AUT Journal of Modeling and Simulation

AUT J. Model. Simul., 55(2) (2023) 359-368
DOI: 10.22060/miscj.2024.22496.5328

Reference Tracking of Nonlinear Dynamic Systems over Additive White Gaussian Noise Channel

Somayeh Dolatkhah Takloo [1], Alireza Farhadi[2], Ali Khaki-Sedigh[1]*

[1] Faculty of Eelectrical Eengineering, K. N. Toosi University of Technology,Tehran, Iran
[2] Department of Electrical Engineering, Sharif University of Technology, Tehran, Iran

ABSTRACT: This paper is concerned with state tracking as well as reference tracking of nonlinear dynamic systems with process and measurement noise over the Additive White Gaussian Noise (AWGN) channel which is subject to transmission noise and transmission power constraints. The AWGN channel is a continuous alphabet channel. Therefore, this channel is very suitable for controlling dynamic systems over wireless communication links. To address these problems, a novel encoder, decoder, and controller are proposed. This method compensates for communication imperfections and maintains real-time reference tracking at the end of the communication link. For identifying the time of linearization in the encoder and decoder, Monte-Carlo approximation is applied. Using the Monte Carlo approximator provides a possible approximation of the estimation error in the encoder and decoder at the same time. The linearization method is based on the variable (optimal) linearization rate. A proper encoder, decoder, and controller for real-time state estimation and reference tracking are proposed. The nonlinear dynamic system which was considered in this paper has process and measurement noises. Simulation results illustrate the satisfactory state tracking and reference tracking performances of the proposed technique; while the variable linearization technique is used.

Review History:

Received: Jun. 24, 2023
Revised: May, 01, 2024
Accepted: May, 28, 2024
Available Online: May, 30, 2024

Keywords:

Autonomous vehicle
Telepresence
Teleoperation
AWGN Channel
The Extended Kalman Filter
Monte-Carlo Approximation

1- Introduction

Networked Control Systems (NCS) have been gaining sharp attention from the research community in recent years. The elimination of unnecessary wiring, effective reduction of system complexity, and wide range of applications are some of the advantages of NCSs. Nevertheless, the design and development of NCSs are subject to some difficulties, such as unavoidable random packet erasure and dropout, communication noise, distortion, etc. A simple example of NCS is the problem of telepresence and teleoperation of Unmanned Aerial Vehicles (UAVs). The wide applications of miniature drones, which are in fact a specific form of UAV is one of the main research motivations for nonlinear networked control systems. The miniature drone is a small unmanned aerial vehicle, which is controlled remotely by a distant controller/operator. Fig. 1 illustrates a basic block diagram for controlling UAVs over communication channels subject to imperfections. The imperfect communication channel is modeled based on the environment where NCS is deployed. Hence, it seems that a proper channel for controlling dynamic systems over a communication channel is the AWGN channel; this channel is a basic model for satellite communication, deep space communication, and particularly when the line of sight is strong.

Reference tracking (teleoperation) and telepresence are the two main goals of remotely controlled autonomous vehicles. Reference tracking means the tracking of a desired trajectory designed by a remote human operator/intelligent control unit; telepresence means providing the states of a remotely controlled vehicle for a remote human operator/ intelligent control unit in real-time so that remote human operator/intelligent control unit can design proper desired trajectory for the satisfactory remote reference tracking. In the teleoperation of miniature UAVs, remote autonomous vehicles should track reference signals generated by a distant controller or operator based on the information received from remote vehicles via a wireless network. Generated control signals should be also communicated to the remote vehicle via a wireless communication network. Because the generated control signals can be communicated to the remote vehicle with high transmission power, effective information transmission can be achieved from the distant controller to the dynamic system. However, the transmission of sensor measurements from the UAV to the distant controller is subject to limited transmission power; because miniature UAVs are small and equipped with limited-capacity onboard batteries. Hence, the communication from a UAV to a distant controller is subject to communication imperfection as shown in Fig. 1.

The transmission of information via the AWGN channel is subject to transmission noise and also the antenna's power constraint. One of the motivations for considering the AWGN

*Corresponding author's email: sedigh@kntu.ac.ir

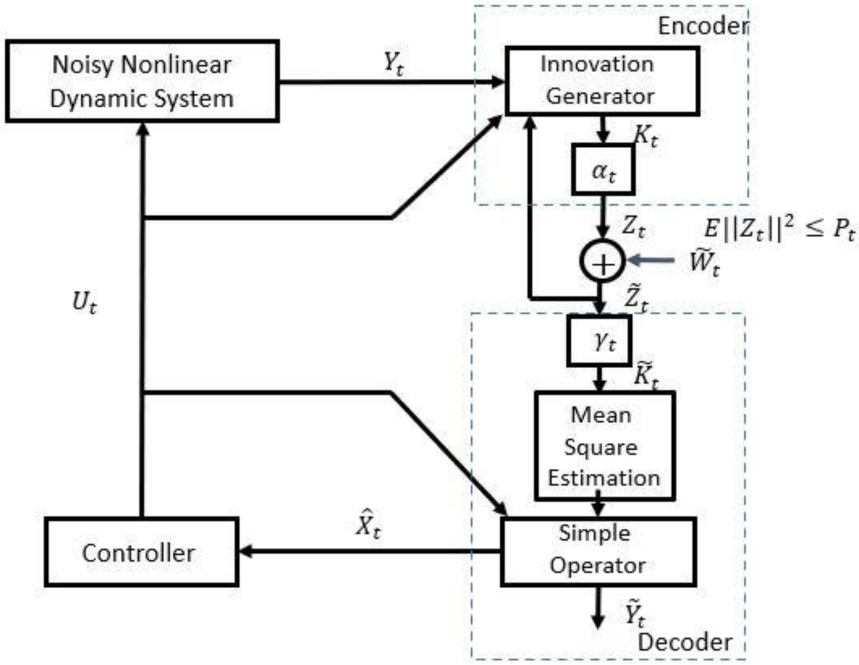

Fig. 1. A nonlinear noisy dynamic system controlled over the AWGN channel

channel in this paper is the availability of relatively cheap with long communication range FM transceivers [1].

In the literature, many works addressed the problem of controlling linear dynamic systems over communication channels [1] - [4]. Most of these papers are concerned with the stability and state tracking of linear dynamic systems over the AWGN channel. For example, [4] addressed the necessary conditions for the stabilization of noiseless linear dynamic systems over the AWGN channel. Dynamic systems in the real world are nonlinear; hence, in recent years many works addressed the problem of controlling nonlinear dynamic systems over the AWGN channel [5] - [9]. In the aforementioned papers, a number of issues, such as estimation, stability, and performance were addressed. [5] - [7] studied the problem of stability and state tracking of nonlinear systems over the AWGN communication link. In [6] a method presented an approximate linear dynamic system with the use of a describing function that can replace the nonlinear time-invariant system. So, the presented method in [6] is applicable only for invariant dynamic systems that respond periodically to input signals. What's more, in this paper measurement and process noise are considered.

In [8] the stability condition for the sampled continuous-time controlled systems is considered. [9] proposes a transmission strategy for a communication system where a sender sends messages over a memoryless Gaussian point-to-point channel to a receiver and receives the output feedback over another Gaussian channel with known variance and unit delay. The sender sequentially transmits the message over multiple times till a certain error performance is achieved. In this paper, although the linearization technique is used, the linearization period is not accurate. In [10] the reference tracking of nonlinear dynamic systems over the AWGN channel using the fixed rate linearization method is addressed.

This paper aims to fill the gap in the literature by addressing the problems of state tracking as well as reference tracking of a quite general form of discrete-time nonlinear dynamic systems subject to both process and measurement noises over the discrete-time AWGN channel, with application in the telepresence and teleoperation of autonomous vehicles. This issue has not been studied before. In this paper, we are concerned with the block diagram of Fig. 1 described by nonlinear dynamic systems and the AWGN channel. Since the encoder and decoder are separated, a new technique is required to determine the linearization time in both at the same time. The innovative method used in this paper to address these problems is presenting a proper encoder, decoder, and controller for mean square reference tracking at the end of the communication link and stability of nonlinear dynamic systems when measurements are sent through the AWGN channel. The decoder for the case of the linearization with the variable rate is the extended Kalman filter with the optimal linearization period. To the best of our knowledge, the extended Kalman filter with the variable linearization period has not been presented in the literature. From a stability perspective of switching systems, dwell time is the minimum required time, which should be considered before switching to the next subsystem. In this paper, the required dwell time is computed based on sampling time, and the linearization time is chosen greater than the dwell time, which is the right time for linearization. Therefore, dwell time is the lower limit for linearization time. For the upper limit, the mean square estimation error in the decoder is used.

In this paper, we implement a linearization method, which is different from our previously proposed method in [12]. The method proposed in [12] and the method proposed in this paper are successful provided the encoder and decoder linearize the nonlinear dynamic system at the same time. In [12] the communication link is the packet erasure channel and therefore the encoder after linearizing the nonlinear dynamic system sends information bits with a different length to inform the decoder of the right time for linearization. However, we focus on a different communication link and therefore the mechanism proposed in [12] is useless in the setup considered in this paper. Also in this paper delay is considered less than sampling time and is ignored.

The satisfactory performance of the theoretical developments is illustrated via computer simulations by applying the proposed encoder, decoder, and controller on the unicycle model.

This paper is organized as follows: Section 1 was the introduction. Section 2 presents the system model and preliminaries. Section 3 introduces theoretical development. Simulation results are given in Section 4 and Section 5 concludes the paper.

2- System Model and Preliminaries

Throughout certain conventions are used: $E[\bullet]$ denotes the expected value, $|\bullet|$ the absolute value, $\|\bullet\|$ the Euclidean norm and V' denotes the transpose of vector/ matrix V . A^{-1} and $\lambda_i(A)$ denote the inverse and eigenvalues of a square matrix A , respectively. $\mathbb{R}$ and $\mathbb{N}$ denote the sets of real numbers and natural numbers, respectively. I is the identity matrix and $X^{(i)}$ denotes the ith element of the vector of X . $\underline{0}$ denotes the zero vector/matrix. $N^+=\{0,1,2,3,\cdots\}$ and $\mathbb{R}^+$ is the set of non-negative real numbers. State and reference tracking of nonlinear dynamic systems over the AWGN channel are addressed in this paper for the block diagram of Fig. 1. The building blocks of Fig. 1 are described below:

2- 1- Communication Channel

The communication channel is a parallel AWGN channel. It is described by:

$$\tilde{Z}_t = Z_t + \widetilde{W}_t \tag{1}$$

where $Z_t \in \mathbb{R}^l$ is the channel input and $\tilde{Z}_t \in \mathbb{R}^l$ is the channel output. $\widetilde{W}_t$ is the channel noise with zero mean and diagonal variance W_c . In addition to suffering from transmission noise, the AWGN channel is subject to limited transmission power $\left|Z_t^{(i)}\right| \le P_t^{(i)}$, where $P_t^{(i)}$ is the upper limit on the antenna's power.

2- 2- Encoder

The encoder is a causal function that maps $\left(Y_t, \tilde{Z}^{t-1}, U^{t-1}\right) \to Z_t$, $\tilde{Z}^{t-1} = \left(\tilde{Z}_0, \tilde{Z}_1, \cdots, \tilde{Z}_{t-1}\right)$ (available for the encoder via the feedback channel) and $U^{t-1} = \left(U_0, U_1, \cdots, U_{t-1}\right)$.

2- 3- Decoder

The decoder is the extended Kalman filter which will be described later.

2- 4- Controller

The controller has the following form with the proper gain $L_t : U_t = L_t \hat{\tilde{X}} + \mu_t$. μ_t is chosen so that the tracking objective is met.

2- 5- Plant

The nonlinear noisy dynamic system in this paper is described as follows: [11]

$$\begin{cases} X_{t+1} = F(X_t, U_t) + W_t \\ Y_t = G(X_t, U_t) + V_t \end{cases} \tag{2}$$

where $t \in N^+$ is the time instant, $F(X_t, U_t) \in \mathbb{R}^n$ is a nonlinear vector function, $X_t \in \mathbb{R}^n$ is the system state variables vector, $Y_t \in \mathbb{R}^l$ is the observation signal vector and $U_t \in \mathbb{R}^m$ is the control signal.

It is assumed that the initial state X_0 is a Gaussian-distributed random variable with a mean $\bar{X}_0$ and variance Q_0 . $W_t \in \mathbb{R}$ is the process noise and $V_t \in \mathbb{R}$ is the measurement noise. Both noises are i.i.d. with normal distribution and zero mean with Q and R variances, respectively. X_0, W_t and V_t are mutually independent. The nonlinear dynamic model in this paper that is used for simulation is the unicycle model, which is an abstract model for autonomous vehicles including UAVs, described as follows: [11]

$$\begin{cases} \dot{X}(t) = \mathrm{v}(t)\cos(\phi(t)) \\ \dot{Y}(t) = \mathrm{v}(t)\sin(\phi(t)) \\ \dot{\phi}(t) = u(t) \end{cases} \tag{3}$$

Here, t is the continuous time index, $\left[x(t) \quad y(t) \quad \phi(t)\right]^{tr}$ denotes the state vector, $\mathrm{v}(t)$ is the forward velocity and $u(t)$ is the orientation rate. The input is $U(t) = \left[\mathrm{v}(t) \quad u(t)\right]^{tr}$ and the output is $Y(t) = \left[x(t) \quad y(t) \quad \phi(t)\right]^{tr}$. Throughout, it is assumed that the above dynamic is subject to the process and measurement noises: $W_t \sim N(0,Q)$ and $V_t \sim N(0,R)$ respectively.

3- Theoretical Development

In this section, an encoder, decoder, and controller for the mean square state tracking as well as the reference tracking of noisy nonlinear dynamic systems over the AWGN channel are presented.

The applied methodology is based on the linearization of the nonlinear noisy dynamic system at operating points, as follows: In the beginning, the nonlinear dynamic is linearized at the initial state $\left(\bar{X}_0, U_0\right), \left(U_0 = \underline{0}\right)$. Then the coding scheme presented in [11] is applied to the extracted linear model in each sampling time. For each element of the measurement vector of the dynamic system (2), we use the coding scheme of Section [11]. The linearized model is a good approximation of the nonlinear dynamic (2) in the beginning; therefore, the mean square estimation error, i.e., $EX_t - \hat{X}_t{}^2$ decreases as time progresses; because the decoder receives more measurements from the dynamic system. But, as time progresses, the nonlinear dynamic system should be linearized at a new operating point so that the best approximation of the nonlinear system is available at all times. In the proposed method the largest possible linearization period that results in a good approximation of the nonlinear system is chosen by the family of the linearized systems. Therefore, the encoder and decoder can approximate the mean square estimation error using the Monte-Carlo and then the best linearization rate is determined. The proposed encoder and decoder rules are in fact the extended Kalman filter for state estimation over the AWGN channel with the optimal linearization rate.

In [13] the stability is shown by determining the linearization period under the dwell time τ_a. T_t is shown in [14] that the average dwell time τ_a, which is a measure of the frequency of switches (here the frequency of updating linearized system), should be greater than or equal to a critical value denoted by τ_a^* a defined as follows. $\tau_a \geq \tau_a^*$;

$\tau_a = \dfrac{1}{N_t}$, $\tau_a^* = \dfrac{\ln h}{\ln \lambda - \ln \lambda^*}$ where N_t is the number of switches that occur in the time interval of $[0,t]$ and h, λ and λ^* are defined as follows:

For all linearized models with the system matrix $A_{[j]}$, there exist $\lambda_1 < 1$ and $\lambda_2 > 1$ such that the following relations hold [14]:

$A_{[j]} < 1, A_{[j]}^t \leq h_j \lambda_1^t$, where $A_{[j]} \geq 1; A_{[j]}^t \leq h_j \lambda_2^t$. Then, $h = \max h_j$, $\lambda \in [\lambda_1, 1]$, $\lambda^* \in [\lambda_1, \lambda]$

is the largest value that satisfies the following inequality for some $c > 0 : X_t \leq c\left(\lambda^*\right)^t X_0$ In order to satisfy the above condition, it is sufficient that the linearization period T_l is much larger than the system sampling period (e.g., $T_l \geq 15T$, where T is the sampling period).

3- 1- Encoder, decoder and controller

For the simplicity of the presentation of the encoder, decoder, and controller, without loss of generality, suppose that the nonlinear dynamic system is controlled over the single input - single output discrete time AWGN channel. Then:

3- 1- 1- Encoder Description:

The innovation generator for each linearized system has the following description (α_t is defined shortly):

$$Z_t = \alpha_t K_t,\ K_t = Y_t - \hat{Y}_t,\ \hat{Y}_t = C\hat{X}_t \tag{4}$$

3- 1- 2- Decoder Description

For each linearized system, the pre-decoding part (i.e., $\tilde{K}_t$) is described by

$$\tilde{K}_t = \gamma_t Z_t \tag{5}$$

where $\tilde{Z}_t$ is the channel output and $\alpha_t, \gamma_t \in \mathbb{R}^+$ are defined as follows:

$$\alpha_t = \sqrt{\frac{\eta_t W_c}{D_v}}\ ,\ \eta_t = 1 - \frac{D_v}{\psi_t},\ \gamma_t = \sqrt{\frac{D_v \eta_t}{W_c}} \tag{6}$$

where $D_v < min_{t \in \mathbb{N}^+} \psi_t$. D_v is a given threshold and ψ_t will be defined shortly. The mean square state estimator, when the linearized system is valid, has the following description:

$$\hat{X}_{t+1} = A\hat{X}_t$$

$$+\frac{1}{\alpha_t . \gamma_t} A\pi_t C^{tr}\left(C\pi_t C^{tr} + DRD^{tr} + \frac{W_c}{\alpha_t^2}\right)^{-1}\tilde{K}_t \tag{7}$$

$$+BU_t \quad \hat{X}_0 = \bar{X}_0 = E[X_0]$$

Where π_t is the mean square state estimation error given by the following Riccati equation:

$$\pi_{t+1} = A\pi_t A^{tr} -$$

$$A\pi_t C^{tr}\left(C\pi_t C^{tr} + DRD^{tr} + \frac{W_c}{\alpha_t^2}\right)^{-1} C\pi_t A^{tr} \tag{8}$$

$$+BQB^{tr} \qquad \pi_0 = \bar{V}_0$$

Then, $\psi_t \triangleq C\pi_t C^{tr} + DRD^{tr}$ and $\tilde{Y}_t = \tilde{K}_t + C\hat{X}_t$. It has been shown in [11] that using this coding the scheme, we have real-time reliable communication up to the distortion level D_v, as follows:

$EY_t - \tilde{Y}_t{}^2 = EK_t - \tilde{K}_t{}^2 = D_v, \forall t \in \mathbb{N}^+$. To achieve this real-time reliable communication by allocating the minimum channel capacity (bandwidth), we should tune the antenna's power as follows:

$$E[Z_t^2] = \alpha_t^2 \psi_t = \frac{\eta_t W_c}{D_v} \psi_t \triangleq P_t \quad (9)$$

3- 1- 3- Controller Description

With the assumptions that the pair $(C^{tr}C, A)$ is detectable and the pair (A, B) is stabilizable at each linearized zone, the stabilizing remote controller also optimizes the following quadratic cost functional

$$\lim_{T\to\infty} \frac{1}{T} \sum_{t=0}^{T-1} E\left[\|X_t\|^2{}_{C^{tr}C} + \|U_t\|^2{}_H\right] (H > 0) \quad (10)$$

Is given by

$$U_t = -\Delta_c \hat{X}_t \quad (11)$$

where $\Delta_c = \left(H + B^{tr} P_\infty B\right)^{-1} B^{tr} P_\infty A$ and P_∞ is the unique positive semi-definite solution of the following

algebraic Riccati equation:

$$P_\infty = A p_\infty A^{tr} - A^{tr} p_\infty B (H + B^{tr} P_\infty B)^{-1} B^{tr} P_\infty A + C^{tr} C \quad (12)$$

So, for our problem, the controller is described in the following:

$$U_t = -L_t \hat{\hat{X}}_t + \mu_t \quad (13)$$

Where $\hat{\hat{X}}_t = \hat{X}_t - R_t$ and for each $t \in \left[(j-1)T_l, jT_l - 1\right]$. $L_t = L_{[j]}$ is chosen such that the matrix $A_{[j]} - B_{[j]} L_{[j]}$ is stable and μ_t is chosen such that

$$\mu_t = -B_{[j]}^{+}\left(\left(A_{[j]} - B_{[j]} L_{[j]}\right)\left(R_t - X_{[j]}\right) + B_{[j]} L_{[j]} R_t - R_{t+1} + F(X_t, U_t)\right) \quad (14)$$

Where $B_{[j]}^{+}$ is the pseudo-inverse of the matrix $B_{[j]}$.

3- 2- Method for variable rate linearization

In the variable linearization case, the encoder and decoder should agree on the time of linearization. Linearizing nonlinear dynamics in each step around the current operating point is used in this paper. Following that, in the beginning, the nonlinear dynamic system (2) is linearized around $(\bar{X}_0 U_0)(U_0 = \bar{0})$. Then for each linearized step, the proposed method in [11] is applied to estimate the states of the linear system. Since in the initial moments of linearization, linear dynamics are a good approximation of non-linear dynamics, the mean square of the estimation error decreases with increasing time due to receiving more measured information from the output. Therefore, to maintain this decreasing trend of the mean square of the estimation error, it is necessary to continuously linearize the nonlinear dynamic. To determine the right time for the linearization of the nonlinear dynamic system at a new operating point, i.e., $\left(\hat{X}_t U_t\right)$, we look at the trend of $EX_t - \hat{X}_t{}^2$. In each linearized zone, the mean square estimation error has either an increasing or decreasing trend as time progresses depending on the value of the mean square estimation error at the beginning of the linearization which is the steady state value for the linearized system. For example, if at the beginning of the linearization, the mean square estimation error is smaller than its steady state value, the mean square estimation error in this linearized zone has an increasing trend. Now, when the linearized system is no longer a good approximation for the nonlinear system, the trend for the mean square estimation error is reversed; and this is the right time for updating the linearized system. To implement this coding scheme, both the encoder and decoder should observe the trend of the mean square estimation error to linearize the nonlinear dynamic system at the new operating point. Note that the encoder and decoder in the proposed method reconstruct the mean square estimation error using the Monte Carlo approximation method by the knowledge of $\Delta\hat{X}_t$ communicated to the encoder by the feedback channel. This method works as follows:

1- At the sample time $t = 0$, the encoder and decoder choose M realization for $X_0 \sim N\left(\bar{X}_0, Q_0\right)$ and Compute $\hat{X}_0 = \frac{1}{M}\sum_{i-1}^{M} X_0^{[i]}$, and then

2- They compute $E_0 = \frac{1}{\sqrt{M}}\left(X_0^{[1]} - \hat{X}_0 \cdots X_0^{[M]} - \hat{X}_0\right)$, where $X_0^{[i]}$ is the ith realization of X_0, and then

3- They compute $\pi_0 = trace\left(E_0 E_0^{tr}\right)$.

4- At the time instant $t = 1$, using the dynamic system model (2) for the nonlinear dynamic system and M realization obtained for X_0 available from the previous time instant as well as M realization for the process noise, the encoder, and decoder compute M realization for X_1 and compute $\hat{X}_1$, and then

5- They compute $E_1 = \frac{1}{\sqrt{M}}\left(X_1^{[1]} - \hat{X}_1 \cdots X_1^{[M]} - \hat{X}_1\right)$, where $X_1^{[i]}$ the ith realization of X_1, and then

6- They compute $\pi_1 = trace\left(E_1 E_1^{tr}\right)$, and then this procedure is repeated:

7- Go to the step 4 for the next time step $(t+1)$.

For the other time instances, π_t is reconstructed at the encoder and decoder. By observing the increasing or decreasing

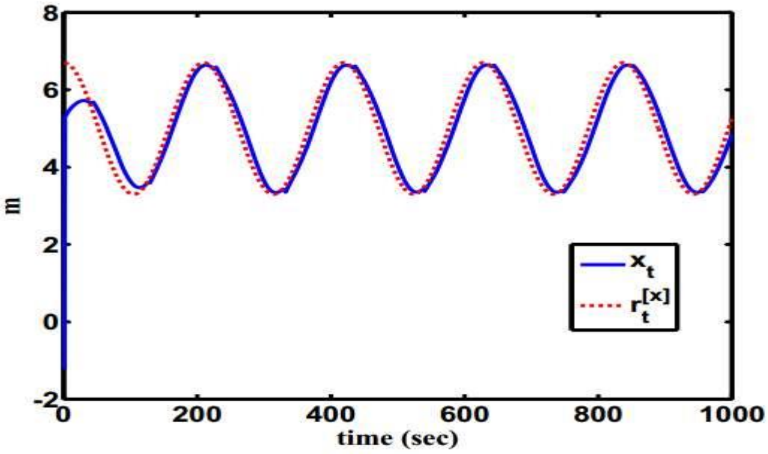

Fig. 2. x_t and $r_t^{[x]}$ for the variable rate linearization method when $W_t, V_t \sim N\left(\underline{0}, \underline{0}\right)$

trend of $trace\left(\pi_t\right)$ encoder and decoder determine the right time for updating the linearized system. As soon as the trend of $trace\left(\pi_t\right)$ changes, the encoder and decoder notice that this is the right time to update the linearized model. So, in this way, the variable (optimal) linearization rate is determined.

4- Simulation Results

In this section, for illustration, we set $T = 0.01, x_0 \sim N(1,1), y_0 \sim N(1,1), \phi_0 \sim N(1,1)$, and apply the proposed encoder, decoder, and controller to the autonomous vehicle dynamic (3). The discrete-time equivalent model is (15), where T is the sampling period.

$$F(X_t, U_t) = \begin{cases} x_{t+1} = x_t + Tv_t \cos(\phi_t) \\ y_{t+1} = y_t + Tv_t \sin(\phi_t) \\ \phi_{t+1} = \phi_t + Tu(t) \end{cases} \tag{15}$$

Denoting the function (15) as F with the input vector U_t in (2), and for $G\left(X_t, U_t\right)$ we have:

$$G(X_t, U_t) = X_t \tag{16}$$

Note also that for this model, the state vector is $X_t = \left[x_t \; y_t \; \phi_t\right]^{tr}$. The discrete-time equivalent dynamic (15) is obtained by applying a control signal to the dynamic (3) using Z.O.H and sampling its outputs using a sampler.

Subsequently, the state space representation of the family of the discrete-time equivalent linearized systems, for the jth linearization have the following state space matrices:

$$A = \begin{bmatrix} 1 & 0 & -Tv_{[j]} \sin(\phi_{[j]}) \\ 0 & 1 & Tv_{[j]} \cos(\phi_{[j]}) \\ 0 & 0 & 1 \end{bmatrix} \tag{17}$$

$$B = \begin{bmatrix} Tcos(\phi_{[j]}) & 0 \\ Tsin(\phi_{[j]}) & 0 \\ 0 & T \end{bmatrix} \tag{18}$$

Now, for tracking a circle with the center located at (5, 3) and a radius of 2, by the autonomous vehicle, we need to choose the elements of the reference vector $R_t = \left[r_t^{[x]} \; r_t^{[y]} \; r_t^{[\phi]}\right]^{tr}$ as follows [10]:

$$\left[r_t^{[x]} \; r_t^{[y]} \; r_t^{[\phi]}\right]^{tr} = \left[5 + 2\cos(3Tt) \;\; 3 + 2\sin(3Tt) \;\; \arctan\left(\frac{r_t^{[y]} - \hat{y}_{t-1}}{r_t^{[x]} - \hat{x}_{t-1}}\right)\right]^{tr} \tag{19}$$

Fig. 2- Fig.5 illustrates the performance of theoretical developments and tracking. Fig.5 illustrates that using the proposed method, tracking is achieved eventually. Also, Fig.6 shows that using the proposed method, tracking faster reference signals is possible. Fig.7 corresponds to the proposed technique in [11] with the fixed rate linearization period. Comparing the simulation result of Fig.6 with that of Fig.7 illustrates that our proposed method has much better performance. Comparing Fig.6 and Fig.7 clearly shows that the method presented in [9], unlike the technique proposed in

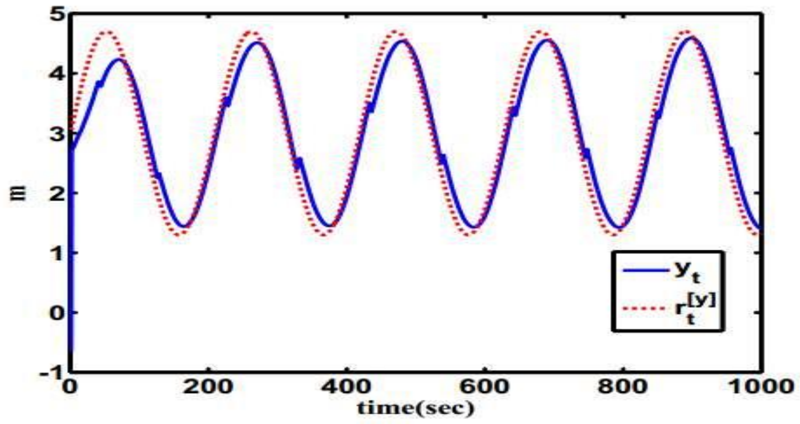

Fig. 3. y_t and $r_t^{[y]}$ for the variable rate linearization method when $W_t, V_t \sim N(\underline{0}, \underline{0})$

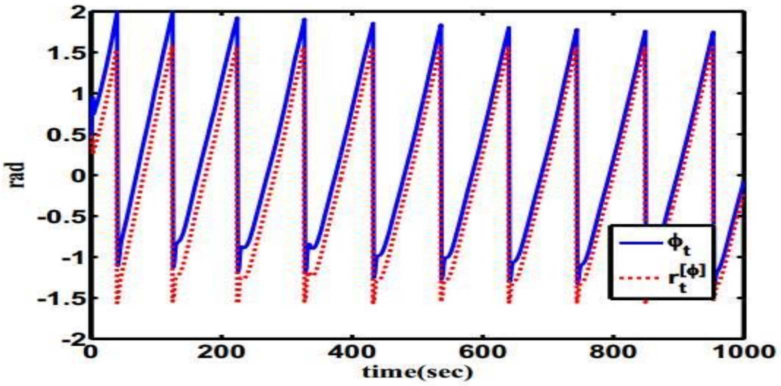

Fig. 4. ϕ_t and $r_t^{[\phi]}$ for the variable rate linearization method when $W_t, V_t \sim N(\underline{0}, \underline{0})$

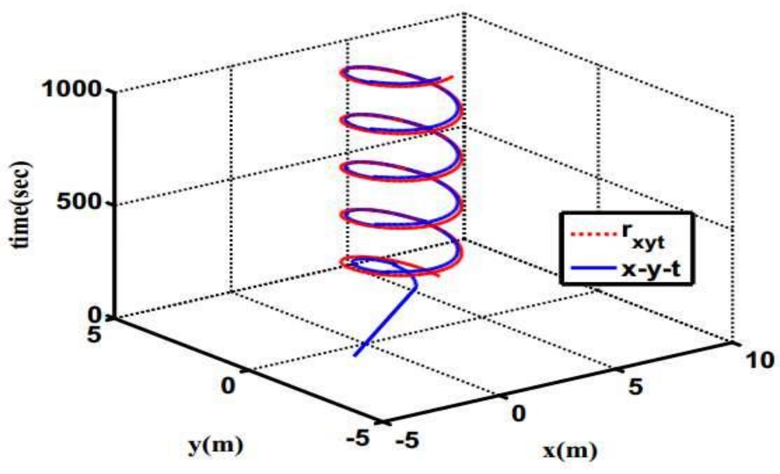

Fig. 5. x_t-y_t- time diagram for the variable rate linearization period when $W_t, V_t \sim N(\underline{0}, \underline{0})$

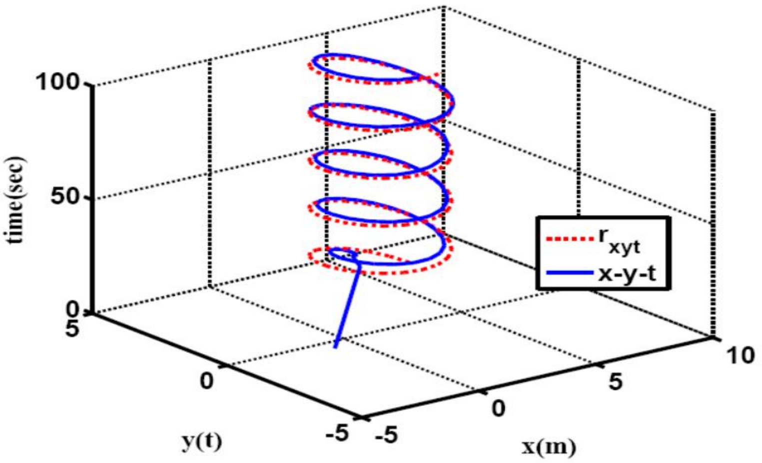

Fig. 6. x_t-y_t- time diagram for the variable rate linearization period when $W_t, V_t \sim N\left(\underline{0}, \underline{0}\right)$

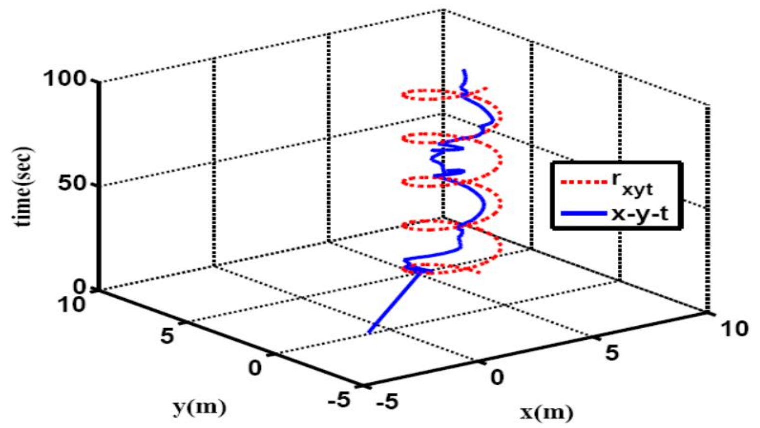

Fig. 7. x_t-y_t- time diagram for the fixed rate linearization period when $W_t, V_t \sim N\left(\underline{0}, \underline{0}\right)$

this article, loses its tracking ability when faced with a fast reference signal.

Now to quantify the performance of the proposed method, we define the Root Sum Square Error (RSSE)

criterion as follows:

$$RSSE = \sqrt{\sum_{t=\frac{start\ time}{T}}^{\frac{end\ time}{T}} \left(x_t - r_t^{[x]}\right)^2 + \left(y_t - r_t^{[y]}\right)^2 + \left(\phi_t - r_t^{[\varphi]}\right)^2} \quad (20)$$

Table 1 is related to Fig.5 which corresponds to slow reference signal for different channel noises. From this table follows that the proposed method is robust against channel noises. Also, in this table, we can see that using the variable rate linearization technique is more accurate and practical. In this way, the number of linearizations could be optimal and RSSE is less than the technique proposed in [10]. Table 2 RSSEs are computed for different processes and measurement noises. It is clear that the proposed technique is robust when there are process and measurement noises.

Covariance analysis:

Covariance analysis has been used to verify the presented method. For illustrating the performance of the proposed method the covariance of error is computed in each iteration. To do this, the covariance of the error at each sampling time (t=1000) is calculated, and then the totals are collected. Finally, we analyzed them by averaging them. The results illustrate that the covariances are in order 10^{-4} and off-diagonal terms are almost zero.

$$cov = \begin{bmatrix} 0.00036 & -0.0004 & 0.0000 \\ -0.00045 & 0.0004 & 0.0000 \\ 0.0000 & -0.0000 & 0.0000 \end{bmatrix}$$

Table 1. RSSES computed for different channel noises for the case $W_t, V_t \sim N\left(\underline{0}, \underline{0}\right)$ for proposed method and method presented in [10]

Case	RSSE in proposed method	RSSE proposed in [10]
$\widetilde{W}_t \sim N(0,0.1.I_3)$	20.3024	21.7861
$\widetilde{W}_t \sim N(0,I_3)$	21.5839	24.8606
$\widetilde{W}_t \sim N(0,2.I_3)$	23.6912	25.6621

Table 2. RSSES Computed for Different process and measurement noises for

Case	RSSE
$W_t, V_t \sim N(0,0.1.I_3)$	20.7421
$W_t, V_t \sim N(0,I_3)$	21.4337
$W_t, V_t \sim N(0,2.I_3)$	24.72

5- Conclusion

In this paper, a new method for state tracking as well as reference tracking of noisy nonlinear dynamic systems over the AWGN channel with applications in the telepresence and teleoperation of autonomous vehicles was presented. The proposed method was based on a linearization technique with a variable rate. A proper encoder, decoder, and controller for tracking the state trajectory of nonlinear dynamic systems at the end of the communication link as well as reference tracking over the AWGN channel were presented. The satisfactory performances of the proposed method were illustrated by implementing this method on the unicycle model, which is an abstract model for representing the autonomous vehicle dynamic. One of the major contributions of this paper is the presentation of an extended version of the extended Kalman filter with the variable linearization rate over the AWGN channel.

References

[1] E. Nuno, I. Sarras and L. Basanez (2014). An adaptive controller for bilateral teleoperation: Variable time-delay case. IFAC Proceedings, 47(3): 9341 - 9346.

[2] C. Liu, J. Guo and P. Poignet (2018). Nonlinear model – mediated teleoperation for surgical applications under time variant communication delay. IFAC Proceedings, 51(22): 493 - 499.

[3] T. Imaida, Y. Yokokohji, T. Doi, M. Oda and T. Yoshikwa (2004). Groundspace bilateral teleoperation of ETS-VII robot arm by direct bilateral coupling under 7-s time delay condition. IEEE Transactions on Robotics and Automation, 20(3): 499 - 511.

[4] L. Xu, Y. Mo, L. Xie, and N. Xiao, "Mean square stabilization of linear discrete-time systems over power-constrained fading channels," IEEE Trans. Autom. Control, vol. 62, no. 12, pp. 6505–6512, 2017.

[5] Parsa A. and Farhadi, A. Tele-operation of autonomous vehicles over additive white Gaussian noise channel, Scientia Iranica, 1592-1605, Winter and Spring 2021

[6] Parsa, A. and Farhadi, A. (2019). Reference tracking of nonlinear dynamic systems over AWGN channel using describing function. Scientia Iranica, Transactions on Computer Science and Engineering and Electrical Engineering, 26(3), 1724 - 1735.

[7] Sanjeroon, V., Farhadi, A., Motahari, A. and Khalaj, B. (2018). Estimation of nonlinear dynamic systems over communication channels. IEEE Transactions on Automatic Control, 63(9), 3024 - 3031.

[8] S. Dasguptal, "Control over bandlimited communication channels: limitations to stabilizability," in IEEE CDC, vol. 1, pp. 176–181, 2003.

[9] R. K. Mishra, D. Vasal and H. Kim, "Linear Coding for AWGN Channels With Noisy Output Feedback via Dynamic Programming," in IEEE Transactions on Information Theory, vol. 69, no. 8, pp. 4889-4906, Aug. 2023, doi: 10.1109/TIT.2023.3260807.

[10] S. D. Takloo, A. Farhadi and A. Khaki-Sedigh, ”Control and Measurement of Nonlinear Dynamic Systems over AWGN Channel with Application in Tele-operation of Autonomous Vehicles” 2022 30th Mediterranean Conference on Control and Automation (MED), Vouliagmeni, Greece, 2022, pp. 835-840,doi: 10.1109/MED54222.2022.9837145.

[11] Charalambous, C. D., and Farhadi, A. (2008). LQG optimality and separation principle for general discrete time partially observed stochastic systems over finite capacity communication channels. Automatica, 44(12), 3181 – 3188.

[12] Parsa, A. and Farhadi, A. (2018). Measurement and control of nonlinear dynamic systems over the Internet (IoT): Applications in remote control of autonomous vehicles, Automatica, 95, 93 - 103.

[13] Lin, H. and Antsaklis, P. J. (2009). Stability and stabilizability of switched linear systems: A survey of recent results. IEEE Transactions on Automatic Control, 54(2), 308 - 322.

[14] Zhai, G. Hu, B., Yasuda, K. and Michel, A. N. (2002). Qualitative analysis of discrete-time switched systems. In Proceedings of the American Control Conference, (pp. 1880 - 1885).

HOW TO CITE THIS ARTICLE

S. Dolatkhah Takloo, A. R. Farhadi, A. Khaki-Sedigh, Reference Tracking of Nonlinear Dynamic Systems over Additive White Gaussian Noise Channel, AUT J. Model. Simul., 55(2) (2023) 359-368.

DOI: 10.22060/miscj.2024.22496.5328