α) variation against the fluid velocity profile $(f'(\eta))$ and temperature profile $(\theta(\eta))$ is depicted in Fig. 4(a, b). The graph reveals a major advancement in the velocity field and a small decline in the temperature field for a greater estimation of α. The inverse relationship between the power-Eyring parameter α and viscosity parameter is responsible for the outcome. An increment in the fluid viscosity results in the Powell-Eyring parameter reduction, which lowers fluid internal resistance by allowing the fluid molecules to travel more freely and thus increases the velocity field.

Analysis of the Powell-Eyring parameter (δ) via fluid velocity distribution $(f'(\eta))$ and temperature distribution $(\theta(\eta))$ is expressed in Fig. 5(a, b). Observations from the Fig. reveal a distinct impact Powell-Eyring parameter δ has on the velocity field and temperature field. The thermal distribution for an Eyring-Powell fluid appreciates for a higher Eyring-Powell parameter $(\delta = 0.2, 0.4, 0.6, 1.0)$ while the fluid flow velocity declines.

Consequence of the magnetic parameter (M) variation against the Powell-Eyring fluid flow velocity $f'(\eta)$ and temperature distribution $\theta(\eta)$ is illustrated in Fig. 6(a, b). It is observed that an augmentation in the magnetic parameter decelerates the velocity gradient of the Eyring-Powell fluid. Greater estimation in magnetic influence raises the resistance force or Lorentz force, which acts perpendicular to the path of fluid flow. As a result, the boundary layer thins as the Lorentz force grows and the impedance force rises in an attempt to resist fluid flow. It is clear that raising the magnetic parameter results in an improvement in the temperature distribution and the sheet temperature $\theta(0)$, which raises the thermal boundary layer thickness.

Fig. 7(a, b) represent the influence of viscosity parameter variation (γ) on the fluid velocity profile $(f'(\eta))$ and the thermal distribution $(\theta(\eta))$ for Eyring-Powell fluid. An exaggerated estimation of viscosity parameter reduces the fluid velocity gradient but raises the temperature field for an Eyring-Powell fluid. It is a fact that fluid's molecules interact more intensely at a higher viscosity, which hinders the molecular movement capability. At greater estimation of viscosity, the fluid particles encounter greater resistance while flowing and this leads to a rise in internal resistance and a slight decrease in the fluid velocity distribution.

The impact of the surface-convection parameter (Δ) on the temperature distribution $(\theta(\eta))$ and velocity distribution $f'(\eta)$ is seen in Fig. 8(a, b). A higher surface-convection parameter raises both the fluid temperature and the sheet temperature $\theta(0)$ due to the direct relationship between the surface-convection parameter and the heat transfer coefficient. Furthermore, as the value of Δ grows, the velocity distribution of the boundary layer substantially increases. Surface-convection parameter is responsible for heat transfer from the fluid to its surroundings. When the surface-convection parameter increases, the enhanced heat transfer causes the fluid's temperature to increase.

The graphical representation of the temperature distribution $\theta(\eta)$ and velocity distribution $f'(\eta)$ of the Eyring-Powell fluid against variation in thermal conductivity parameter (ε) is shown in Fig. 9(a, b). The fluid velocity field is indirectly affected by the thermal conductivity parameter, which results in a little increase in the fluid velocity gradient. Furthermore, advancement in thermal conductivity parameter implies a higher molecular kinetic energy which results in an increase in molecular collisions and greater thermal boundary layer thickness away from the sheet.

The relationship between the Powell-Eyring fluid velocity field $f'(\eta)$, temperature distribution $(\theta(\eta))$ and radiation parameter (R_d) are presented in Fig. 10(a,b). In Fig. 10(a) we notice the impact of the radiation parameter on the temperature and found that the temperature field escalates when the radiation parameter is boosted. Also, the reaction of velocity field to increasing radiation parameter is illustrated in Fig. 10(b). The graph indicates that for every increase in the radiation parameter, fluid velocity accelerates.

Fig. 11(a, b) show how the Eyring-Powell fluid temperature distribution $\theta(\eta)$ and flow profile $f'(\eta)$ are affected by the heat generation/absorption parameter (Q). For both the heat distribution and the fluid velocity in Fig. 11(a) and 11(b), a consistent trend is observed. An improvement in the heat generation/absorption parameter $(Q = 0.1, 0.2, 0.3, 0.4)$ increases both the temperature profile and the velocity field.

Fig. 12(a, b) display the reaction of Eyring-Powell thermal profile $(\theta(\eta))$ and momentum profile $f'(\eta)$ to varying Eckert number (Ec). An Eckert number is the process by which kinetic energy is transformed into stored energy and subsequently dispersed as heat when work is done in proportion to a viscous fluid's stresses. Therefore, a rise in Eckert number significantly improves the temperature distribution and sheet temperature and indirectly raises the fluid velocity gradient.

Fig. 13(a,b) show the effect of the thermal Grashof number (G_{rt}) on the velocity field $(f'(\eta))$ and temperature field $(\theta(\eta))$ for an Eyring-Powell fluid. Fig. 13(a) represent the change in velocity profile under the influence of thermal Grashof number. For a greater estimation of the thermal Grashof number $Grt = 0.5, 1.0, 2.5, 5.0$, the fluid velocity improves. The behavior of the fluid temperature field towards an enhanced thermal Grashof number is found in Fig. 13(b). We observed that the temperature profile for Eyring-Powell fluid decreases when enhancing the thermal Grashof number

Fig. 14(a, b) illustrates the consequence of Prandtl number (Pr) variation against the fluid velocity profile $(f'(\eta))$ and temperature profile $(\theta(\eta))$. Fig. 14(a) represents the change in velocity profile under the influence of Prandtl number. An increase in the estimation of Prandtl number causes a decline in velocity profile. Fig. 14(b) depicts effect of Prandtl number on thermal field. For increasing Prandtl number, thermal diffusivity decreases, meaning that fluid particles move more slowly from the hot to the cold side which consequently reduces the temperature profile of an Eyring-Powell fluid.

4- 1- Phyisical Quantities

Table 2 presents an overview of the impact of some pertinent varying fluid properties on the skin friction and Nusselt number of an Eyring-Powell fluid. The computed values shown in the table reveal that advancement in either the thermal relaxation time parameter (ξ), Eyring-Powell parameter (α), or thermal conductivity parameter (ε) thickens the skin friction of the fluid. In contrast, a higher value of the Powell-Eyring parameter (δ), viscosity parameter variation (γ), slip velocity parameter (λ), surface-convection parameter (Δ), radiation parameter (Rd), or heat generation/absorption parameter (Q) causes the skin friction to wane. Further, fluid variables such as the thermal relaxation time parameter (ξ), Eyring-Powell parameter (α), slip velocity parameter (λ), surface-convection parameter (Δ), or radiation parameter (Rd) boost the Nuselt number when either of the parameters is increased. However, the augmentation of the Powell-Eyring parameter (δ), viscosity parameter variation (γ), thermal conductivity parameter (ε), or heat generation/absorption parameter (Q) reduces the Nuselt number.

5- Conclusion

An insight into the consequences of the varying fluid properties on the flow and heat transfer characteristics of Eyring-Powell fluid over a convectively heated stretched sheet in the presence of Cattaneo–Christov heat flux model is hereby presented below:

Enhancement of the Powell-Eyring parameter (α), thermal relaxation parameter (ξ), slip velocity parameter (λ), thermal Grashof number (G_{rt}) or Prandtl number (Pr) against the Eyring-Powell fluid temperature profile $(\theta(\eta))$ reveals a decline in the fluid temperature field

Augmentation of thermal relaxation parameter (ξ), slip velocity parameter (λ), Powell-Eyring parameter (δ), magnetic parameter (M), viscosity parameter (γ), or Prandtl number (Pr) decelerates the velocity gradient of Eyring-Powell fluid.

The thermal boundary layer thickness of an Eyring-Powell fluid appreciates for a higher Powell-Eyring parameter (δ), magnetic parameter (M), viscosity parameter (γ), surface-convection parameter (Δ), thermal conductivity parameter (ε), radiation parameter (R_d), heat generation/absorption parameter (Q), or Eckert number (E)

Advancement in the Eyring-Powell fluid velocity is observed when the Powell-Eyring parameter (α), surface-convection parameter (Δ), thermal conductivity parameter (ε), radiation parameter (R_d) heat generation/absorption parameter (Q) thermal Grashof number (G_{rt}) or Eckert number (E) is boosted.

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