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# Friction Compensation for Dynamic and Static Models Using Nonlinear Adaptive Optimal Technique

M. Nazari Monfared [1],and M. J. Yazdanpanah [2*]

1- M.Sc. student of Control Engineering in the Department of Electrical Engineering, Faculty of Electrical, Biomedical, and Mechatronic, Qazvin Branch, Islamic Azad University, Qazvin, Iran
2- Professor, Control & Intelligent Processing Center of Excellence, School of Electrical and Computer Engineering, University of Tehran, Tehran, Iran

***ABSTRACT***

Friction is a nonlinear phenomenon which has destructive effects on performance of control systems. To obviate these effects, friction compensation is an effectual solution. In this paper, an adaptive technique is proposed in order to eliminate limit cycles as one of the undesired behaviors due to presence of friction in control systems which happen frequently. The proposed approach works for nonlinear dynamic and static friction models and is applicable to a wide range of different mechanical systems. It is also applied to a simple inverted pendulum on a cart as a highly nonlinear under-actuated system. A nonlinear optimal controller based on the approximate solution of Hamilton-Jacobi-Bellman partial differential equation is designed to fulfill our control objectives and achieve preferable performance compared to those of the linear optimal controllers. It causes to have more accuracy in system's response and positioning in the presence of friction. Simulation result approve the effectiveness of both the presented technique and controller.

***KEYWORDS***

Adaptive technique, friction compensation, HJB partial differential equation, inverted pendulum on a cart, nonlinear optimal controller.

* Corresponding Author, Email: yazdan@ut.ac.ir

## 1. INTRODUCTION

Friction is a physical phenomenon which arises in mechanical systems with moving parts. It is highly nonlinear and has malicious effects on control systems performance. Wear, loss of energy, occurrence of limit cycles, steady state errors, and even instability are some of the results of friction presence [1], [2], [4]. On the other hand, generally, this phenomenon is not considered in systems mathematical modeling and controllers design procedure. Therefore, friction compensation is an inevitable process from control theory point of view to achieve a desired behavior.

Many researchers have put these different aspects of friction into their perspective. True nature perception and mathematical modeling of this phenomenon, analysis of its impacts on control systems behavior, and proposition of various techniques for compensation are some of these aspects.

Mathematical modeling of friction is dependent on the realization of its behaviors in practice. Presliding displacement, Stribeck effect, spring like behavior, friction lag, varying break-away force, and stick-slip motion are some of the known behaviors [2], [4]. As a result, different models have been presented such as Classic, Exponential, Karnopp, Dahl, Armstrong Seven Parametric, Generalized Maxwell Slip, and LuGre that encompass entire or a partial of the behaviors [2-6].

The designed controllers for systems fail to do their missions perfectly in the presence of friction. Hence, during the past decades, great efforts have been done to propose diverse friction compensation techniques [1], [3], [7]-[13]. Compensation approaches are used to counteract friction force which leads to achieve the closed loop desired behavior.

One of the main ideas for compensation is based on the estimation of friction force which requires to model parameters identification. Since models are highly nonlinear, process of identification is not straightforward and is a challenging part of compensation techniques. Generally, off-line identification techniques are not as complex as on-line ones, but they need some special conditions which are not feasible in some cases [11], [12]. For more details see [7].

The main contribution of this paper is proposition of an online adaptive identification approach as a new compensation technique. It is utilized to estimate parameters which are not related to friction models, but are imperative to be known in compensation process. The technique can be used for high accuracy positioning in mechanical systems.

The proposed method is applied to simple inverted pendulum on a cart (SIPC) system. To control this system, a nonlinear optimal controller based on Taylor series expansion approximate solution of Hamilton-Jacobi-Bellman partial differential equation (HJB PDE) is designed. This controller leads to considerable improvements in system performance and elimination of friction effects in some special situations.

The rest of this paper is organized as follows: In section II, mathematical model of the SIPC system and models of friction are presented. Then, a linear optimal controller is designed for the system and its performance in facing with friction is evaluated. Section III is devoted to design a nonlinear optimal controller for the system and for investigation of its performance characteristics. The adaptive friction compensation technique is presented in section IV and its ability for elimination of friction effects is assessed. Finally, the proposed technique is compared with other techniques in Section V and conclusion is drawn in Section VI.

## 2. EFFECTS OF FRICTION ON CONTROL SYSTEMS

To deal with friction, it is necessary to have models which embody known behaviors as far as possible. The proposed models are classified in two main groups, dynamic and static [2], [4]. In this section, SIPC model description and an overview on some significant models are presented. Then, the effects of friction on the closed loop system's response with linear optimal controller are analyzed.

### A. Description Of Sipc System Model

The SIPC system is one of the best test beds for evaluation of the proposed control ideas and has different applications [14]. Fig. 1 depicts the schematic of the system. The cart can move horizontally and the pendulum is mounted by a joint on the cart and can rotate freely. Differential equations governing on the system are given by [14]:

$$\begin{aligned}&(M_1+M_2)\ddot{X}+M_2l\ddot{\theta}\cos(\theta)-M_2l\dot{\theta}^2\sin(\theta)=u-F\\&\frac{4}{3}M_2l^2\ddot{\theta}-M_2gl\sin(\theta)+M_2l\ddot{X}\cos(\theta)=0\end{aligned} \tag{1}$$

where $u$ is control signal and friction force is considered between cart and ground. Other parameters are described in Table 1. The state space equation of system is obtained by taking $x^T=\begin{bmatrix}X & \theta & \dot{X} & \dot{\theta}\end{bmatrix}^T$ as state vector. In this paper, control objective is to stabilize the unstable upward equilibrium point to origin asymptotically.

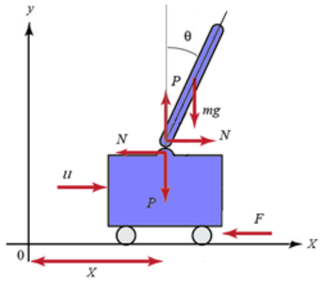


**Fig. 1. Schematic of SIPC system with exerted forces on it [1].**

**TABLE 1.** SIPC SYSTEM PARAMETERS DESCRIPTION

| Parameter | | |
|---|---|---|
| *Symbol* | *Quantity* | *Magnitude[a, b]* |
| *M* | *mass of cart* | *0.815 kg* |
| *m* | *mass of pendulum* | *0.210 kg* |
| *L* | *length of pendulum from pivot to center of mass* | *0.305 m* |
| *g* | *gravitational acceleration* | *9.81 m/s$^2$* |
| *X* | *cart displacement* | *- m* |
| *θ* | *angular position of pendulum* | *- rad* |

Magnitude of parameters are based on [14]
kg: kilogram, rad: radian, m: meter, s: second

### B. Dynamic And Statics Models Of Friction

Different models of friction have been presented in papers by researchers, but in this paper we want to focus on three known models [1]. The first presented model is a classic given as [4]:

$$F = F_c sign(v) + F_v v \tag{2}$$

where $v$ and $F$ are relative velocity of rubbing objects and friction force between them, respectively. Description and value of all models parameters in our problem are given in Table 2.

Exponential model given in [4] is one of the most comprehend static models which cover static behaviors of friction in practice:

$$F = \left( F_c + (F_s - F_c) e^{-\left(\frac{v}{v_s}\right)^2} \right) sign(v) + F_v v \tag{3}$$

Dynamic models cover friction dynamic behaviors in addition to static ones. Hence, dynamic models are richer than static ones. LuGre is a prominent dynamic model and is based on bristle conception of surfaces [3], [4]. This model is described by the following equation:

$$F = \sigma_0 z + \sigma_1 \dot{z} + F_v v \tag{4}$$

where is the state variable of model and given by:

$$\dot{z} = v - \frac{|v|}{g(v)} z \tag{5}$$

and

$$g(v) = \frac{1}{\sigma_0} \left( F_c + (F_s - F_c) e^{-\left(\frac{v}{v_s}\right)^2} \right) \tag{6}$$

**TABLE 2.** . FRICTION MODELS PARAMETERS DESCRIPTION

| Parameter | | |
|---|---|---|
| *Symbol* | *Quantity* | *Magnitude[c, d]* |
| $F_v$ | *viscous friction coefficient* | *3 N-s/m* |
| $F_c$ | *coulomb friction coefficient* | *0.431 N* |
| $F_s$ | *static friction coefficient* | *0.844 N* |
| $v_s$ | *Stribeck velocity* | *0.105 m/s* |
| $\sigma_0$ | *stiffness coefficient* | *121 N/m* |
| $\sigma_1$ | *damping coefficient* | *70 N-s/m* |

[a.] Magnitude of parameters are considered for simulation and are based on [14]
[b.] N: Newton, S: second, m: meter

The LuGre model is characterized by six parameters while Exponential and Classic models are characterized by four and two parameters, respectively. To estimate friction using these models, all parameters should be identified which with respect to their nonlinear structure is not an easy task.

### C. Effects Of Friction On Closed Loop System

To achieve our control objectives a Linear Quadratic Regulator is designed for the SIPC system in the absence of friction [1]. This controller ensures the asymptotic stability of system's unstable equilibrium point while minimizing the following energy-based cost function:

$$J = \int_0^\infty (x^T Q x + u^T R u) dt \tag{7}$$

State feedback control signal is achieved using the solution of Riccati algebraic equation [15]. The controller is designed for the linearized system while friction is not considered in system equations, but it is exerted to the closed loop system (CLS) in simulations. Fig. 2 shows the structure of CLS in the presence of friction [1]. Responses of cart and pendulum to initial conditions in CLS with LQR controller are shown in Fig. 3 and Fig. 4.

Fig. 3, easily shows that the LQR as a modern control technique stabilizes system to origin asymptotically in the absence of friction, but in the presence of friction between cart and ground it is unsuccessful and limit cycle behavior is emerged. Unfortunately, modifications on controller design arbitrary parameters cannot eliminate limit cycles or even decrease its frequency or amplitude substantially. On the other hand, applying each of friction models, dynamic or static, leads to occurrence of limit cycle behavior with different amplitudes and frequencies. Therefore, a nonlinear optimal controller is designed for the system and its performance is evaluated in the absence and presence of friction.

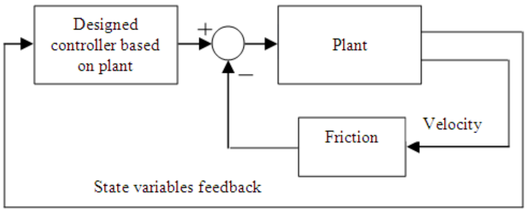


**Fig. 2. Structure of closed loop system in presence of friction [1].**

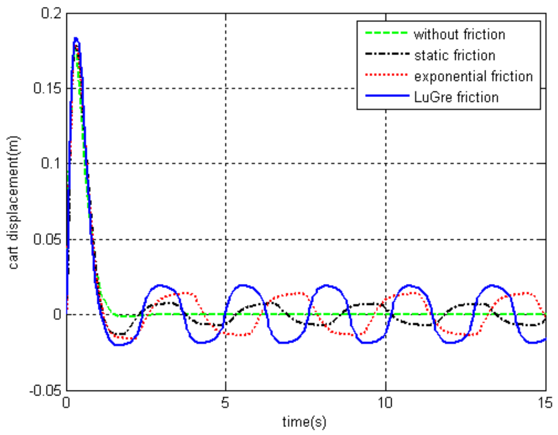


**Fig. 3. Cart displacement response to initial condition** $x_0^T = [0 \quad 0.2325 \quad 0 \quad 0]$ **in closed loop system.**

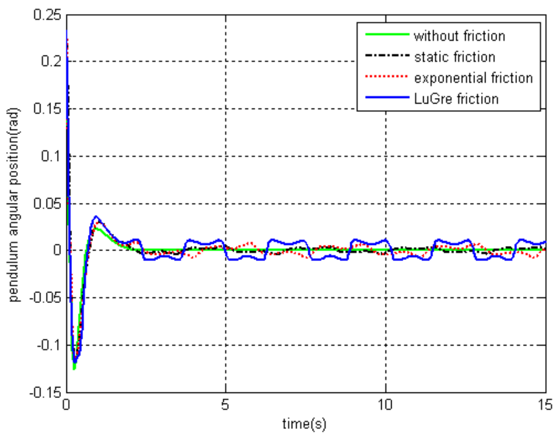


**Fig. 4. Pendulum angular displacement response to initial condition** $x_0^T = [0 \quad 0.2325 \quad 0 \quad 0]$ **in closed loop system.**

## 3. Nonlinear Optimal Controller Design

Dynamic programming problem leads to HJB PDE which based on its approximate solution for the system nonlinear optimal controller is designed. Exact solution of HJB PDE is not possible, therefore, various approaches have been presented to solve it approximately [16]-[20]. Using power series expansion (PSE) is one of the useful solution techniques which is known as Albrecht method [16], [17]. In this paper, Taylor series expansion (TSE) is used to solve HJB PDE for the SIPC system and design a nonlinear optimal controller.

### A. General Overview On Tse Approach

Solution of HJB PDE based on PSE is possible for nice optimal control problems [17]. Objective in optimal control problems is minimizing the following infinite horizon cost function [15]:

$$J = h\left(x\left(t_f\right), t_f\right) + \int_{t_0}^{t_f} g\left(x\left(\tau\right), u\left(\tau\right), \tau\right) d\tau \qquad (8)$$

Subject to a nonlinear dynamic system which is given by:

$$\dot{x} = f\left(x, u\right) \qquad (9)$$

where $t_0$ and $t_f$ are fixed time points. It is supposed that the integrant term of (8) and dynamic system (9) could be written in TSE form with the following formats [17]:

$$L\left(x, u\right) = \frac{1}{2} x^T Q X + \frac{1}{2} u^T R u + l^{(3)}\left(x, u\right) + l^{(4)}\left(x, u\right) + \ldots \qquad (10)$$

and

$$\dot{x} = Ax + Bu + f^{(2)}\left(x, u\right) + f^{(3)}\left(x, u\right) + \ldots \qquad (11)$$

Symbol $(.)^{(n)}$ denotes functions which are composed of terms with degree.

In this paper, the problem is considered infinite horizon ($t_f = \infty$), hence the HJB PDEs are written and described by [17]:

$$\frac{\partial v\left(x\right)}{\partial x} f\left(x, u\right)\Big|_{u=u^*} + L\left(x, u\right)\Big|_{u=u^*} = 0 \qquad (12)$$

$$\frac{\partial v\left(x\right)}{\partial x} \frac{\partial f}{\partial u}\left(x, u\right)\Big|_{u=u^*} + \frac{\partial L\left(x, u\right)}{\partial u}\Big|_{u=u^*} = 0 \qquad (13)$$

where $u^*$ is the optimal control signal which can minimize the cost function (8) and $v\left(x, t\right)$ is optimal cost. To solve HJB PDE using TSE, optimal cost and optimal control signal are supposed to be according (14) and (15), respectively [17].

$$v\left(x\right) = \frac{1}{2} x^T P x + v^{(3)}\left(x\right) + v^{(4)}\left(x\right) + \ldots \qquad (14)$$

$$u^*\left(x\right) = Kx + K^{(2)}\left(x\right) + K^3\left(x\right) + \ldots \qquad (15)$$

Now, by substituting (10), (11), (14), and (15) in (12) and (13), separating terms with identical degree and putting them equal to zero, unknown terms $v^{(n)}\left(x\right)$ and $K^{(n)}\left(x\right)$ are obtained.

### B. Approximate Solution Of Hjb Pde For The Sipc System

In this problem, the cost function is considered according to (7). The optimal control problem is nice; hence, using TSE for solution of HJB PDE is possible. To find the first unknown parameter in control signal (15), $K$, TSE of functions are substituted in (12) and (13). Then, terms with degree two and one into (12) and (13), respectively, are separated and put equal to zero. This process leads to Riccati algebraic equation which is the same as the LQR controller design strategy and state feedback gain is obtained.

Since the function in (11) is zero, the second unknown term in (15) would be equal to zero. In the next step, by separating terms with order three in (13), the following equation for is procured.

$$K^{(3)}(x) = -R^{-1}\left(x^T P \frac{\partial f^{(3)}(x,u)}{\partial u} + \frac{\partial v^{(4)}(x)}{\partial x} B\right) \qquad (16)$$

where $v^{(4)}(x)$ is unknown. To acquire it, terms with order four in (12) are isolated and put equal to zero. Hence, we would have:

$$x^T P f^{(3)}(x, Kx) + \frac{\partial v^{(4)}(x)}{\partial x} Ax + \frac{\partial v^{(4)}(x)}{\partial x} BKx = 0 \qquad (17)$$

To solve (17), the following linear operator which is invertible [17], is employed:

$$v^{(4)}(x) \mapsto \frac{\partial v^{(4)}(x)}{\partial x}(A + BK)x \qquad (18)$$

Therefore, $v^{(4)}(x)$ is obtained as:

$$v^{(4)}(x) = -x^T P f^{(3)}(x, Kx) \qquad (19)$$

In this problem, the first three terms in (15) are only obtained.

### C. Performance Of Nonlinear Optimal Controller

The designed nonlinear optimal controller (NOC) is assessed in three different aspects. At first, response characteristics of CLS in the absence of friction is evaluated, hence system is actuated by the initial condition $x_0^T = [0 \quad 0.5232 \quad 0 \quad 0]$. Fig. 5 and Fig. 6 show position of cart and pendulum and the performance of NOC based on error criteria. Error signal is described by difference between position of cart and pendulum with the origin as the desired point and error criteria is considered as the energy of error signal. In fact, the performance of NOC is compared with LQR as a linear optimal controller. NOC has faster convergence to the origin and its error signal energy is about four times less than that of the LQR controller.

Presence of friction is another factor which challenges the operation of controllers. NOC in some conditions eliminates effects of friction without using friction compensation loop. Fig. 7 demonstrates that NOC can prevent emergence of limit cycle behavior when Classic and Exponential are considered as exerted friction models to the system, however, the LuGre dynamic model impacts on performance of the nonlinear controller.

Robust analysis is the last factor which is utilized to assess NOC. It is supposed that there are uncertainties in system parameters values such as cart and pendulum mass. A disturbance signal is also exerted to the cart. Response of CLS to the initial condition $x_0^T = [0 \quad 0.4534 \quad 0 \quad 0]$ in the presence of friction is shown in Fig. 8. The disturbance signal is a pulse in time interval 4 to 6(s) with an amplitude equal to one. The uncertainties for the cart and pendulum mass are considered equal to 20 and 10 percent, respectively. In these conditions asymptotic stability of system is satisfied.

As a result, the better performance of NOC with respect to that of the LQR is concluded. Elimination of friction effects in some cases, faster convergence, low energy of error signal, and robustness are some of the NOC advantages.

In the next section, an adaptive compensation technique is used to eliminate friction effects in cases in which the nonlinear controller is not capable to eliminate destructive effects of the friction.

## 4. Friction Compensation

Different techniques have been presented to compensate for friction and eliminate its effects. The occurrence of limit cycle behavior is one the major problems that emerges in closed loop systems in the presence of friction. A general scheme of compensation techniques in other papers is shown in Fig. 9. Their idea is based on reconstructing a force which is the estimated friction force, $F_{estimated}$, and adding it to the system where friction force, F, is exerted. This procedure makes to these two forces counteract each other. These approaches require a friction model, estimation of model parameters, and state observer in cases where dynamic models are used. Hence, utilizing these methods is complex and has special difficulties.

In this paper, a new friction compensation technique using Adaptive Noise Cancellation (ANC) idea is proposed. ANC approach is utilized to counteract noise using a signal whose amplitude is identical with that of the noise, but in an opposite direction [21], [22]. We use this technique to eliminate friction effects.

### A. Adaptive Friction Compensation Technique

The desired behavior of the closed loop system is to converge to the origin asymptotically, but friction causes to emerge limit cycle as an undesired behavior. The difference between desired and undesired response in CLS is considered as a noise signal which should be eliminated.

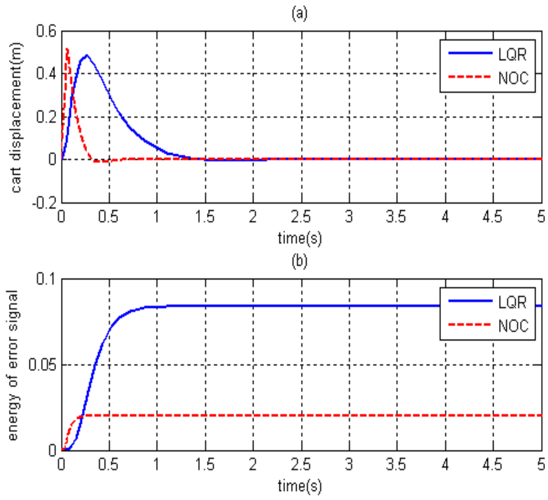


**Fig. 5. Performance of NOC on cart position in closed loop system in comparison with LQR. (a): cart position, (b): energy of error signal.**

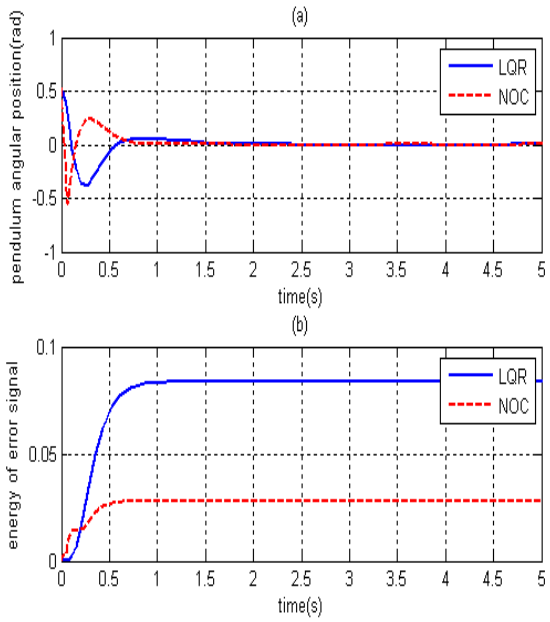


**Fig. 6. Performance of NOC on pendulum angular position in closed loop system in comparison with LQR. (a): pendulum angular position, (b): energy of error signal**

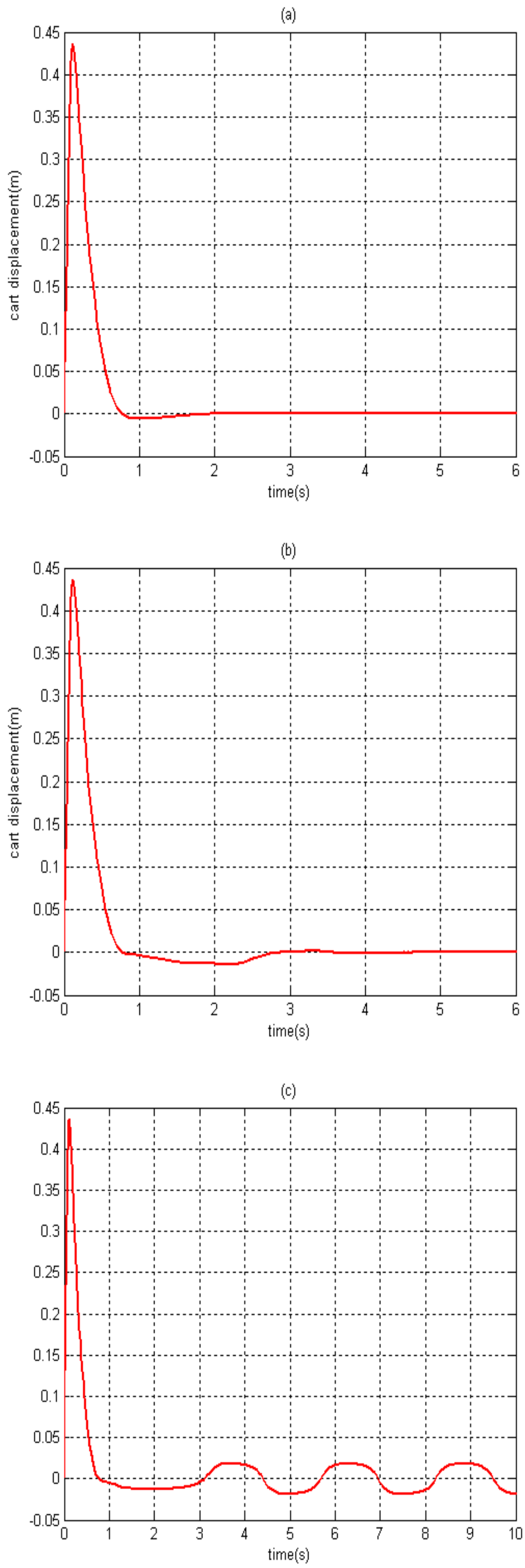


**Fig. 7. . Capability of NOC in encountering with different models of friction. (a): static model, (b): exponential mode, (c): LuGre dynamic model. System is actuated by initial conditions**

$$x_0^T = [0 \quad 0.4534 \quad 0 \quad 0].$$

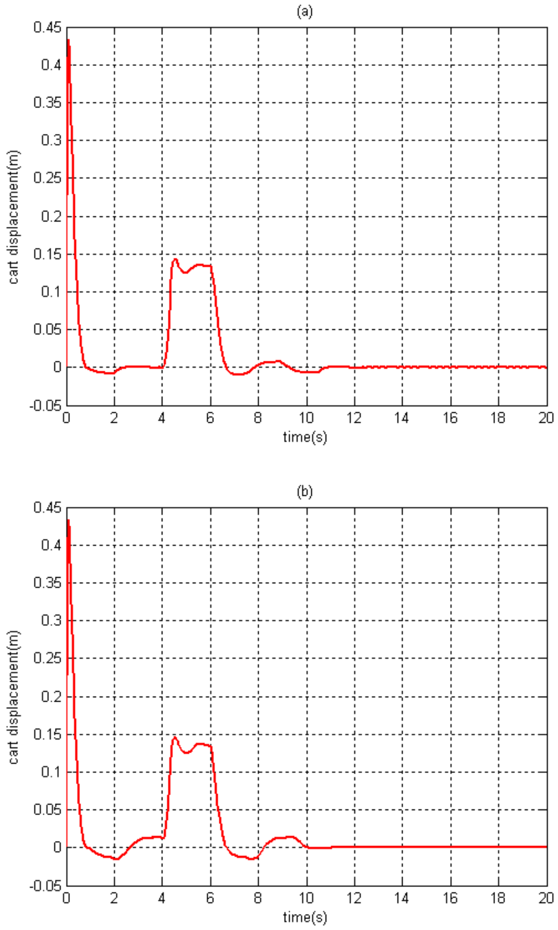


**Fig. 8. . Response of cart position in closed loop system in presence of friction with uncertainty for cart mass (20 percent) and pendulum mass (10 percent) and exerted disturbance signal, (a): static model,(b): exponential model.**

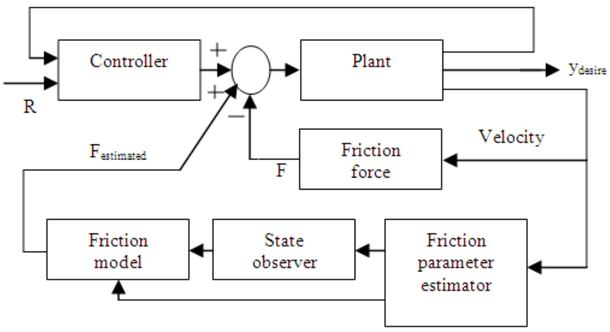


**Fig. 9. General procedure for friction compensation [1].**

Based on the ANC approach, we require to construct a time signal which has identical amplitude with noise signal, but in opposite direction. We call it spurious noise signal (SNS). The noise signal denoted by "$N$" is considered as summation of some sine signals with unknown different amplitudes ($A_i$), frequencies ($\omega_i$), and phases ($\varphi_i$).

$$N = \sum_{i=1}^{n} A_i \sin(\omega_i t + \varphi_i) \tag{20}$$

To create SNS, the unknown parameters $A_i$, $\omega_i$, and $\varphi_i$ should be estimated. Fig. 10 shows an overall view for the proposed adaptive compensation technique.

Using Fourier transform, spectrum frequency of the noise signal is obtained which dominant frequencies are considered as unknown frequency components in (20). To estimate two other groups of unknown parameters, Gradient Algorithm (GA) as an online estimation adaptive technique is used. First, (20) is rewritten as:

$$\begin{aligned} N &= \alpha_1 \sin\omega_1 + \alpha_2 \sin\omega_1 + \alpha_3 \sin\omega_2 \\ &\quad + \alpha_4 \sin\omega_2 + \alpha_5 \sin\omega_3 + \alpha_6 \sin\omega_3 + \cdots \end{aligned} \tag{21}$$

where

$$\begin{cases} \alpha_1 = A_1 \sin\varphi_1 \\ \alpha_2 = A_1 \sin\varphi_1 \end{cases} \begin{cases} \alpha_3 = A_2 \sin\varphi_2 \\ \alpha_4 = A_2 \sin\varphi_2 \end{cases} \begin{cases} \alpha_5 = A_3 \sin\varphi_3 \\ \alpha_6 = A_3 \sin\varphi_3 \end{cases} \tag{22}$$

Now, by separating known and unknown terms in (21), static parametric model form (SPM) is obtained as [21]:

$$Z = (\theta^*)^T \psi \tag{23}$$

where $Z$ and $\psi$ are known, but $\theta$ comprises unknown parameters given by:

$$Z = N, \quad \theta^* = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \\ \alpha_5 \\ \alpha_6 \\ \vdots \end{bmatrix}, \quad \psi = \begin{bmatrix} \sin\omega_1 t \\ \cos\omega_1 t \\ \sin\omega_2 t \\ \cos\omega_2 t \\ \sin\omega_3 t \\ \cos\omega_3 t \\ \vdots \end{bmatrix} \tag{24}$$

Another SPM model is presented as:

$$\bar{Z}(t) = (\bar{\theta}(t))^T \psi \tag{25}$$

where $\bar{\theta}(t)$ and $\bar{Z}(t)$ are estimates of $\theta$ and $Z$, respectively. Our objective is convergence of $\bar{\theta}(t)$ to $\theta$ such that $\bar{Z}(t)$ converge to $Z$. To achieve the objective, the following adaptive law is presented:

$$\dot{\theta} = \Gamma \varepsilon \psi \tag{26}$$

where $\Gamma = \Gamma^T \succ 0$ is adaptive arbitrary gain and $\varepsilon$ is the normalized error between real and estimated vales of $Z$. As a results, the unknown parameters, $A_i$ and $\varphi_i$, are estimated and SNS are constructed.

## B. Simulation Results

According to the pervious section, noise signal is considered as difference between undesired and desired behaviors and its frequency components are calculated using Fourier transform in Matlab Software. Fig. 11 shows noise signal and its frequency spectrum. Four frequencies are selected as dominant frequencies which based on (20) and (21) eight unknown parameters should be identified to create SNS. The constructed SNS using adaptive law (26) for estimation of unknown parameters and its summation with noise signal are shown in Fig. 12. As it is obvious, amplitude of SNS is identical with that of noise signal, but in an opposite direction and their summation is equal to zero.

Cart position in the presence of friction and adaptive friction compensation loop is shown in Fig. 13. The proposed technique decreases the amplitude of fluctuations substantially. The amplitude has decreased from 0.02(m) to 0.00005(m), hence, positioning is done with very high accuracy.

Fig. 14 and Fig. 15 show four out of eight estimated parameters. Input signal to friction compensation loop, noise signal, is not necessarily persistently excited; hence parameters do not converge to their exact values, but without loss of generality the SNS is constructed properly and objective of compensation is satisfied.

In this section, the exerted friction force is considered based on the LuGre dynamic model, but the proposed approach works for other dynamic and static models.

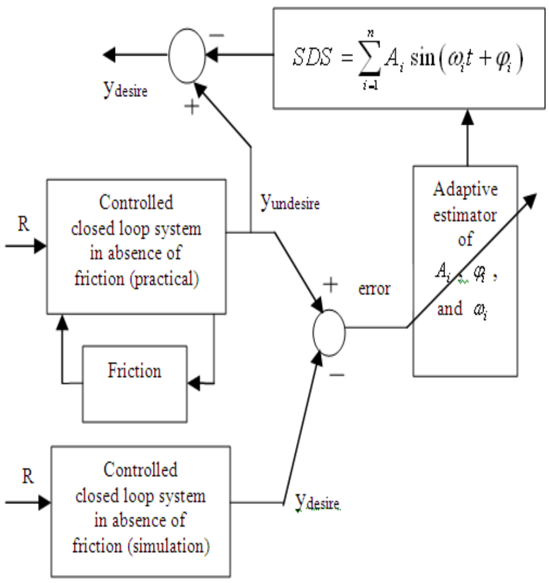


**Fig. 10. Fig. 10. Overall scheme of proposed adaptive friction compensator technique [1].**

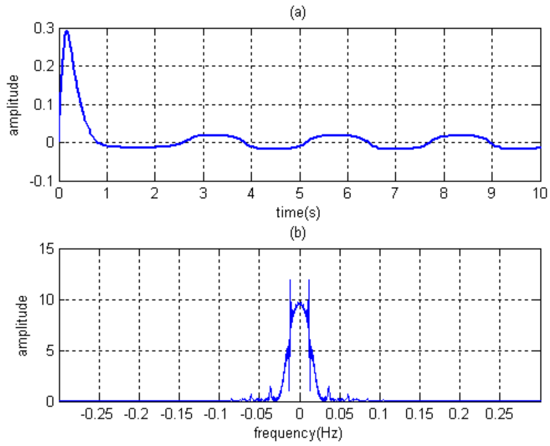


**Fig. 11. Fig. 11. (a): Nosie signal which is difference between response of cart displacement in presence and absence of friction to initial condition $x_0^T = [0 \quad 0.3488 \quad 0 \quad 0]$, (b): Frequency spectrum of noise signal.**

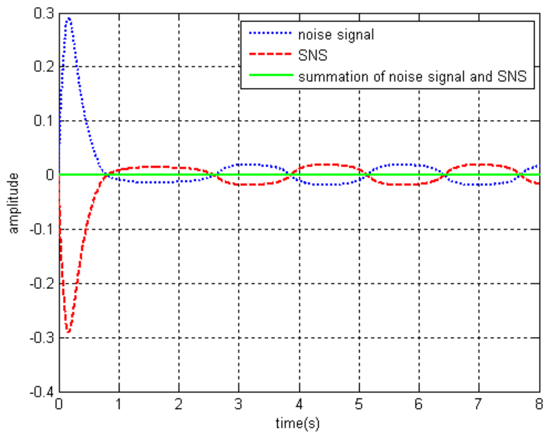


**Fig. 12. Fig. 12. Comparison between SNS and noise signal.**

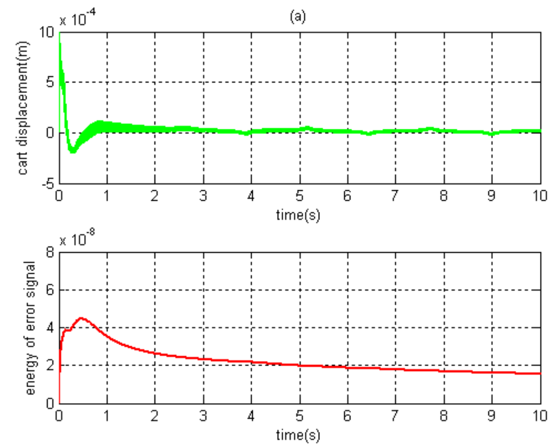


**Fig. 13. Fig. 13. (a): cart modified position response to initial condition $x_0^T = [0 \quad 0.3488 \quad 0 \quad 0]$ in presence of friction compensation loop and nonlinear optimal controller,(b): energy of error signal**

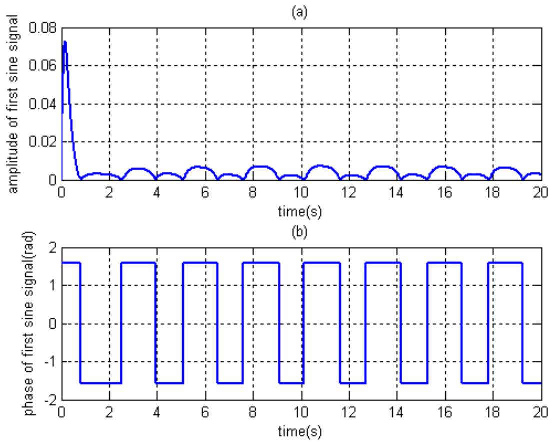


**Fig. 14. Fig. 14. Estimation of unknown parameters for first sine signal, (a): amplitude, (b): phase.**

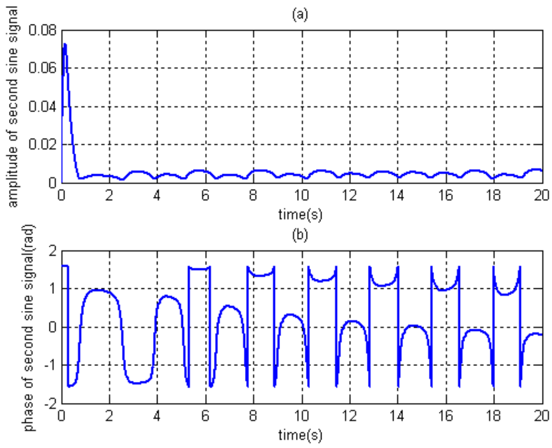


**Fig. 15. Fig. 15. Estimation of unknown parameters for second sine signal, (a): amplitude, (b): phase.**

## 5. Comparison With Other Techniques

Nonlinear optimal controller has preference than the LQR controller. In [1], when adaptive compensation technique is applied to the closed loop system with LQR controller, the amplitude of fluctuations decreases from 0.02(m) in the absence of friction compensation loop to 0.0001(m) while, in this paper, with nonlinear optimal controller, it has decreased to 0.00005(m) and positioning with very high accuracy is achieved. It shows 50 percent improvement in system performance.

The suggested technique is a combination of online and offline approaches and requires just one simple experiment, while many other proposed approaches require some special conditions and experiments for parameter identification which are not feasible in some cases [7], [8], [11], [12]. Estimated parameters in this technique are not related to models; hence, we do not require utilizing complex nonlinear estimation approaches because of highly nonlinear models.

Since this technique deals with friction indirectly and eliminates its effects, it works for different static and dynamic models of friction and no state observer is needed [3].

On the other hand, friction behavior is investigated in two phases, stiction and kinematic [4]. Based on this phases, some of the authors have proposed approaches to estimate parameters and compensate for friction in one special phase or ,generally, their proposed techniques work for one special model [8], [13]. For instance, in [8], LuGre model is linearized and identification is done for stiction regime. In identification process, one experiment which is applying an input with very small amplitude such that system remains in its linear domain is essential. Satisfying this condition is not an easy task. In [11], parameters are classified in two, static and dynamic, groups and estimated separately. Estimation of static parameters require about twenty experiments with conditions on system velocity and dynamic parameters estimation require special condition on the input, while in our proposed adaptive technique, parameters are estimated entirely without any special condition [1].

## 6. Conclusion

Friction is a nonlinear physical phenomenon which is not considered in mathematical modeling of system, but it has destructive effects on its performance. Therefore, the friction compensation problem has been addressed in this paper. At first, a nonlinear optimal controller based on HJB PDE approximate solution was designed for the SIPC system.Then, an adaptive compensation technique which, generally, is used to cancel noise signal was designed and applied to the system. The effectiveness and high performance of the controller and the proposed compensation approach in positioning with very high accuracy and elimination of friction effects were shown by simulations.

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