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Second Order Sliding Mode Control With Finite Time Convergence

V. Behnamgol[1], A. R. Vali[2*], and I. Mohammadzaman[3]

1- PhD Student, Department of Control Engineering, Malek Ashtar University of Technology
2- Associate professor, Control Engineering Department, Malek Ashtar University of Technology
3- Assistant professor, Control Engineering Department, Malek Ashtar University of Technology

ABSTRACT

In this paper, a new smooth second order sliding mode control is proposed. This algorithm is a modified form of Super Twisting algorithm. The Super Twisting guarantees the asymptotic stability, but the finite time stability of proposed method is proved with introducing a new particular Lyapunov function. The Proposed algorithm which is able to control nonlinear systems with matched structured uncertainty, is able to guarantee the finite time stability. The main advantage of this second order sliding mode control is reaching to sliding surface with high precision without chattering in control signal. In simulation section, the proposed algorithm is compared with the boundary layer sliding mode control and then is applied to designing a finite time nonlinear guidance law that is robust with respect to target maneuvers. Simulation results show that the control input in this algorithm is smooth and has no chattering and by applying this method, sliding variables will converge to zero in a given desired finite time.

KEYWORDS

Second Order Sliding Mode, Finite Time Stability, Chattering, Uncertainty.

*Corresponding Author, Email: ar.vali@aut.ac.ir

1. INTRODUCTION

Sliding mode control is known to be robust against parameter uncertainties and external disturbances. This approach has been recognized as one of the efficient tools to design robust controllers for complex high-order nonlinear dynamic plants operating under various uncertain conditions. The major advantage of sliding mode is the low sensitivity to plant parameter variations and disturbances. Sliding mode control enables the decoupling of the overall system motion into independent partial components of lower dimensions, which reduces the complexity of feedback design. Traditional sliding mode control or variable structure control has some intrinsic problems, such as discontinuous control that often yields chattering in control input [1]. Chattering may excite unmolded high frequency dynamics, which degrades the performance of the system and may even lead to instability [2].

One approach to eliminate chattering in control signal is to use a continuous approximation of the discontinuous sliding mode controller. Continuous approximation leads to tracking to within a guaranteed precision rather than perfect tracking [1]. Another approach to cope with this problem and achieve higher accuracy is high order sliding mode control. HOSM has two important features that make it a better choice in designing proper controllers. It improves the accuracy of the design, which is a very important issue, and may provide a continuous control signal. Higher order sliding mode generalizes the standard SM idea, seeking to zeroing not just the sliding variable but also some of its time derivatives. In particular, second order sliding modes (2-SM) would provide for zeroing the sliding variable and its first time derivative. Hence, 2-SM algorithms synthesize a discontinuous control $\dot{u}$ with u continuous; therefore, reducing chattering and avoiding strong mechanical efforts while preserving 1-SM advantages. As shown in Fig. 1, in the first order sliding mode algorithm sliding takes place on the S = 0 axis, while Fig. 2 shows that in the second order sliding mode algorithm sliding will take place at the origin only [3], [4].

In [5] higher order sliding mode control techniques, in specific ‘‘prescribed convergence law’’, ‘‘quasi continuous’’ and ‘‘super twisting’’ control algorithms, are used to robustly stabilize the glucose concentration level of a diabetic patient in the presence of parameter variations and meal disturbance. In [6], a new smooth second order sliding mode method for systems with relative degree one was proposed and asymptotic stability of this algorithm was proved. In [7] finite time second order sliding mode was proposed for removing chattering in systems with relative degree two. In [8] a second order sliding mode controller was designed for uncertain linear systems with parametric uncertainty and [3] studied the applicability of ‘‘sub-optimal’’, ‘‘twisting’’, ‘‘super-twisting’’ and ‘‘with a prescribed law of variation’’ algorithms of second order sliding mode control strategies to a wind energy conversion system, which present finite time convergence, robustness, chattering and mechanical stress reduction, and are of quite simple online operation and implementation. In [9] a second order discrete sliding mode control approach was used for the temperature control of a chemical reactor.

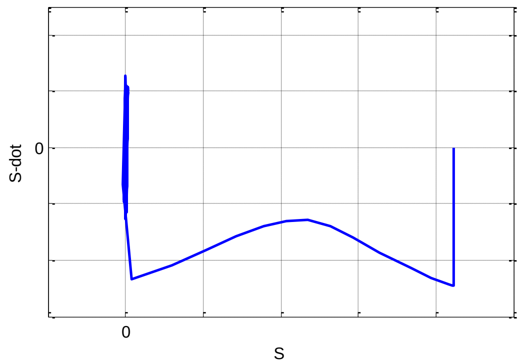

Fig. 1. 1-SMC reaching phase (Sliding takes place on the S = 0 axis)

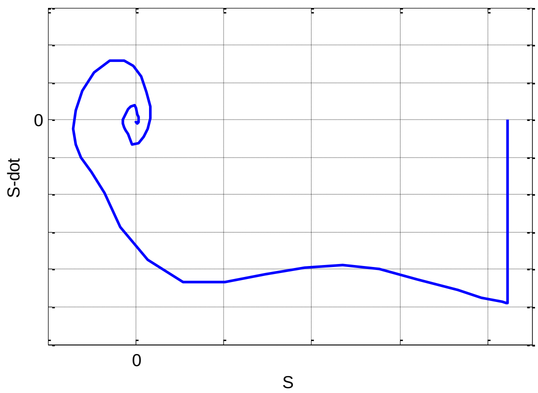

Fig. 2. 2-SMC reaching phase (Sliding takes place in the origin only)

Recently finite time stability and finite time control have been constructed for some systems [10]. Finite time stable systems might enjoy not only faster convergence but also better robustness and disturbance rejection properties [11]. In [12], a Lyapunov stability theorem has been presented for testing finite time stability of a double integrator system by continuous, unbounded or bounded, state feedback control law. Finite time stability of guidance systems have been demonstrated in [13]. This paper cannot guaranty the convergence of the LOS rate to zero in the finite time and only analyze the stability of the guidance system in a short time. In [14], a nonlinear guidance scheme with finite time convergence based on Lyapunov scalar differential inequality was developed. In [15], a new smooth second order sliding mode was proposed and its finite time stability was proved using homogeneity based technique for interceptor guidance system. In this reference the control signal needs more complicated calculations and the sliding surface is defined specially using relative range that being unable to meet zero until zeroing range and has not any benefits of the finite time stability.

In this paper, a new finite time second order sliding mode algorithm is proposed. Finite time convergence is proved using a new Lyapunov function. This algorithm out performs the boundary layer method and its finite time stability is verified through a simulation example. This algorithm is used for design a smooth finite time guidance law.

The paper is organized as follows. In section 2, the conventional first order and second order sliding mode control algorithms is introduced. Section 3 presents the new finite time second order sliding mode control. In section 4 simulation results are presented. Conclusions are reported in section 5.

2. Sliding Mode Control

Consider a first order nonlinear system

$$\dot{x} = f(x) + u + w \quad , |w| \le \alpha \tag{1}$$

where $f(x)$ is a known nonlinear part, and w is a bounded uncertainty. With considering $x_e = 0$ as equilibrium point, the control objective is zeroing state variable. Let us introduce a sliding variable [1]

$$S = x \tag{2}$$

Assume that u in equation (1) is a control input, and must be designed such that this system has the desired properties. Due to introducing sliding variable (2), if $S=0$, then $x=0$ is obtained, and finally the variable x has the desired properties. Conventional first order sliding mode control makes variable S equal to zero in finite time and then maintain the condition $S=0$ for all future time. Typical sliding mode control consists of a reaching mode, during which the sliding variable S moves to the sliding surface $S=0$, and a sliding mode, during which the sliding variable is confined to the sliding surface and the sliding variable has no variation from sliding surface in system without uncertainty. Let us take the control input as [1]:

$$u = u_{eq} + u_{reaching} \tag{3}$$

u_{eq} is the equivalent control determined to cancel the known terms on the first derivation of sliding variable in system without uncertainty as [1]:

$$\dot{S} = f(x) + u_{eq} = 0 \Rightarrow u_{eq} = -f(x) \tag{4}$$

If there is no uncertainty in the system, $u = u_{eq}$ will maintain the system in the sliding surface. Now, let us consider the case where uncertainties exist. In conventional sliding mode control the reaching control is selected by [1]:

$$u_{reching} = -KSign(S) \tag{5}$$

A sufficient condition to guarantee the finite time attractiveness of sliding surface $S=0$ for $S \neq 0$, is to ensure [16]:

$$\dot{V} = S\dot{S} \le -\eta |S| \tag{6}$$

where η is a strictly positive constant, which implies that [16]:

$$t_{reach} \le \frac{|S(0)|}{\eta} \tag{7}$$

In order to satisfy the sliding condition (6) despite uncertainty on the dynamics system (1), substituting equations (2) and (5) into equation (6) to obtain

$$K \ge \eta + \frac{x}{|x|}(w) \tag{8}$$

By choosing K in (8) to be large enough, we can now guarantee that (8) is verified. So we let

$$K = \eta + \alpha \tag{9}$$

where α denotes the bound of w. By substituting equations (4), (5) and (9) into equation (3), the control input signal is given as

$$u = -f(x) + (\eta + \alpha) Sign(x) \tag{10}$$

It is certain from equation (7) that the trajectory of x converges to the manifold $S=0$ at some finite time and it will be confined to that manifold for all the future time [1].

The sliding mode controller (10) contains the discontinuous nonlinear function $Sign(.)$. This nonlinearity can cause the chattering problem due to delays or imperfections in the switching devices [1], [2]. To eliminate the chattering in control input, the higher order sliding modes generalize the conventional sliding mode idea, seeking to zero not just the sliding variable but also some of its time derivatives. In particular, second order sliding mode would provide for the zeroing of the sliding variable and its first time derivative in finite time, through discontinuous control action acting on its second time derivative, being sliding variable of relative degree 2 or 1. Hence, second order sliding mode algorithms synthesis a discontinuous controller $\dot{u}(t)$ which makes $S=\dot{S}=0$, with controller $u(t)$ continuous; therefore, reducing chattering and avoiding strong mechanical efforts while preserving conventional sliding mode control advantages [5], [3] and [4].

Prescribed law of variation (PLV) in the class of second order sliding mode algorithms is the control action depends on a function $g(S)$ determined during the design process, and the convergence properties are strictly related to it. The function must be continuous and smooth everywhere. The control law and the chosen function are [5]:

$$\begin{aligned} &\dot{u}_{reching} = -V_M Sign(\dot{S} - g(S)) \\ &g(S) = -\gamma |S|^{1/2} Sign(S), \gamma > 0 \end{aligned} \qquad (11)$$

It should be noticed that the algorithm needs not just S to be known, but also its time derivative and the value of $g(S)$. This controller trajectory in the phase plane is shown in Fig. 3.

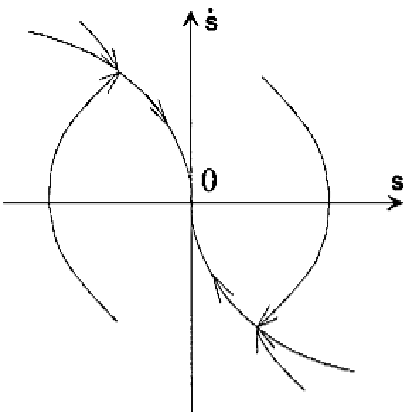

Fig. 3. PLV controller trajectory in the phase plane

In Sub-optimal (SO) algorithm, trajectories in the $S-\dot{S}$ plane are confined within limit parabolic arcs which include the origin, so the convergence behavior may include twisting around the origin, ''bouncing'' on the S axis or a combination of both. The control law may be written as [3]:

$$\dot{u}_{reching} = -\gamma V_M Sign(S - S_M/2) \qquad (12)$$

where S_M is variable and corresponds to the last external value of the sliding variable. This algorithm requires the ability to detect the zeros of $\dot{S}$, and the corresponding values of S in those instants, S_M. The two possible trajectories of the sub-optimal algorithm in the phase plane are shown in Fig. 4.

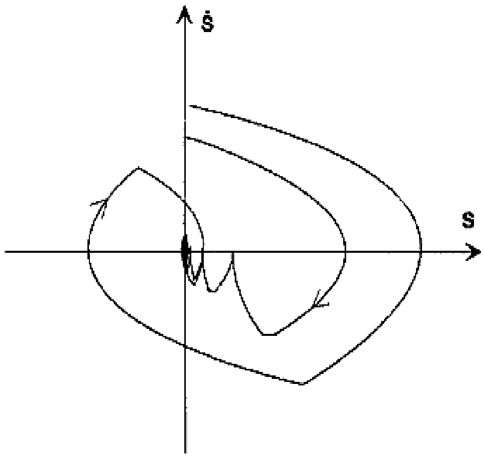

Fig. 4. The two possible trajectories of the sub-optimal algorithm in the phase plane

The twisting (T) algorithm is characterized by the manner in which its trajectories converge to the origin in the sliding plane $S-\dot{S}$. The knowledge of the signs of S and $\dot{S}$ is needed, and the control law is given by [3]:

$$\dot{u}_{reching} = -r_1 Sign(S) - r_2 Sign(\dot{S}), r_1 > r_2 > 0 \qquad (13)$$

The trajectory of the twisting algorithm in the phase plane is shown in Fig. 5.

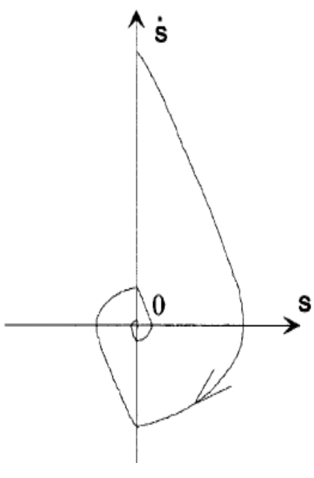

Fig. 5. Twisting controller trajectory in the phase plane

Super twisting is a second order algorithm developed to reduce chattering in systems with Relative Degree 1, by using an integral state in the control, and with the great advantage of not requiring information of $\dot{S}$. The trajectories converge to the origin of the sliding plane turning around it in a typical way. The control law is described by two terms, one discontinuous defined by its time derivative, and the other, a continuous function of sliding variable activated only during the reaching phase [3]:

$$\begin{aligned} u_{reaching} &= -u_1(t) - k_1 |S|^{\gamma}\, Sign(S); 0<\gamma<1 \\ \dot{u}_1(t) &= k_2 Sign(S) \end{aligned} \tag{14}$$

However, control signal in the Super Twisting control algorithm is not smooth and guarantee the asymptotic stability. This controller trajectory in the phase plane is shown in Fig. 6 [4].

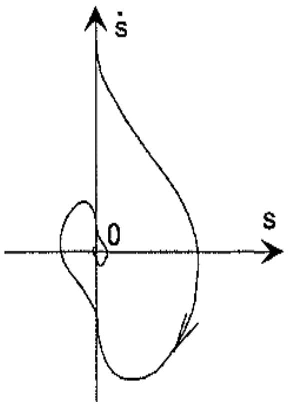

Fig. 6. Super Twisting controller trajectory in the phase plane

3. NEW SECOND ORDER SLIDING MODE

We introduce a new finite time stable second order algorithm. To guarantee the finite time attractiveness of $S=0$ for $S\neq 0$, we use the following Theorem.

Theorem 1: The reaching term of control input

$$u_{reching} = -\beta_1 Sign(S) - \beta_2 \int Sign(S)d\tau \tag{15}$$

with $\beta_2(\alpha-\beta_1)+\eta=0$ provides for the finite time convergence sliding variable.

Proof: Substituting equations (4) and (15) into the (3) and by putting the resulted equation in the (1) and (2), the sliding variable dynamics yields

$$\dot{S} = -\beta_1 Sign(S) - \beta_2 \int Sign(S)d\tau + w \tag{16}$$

The system (16) can be equivalently presented by a system of two first order equations:

$$\begin{cases} \dot{\xi}_1 = -\beta_1 Sign(\xi_1) + \xi_2 + w \\ \dot{\xi}_2 = -\beta_2 Sign(\xi_1) \end{cases}; \quad \xi_1 = S \tag{17}$$

Let a Lyapunov function candidate be:

$$V(\xi) = \beta_2 |\xi_1| + \frac{1}{2}\xi_2{}^2 \tag{18}$$

Then the Lyapunov function derivative will be

$$\begin{aligned} &\dot{V} = \beta_2 \frac{\xi_1}{|\xi_1|}\dot{\xi}_1 + \xi_2\dot{\xi}_2 = \\ &\beta_2 \frac{\xi_1}{|\xi_1|}\left(-\beta_1 Sign(\xi_1) + \xi_2 + w\right) + \xi_2\left(-\beta_2 Sign(\xi_1)\right) = \\ &-\beta_1\beta_2 + \beta_2 w Sign(\xi_1) \end{aligned} \tag{19}$$

To guarantee the finite time attractiveness of sliding surface for $S(0)\neq 0$, a sufficient condition is to ensure

$$\dot{V} \le -\eta \tag{20}$$

where η is a strictly positive constant. Let t_{reach} be the time required to hit the sliding surface $S=0$. Integrating (20) between $t=0$ and $t=t_{reach}$ leads to

$$\begin{aligned} &\int \dot{V} \le \int -\eta \Rightarrow \int \frac{dV}{dt} \le \int -\eta \Rightarrow \\ &\int_{V(t=0)}^{V(t=t_{reach})} dV \le \int_{t=0}^{t=t_{reach}} -\eta dt \Rightarrow \\ &V(t=t_{reach}) - V(t=0) \le \\ &-\eta(t=t_{reach}) + \eta(t=0) \Rightarrow \\ &\beta_2 |\xi_1(t_{reach})| + \frac{1}{2}\left(\xi_2(t_{reach})\right)^2 - \\ &\beta_2 |\xi_1(0)| - \frac{1}{2}\left(\xi_2(0)\right)^2 \le -\eta t_{reach} \end{aligned} \tag{21}$$

where $\xi_1 = S$, $\xi_2 = -\beta_2 \int_0^t Sign(S)dt$ and $S(t_{reach})=0$. Finally:

$$\begin{aligned} &\beta_2 |S(t_{reach})| - \frac{1}{2}\left(\beta_2 \int_0^{t_{reach}} Sign\left(S(t_{reach})\right)\right)^2 \\ &-\beta_2 |S(0)| + \frac{1}{2}\left(\beta_2 \int_0^0 Sign\left(S(0)\right)\right)^2 \le -\eta t_{reach} \Rightarrow \\ &-\beta_2 |S(0)| \le -\eta t_{reach} \Rightarrow t_{reach} \le \frac{\beta_2 |S(0)|}{\eta} \end{aligned} \tag{22}$$

Therefore, (20) is a finite time condition with finite reaching time (22).

Now, in order to satisfy the sliding condition (20) despite uncertainty on the dynamics system (17), substituting equation (19) into (20) yields

$$-\beta_1\beta_2+\beta_2 wSign(\xi_1)+\eta\le-\beta_1\beta_2+\beta_2\alpha+\eta\le 0 \qquad (23)$$

By choosing $\beta_2(\alpha-\beta_1)+\eta\le 0$ in (23), we can now guarantee that (20) is verified. So, we proved finite time convergence $\xi_1=S$ and ξ_2 to zero. In the other words we have a finite time second order sliding mode controller.

By substituting equations (4) and (15) into equation (3), control input becomes

$$u=-f(x)-\beta_1 Sign(x)-\beta_2\int Sign(x)d\tau \qquad (24)$$

The switching bias terms in this control law act as an estimate of the uncertainty, when the system is not in sliding mode steady state. This algorithm is proposed for any nonlinear systems with matched structured uncertainty, but only is able to guarantee the reaching phase in finite time.

4. Simulation Examples

In this section, first the proposed algorithm is compared with the boundary layer method in a simple nonlinear example and its finite time stability is checked. Then, the new second order sliding mode is applied to designing guidance law for generating a smooth acceleration for intercepting with a maneuvering target. In this example removing chattering the $Sign(x)$ function in (24) is replaced with $Tanh(cx)$ function. Therefore the proposed control law is given as

$$u=-f-\beta_1 Tanh(c_1 x)-\beta_2\int Tanh(c_2 x)d\tau \qquad (25)$$

where c_1 and c_2 are positive constants parameter for adjusting the smoothness of control signal. The response of proposed continuous function with deferent values for c, saturation and discontinuous Signum functions are shown in Fig. 7. As shown in this Figure with increasing value of c we are able to adjust the precision of the proposed function.

A. Comparison With The Boundary Layer Smc

For verifying the performance of the proposed SMC, we apply the algorithm to a simple nonlinear model. The nonlinear system equation is:

$$\ddot{x}=-2x\dot{x}+w+u \quad ,|w|\le 1 \qquad (26)$$

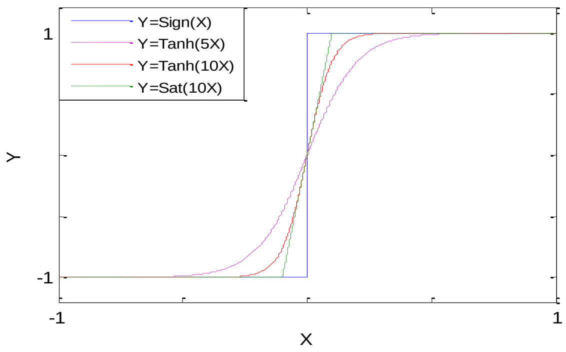

Fig. 7. The responses of Tanh(.), Sat(.) and Sign(.) functions.

Assume that the desired state is $(x,\dot{x})=(0,0)$. For designing a proper control law, sliding variable is introduced as:

$$S=\dot{x}+\lambda x \qquad (27)$$

An equivalent control is determined to cancel the known terms on the right hand side of equation $\dot{S}$ that guarantee sliding along manifolds $S=0$:

$$\begin{aligned}&\dot{S}=\ddot{x}+\lambda\dot{x}=-2x\dot{x}+w+u+\lambda\dot{x}\Rightarrow\\&u_{eq}=2x\dot{x}-\lambda\dot{x}\end{aligned} \qquad (28)$$

If there is no uncertainty ($w=0$) in the system, then the equivalent control will maintain the system in the sliding surface $S=0$.

Now, let us consider the case where uncertainty exists. To guarantee the finite time attractiveness of $S=0$, we apply Theorem 1. Therefore, for the reaching law given in (15), the control input is

$$\begin{aligned}&u=2x\dot{x}-\lambda\dot{x}-\beta_1\,\mathrm{Tanh}\left(c_1\left(\dot{x}+\lambda x\right)\right)\\&-\beta_2\int\mathrm{Tanh}\left(c_2\left(\dot{x}+\lambda x\right)\right)d\tau\end{aligned} \qquad (29)$$

Now we consider the situation in which the initial condition in (26) is $(x(0),\dot{x}(0))=(-1,2)$, $\lambda=1$ and uncertainty as shown in Fig. 8. We apply then the proposed second order sliding mode controller (29) with $\beta_1=2$, $\beta_2=5$, $c_1=10$ and $c_2=100$.

Figs. 9 and 10 show the control input and reaching phase in the proposed SOSMC and conventional SMC. As shown in these Figures, the control input in the proposed SOSMC is smooth and in conventional SMC has chattering. Also in conventional SMC, sliding takes place on the S = 0 axis in reaching phase, But in second order sliding mode algorithm sliding takes place in the origin only.

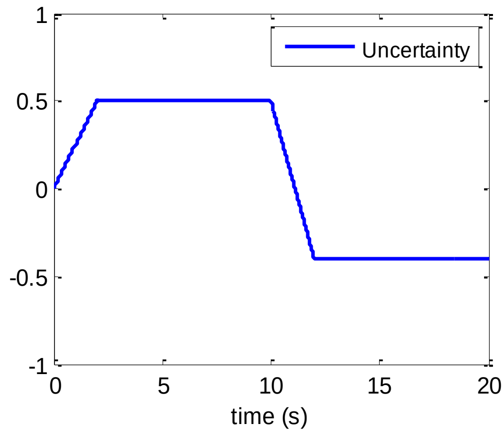

Fig. 8. Uncertainty (W).

Control input and reaching phase in the proposed and boundary layer SMC are shown in Figs. 11 and 12. These Figures show that the control input in the boundary layer is smooth and has no chattering, but sliding takes place on the S = 0 axis in reaching phase.

Sliding variable in the proposed and the boundary layer SMC is shown in Figs. 13 and 14. These Figures show that the S-variable in the proposed algorithm reaches the sliding surface in shorter time and higher precision than boundary layer SMC. Therefore, using boundary layer method the chattering is removed but the precision is decreased.

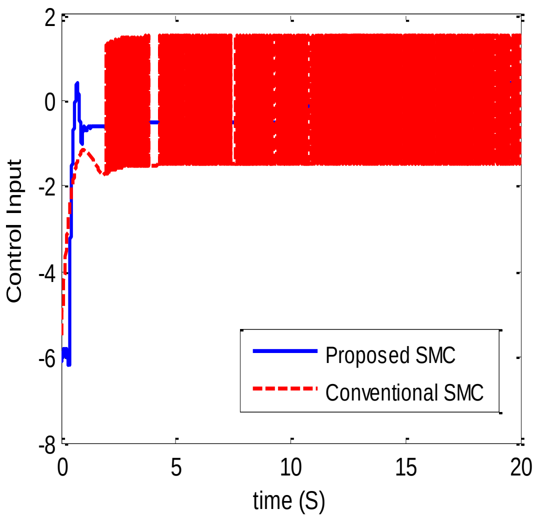

Fig. 9. Control Input Signal using Proposed SOSMC and Conventional SMC.

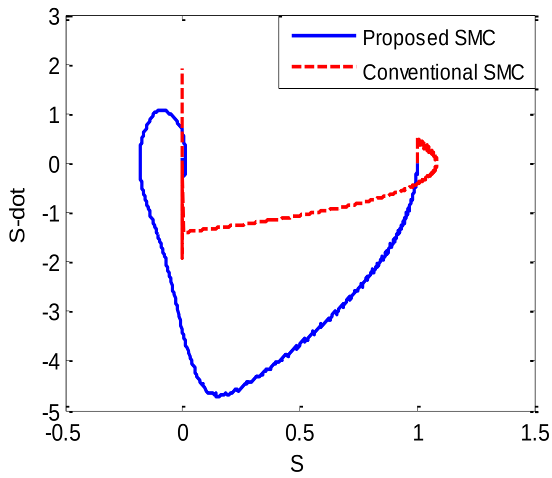

Fig. 10. Reaching Phase using Proposed SMC and Conventional SMC.

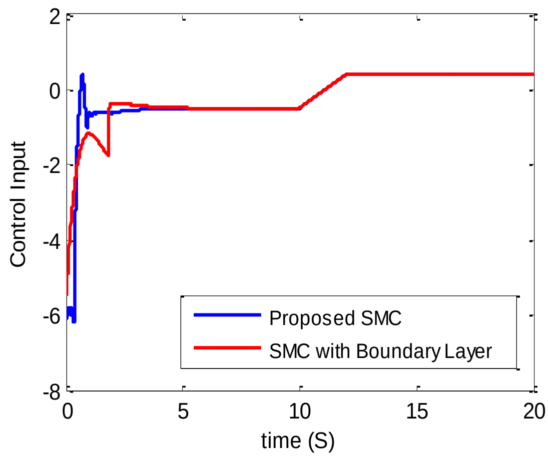

Fig. 11. Control Input Signal using Proposed SMC and Boundary Layer SMC.

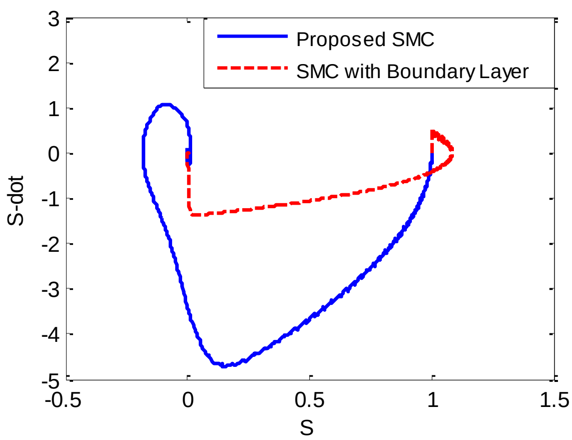

Fig. 12. Reaching Phase using Proposed SMC and Boundary Layer SMC.

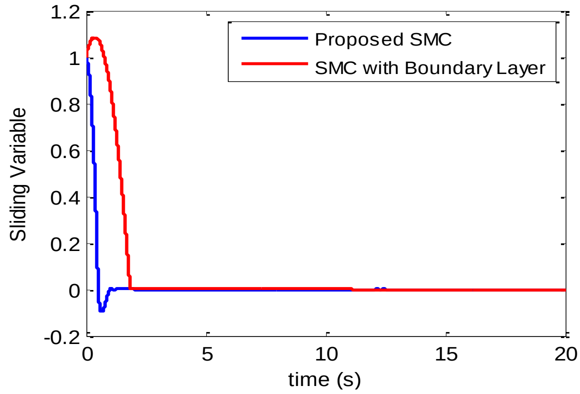

Fig. 13. Sliding Variable using Proposed SMC and Boundary Layer SMC.

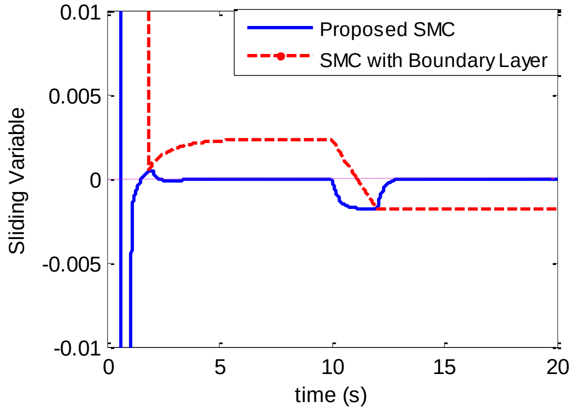

Fig. 14. Sliding Variable using Proposed SMC and Boundary Layer SMC (Sliding Phase).

As shown in Figs. 15 and 16 the state variables in the proposed algorithm converge to zero with a lower overshoot than that of the boundary layer SMC.

The system trajectories using the proposed SMC and boundary layer SMC in the Phase Plane are shown in Fig. 17. This Figure shows that the reaching phase in proposed sliding mode algorithm is shorter than that of the boundary layer method. In other word using proposed method the system trajectory selects a shorter way to reaching the sliding surface.

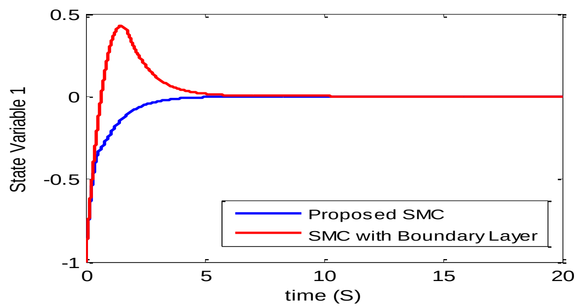

Fig. 15. State Variable 1 using Proposed SMC and Boundary Layer SMC.

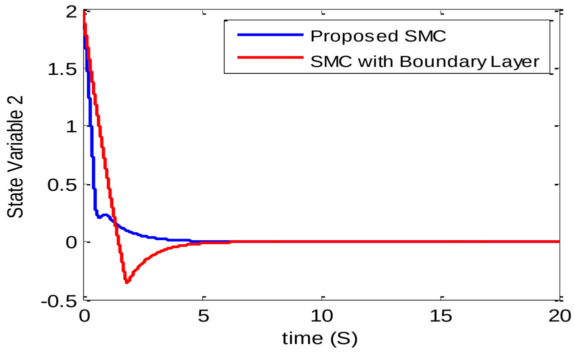

Fig. 16. State Variable 2 using Proposed SMC and Boundary Layer SMC.

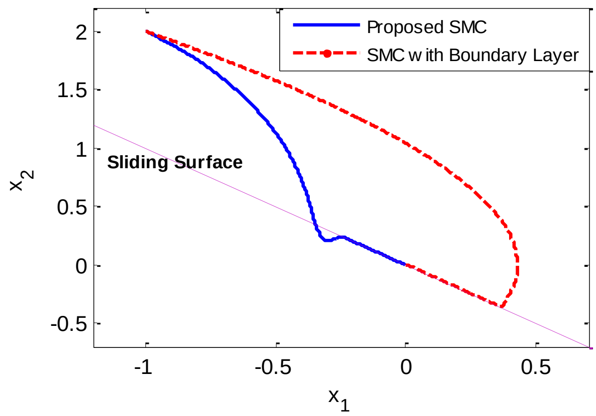

Fig. 17. System Trajectory using Proposed SMC and Boundary Layer SMC in the Phase Plane.

Now we consider the situation in which the initial condition in (26) is $(x(0), \dot{x}(0)) = (-1, -1)$, $\lambda = 1$ and constant uncertainty $w = 0.4$. We apply the proposed controller (29) for $t_r = 2,5,10$. The controller gains are determined using (22) and (23) as shown in Table 1. Note that in this case the constant parameters are $\alpha = 0.5$, $C_1 = 200$, $C_2 = 250$ and $S_0 = -2$.

TABLE 1. CONTROLLER GAINS FOR $t_r = 2,5,10$.

t_r	2	5	10
β_1	1.5	0.9	0.7
β_2	1	0.6	0.3
η	1	0.24	0.06

Figs. 18 and 19 show the control input and reaching phase using the proposed and conventional SMC. As shown in these Figures for decreasing the reaching time the initial amplitude of the control input should increase. Also, sliding takes place at the origin.

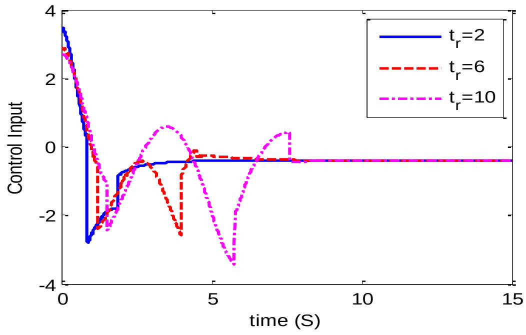

Fig. 18. Control Input Signal using Proposed SMC with $t_r = 2{,}5{,}10$.

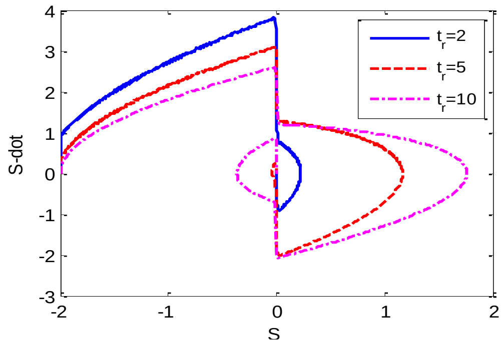

Fig. 19. Reaching Phase using Proposed SMC with $t_r = 2{,}5{,}10$.

As shown in Figs. 20 and 21 sliding variable reaches zero in the desired time.

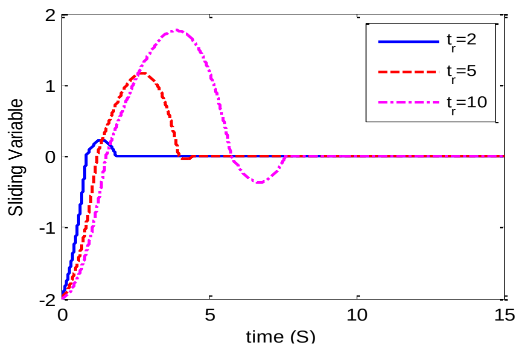

Fig. 20. Sliding Variable using Proposed SMC with $t_r = 2{,}5{,}10$.

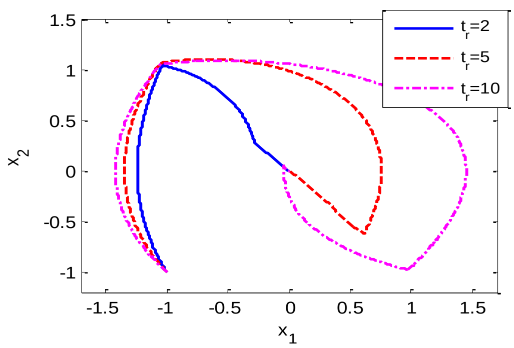

Fig. 21. System Trajectory using Proposed SMC with $t_r = 2{,}5{,}10$ **in the Phase Plane.**

B. Guidance Application

In this section, we apply the proposed algorithm for designing guidance law. For this, consider a two dimensional interceptor and target engagement as shown in Fig. 22. The kinematic relation between the target and interceptor motion is described as:

$$\frac{d}{dt}(\dot{r}) = r\dot{\sigma}^2 - A_{m1} + A_{t1} \tag{30}$$

$$\frac{d}{dt}(r\dot{\sigma}) = -\dot{r}\dot{\sigma} - A_{m2} + A_{t2} \tag{31}$$

where r is the relative range between target and interceptor, σ denotes the line of sight angle with respect to a reference axis, $r\dot{\sigma}$ is the relative lateral velocity between interceptor and target, A_{t1} and A_{t2} are the radial and tangential components of target acceleration and A_{m1} and A_{m2} are the radial and tangential components of interceptor acceleration, respectively [16, 17 and 18].

Note from (30) and (31) that A_{t1}, and A_{t2} can be treated as bounded uncertainties of the system. In equation (30) the control input is A_{m1} and control variable is $\dot{r}$. In equation (31) the control input is A_{m2} and control variable is $r\dot{\sigma}$.

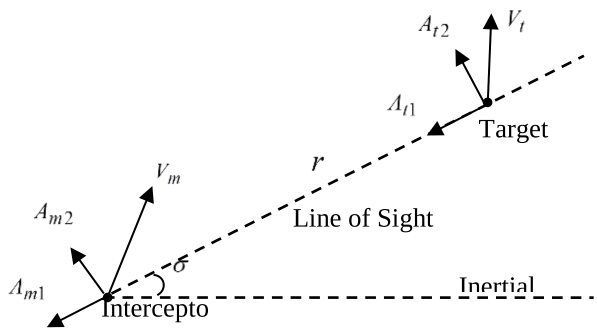

Fig. 22. Interceptor - target engagement geometry

For designing guidance law, closing velocity error and relative lateral velocity could be considered in the sliding variable as

$$\begin{cases} S_1 = \dot{r} - \dot{r}_d \\ S_2 = r\dot{\sigma} \end{cases} \tag{32}$$

Maintaining the closing velocity in a desired value and nulling the relative lateral velocity provide the motivation for these sliding variables. By a suitable choice of controls A_{m1} and A_{m2}, if we are able to achieve $S_1, S_2 = 0$, thus interception is guaranteed.

The equivalent guidance command for the known dynamics is the equivalent control determined to cancel the known terms on the right hand side of Eqs. (30) and (31) that guarantee sliding along manifolds $S_1, S_2 = 0$. Therefore the first derivative of sliding surfaces must be zero and these equivalent controls are achieved as:

$$\begin{cases} A_{m1_{eq}} = r\dot{\sigma}^2 \\ A_{m2_{eq}} = -\dot{r}\dot{\sigma} \end{cases} \tag{33}$$

If there is no uncertainty (target does not maneuver) in the system, then equivalent control will maintain the system in the sliding surfaces.

Now, let us consider the case where target accelerating exist. To guarantee the finite time attractiveness of $S_1, S_2 = 0$, we use Theorem 1. Therefore the guidance commands are designed as follow:

$$A_{m1} = r\dot{\sigma}^2 + \beta_1 sign(\dot{r} - \dot{r}_d) + \beta_2 \int sign(\dot{r} - \dot{r}_d) d\tau \tag{34}$$

$$A_{m2} = -\dot{r}\dot{\sigma} + \beta_3 sign(r\dot{\sigma}) + \beta_4 \int sign(r\dot{\sigma}) d\tau \tag{35}$$

with $\beta_2(\alpha_1 - \beta_1) + \eta_1 = 0$ and $\beta_4(\alpha_2 - \beta_3) + \eta_2 = 0$, which provides for the finite time convergence of the closing velocity and relative lateral velocity where α_1 is the bound of A_{t1} and α_2 is the bound of A_{t1}.

The switching bias terms in this guidance law act as estimates of the target acceleration, when the system is not in steady state sliding mode. In other words, the guidance law (35) behaves like the APN guidance law. This guidance law has been named as the Finite Time Second Order Sliding Mode Guidance. This guidance law can also be viewed as a form of the Modified Proportional Navigation (MPN). However, the main advantage of this guidance law over APN and MPN is that it does not require any explicit target maneuver estimation.

We apply the proposed controllers (34) and (35) for guidance system with equations (30) and (31). we consider the situation in which the initial relative distance is 30 km, the closing velocity is 700 m/s, and the lateral relative velocity is 423 m/s and in the proposed guidance laws for the purpose of implementation, the *Sign*(.) function is replaced with *Tanh*(.) function. Also, the target maneuvers as shown in Figs. 23 and 27 normal to the line of sight. Note that, the bounds of target accelerations (uncertainty) are $\alpha_1 = 5$ and $\alpha_2 = 20$ that are used in (34) and (35).

First, we apply the proposed guidance law (34) in equation (30). Fig. 23 shows the guidance command 1 (control input 1). As seen in this Figure, the control input is smooth and has no chattering, but with increasing the value of η_1, the maximum magnitude of the missile acceleration is increased,

Fig. 24 shows the closing velocity (state variable 1) and Fig. 25 shows the sliding variable 1. As shown in these Figures, the control variable 1 in desired finite time reaches to desired value and sliding variable 1 reaches to sliding surface $S_1 = 0$. Also with varying the value of parameter η_1, we are able to adjust the time of zeroing sliding variable.

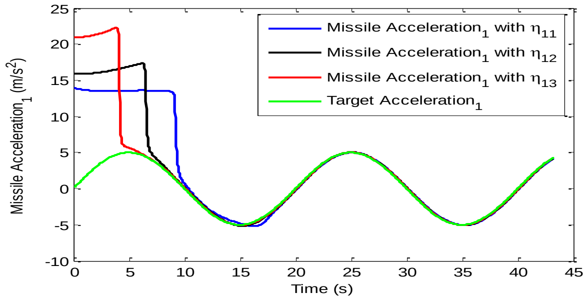

Fig. 23. missile acceleration (control input 1) and target acceleration (uncertainty) with $\eta_{11} < \eta_{12} < \eta_{13}$.

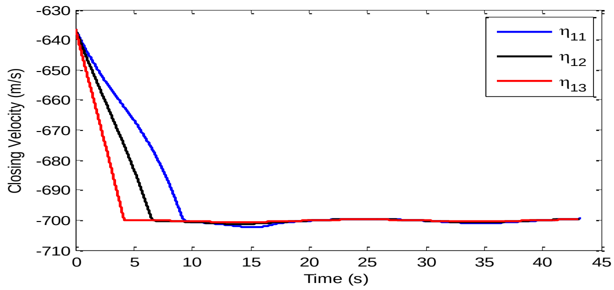

Fig. 24. closing velocity (state variable 1) with different value for η_1 ($\eta_{11} < \eta_{12} < \eta_{13}$).

Fig. 26 presents the phase plane obtained by using the proposed guidance law. The plots confirm the expected behavior of the sliding variable 1 and the phase plane converging to zero in a finite time.

Therefore, the finite time second order sliding mode controller (34) is able to eliminate chattering in control signal and finite time stabilizing the system (30). This controller is further robust against to uncertainty (target maneuvers A_{t1}).

Now we apply the proposed guidance law (35) in (31). Fig. 27 shows the guidance command input 2 (control input 2). As seen in this Figure, the control input is smooth and has no chattering, but by increasing the value of η_2, the maximum magnitude of the missile acceleration is increased,

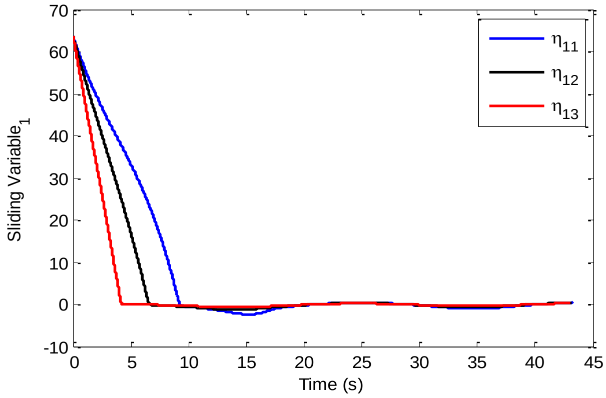

Fig. 25. sliding variable 1 with $\eta_{11} < \eta_{12} < \eta_{13}$.

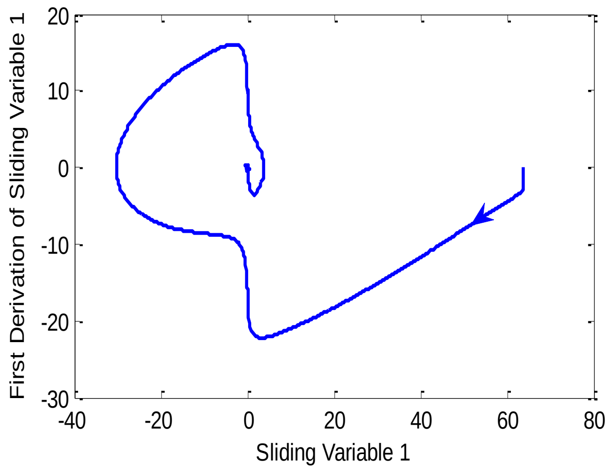

Fig. 26. phase plane of sliding variable 1.

Fig. 28 shows the relative lateral velocity (state variable 2). As shown in this Figure, the control variable 2 in desired finite time goes to zero and the sliding variable 2 reaches to sliding surface $S_2 = 0$. Also with varying the value of parameter η_2, we are able to adjust the time of zeroing sliding variable.

Fig. 29 presents the phase plane obtained by using the proposed guidance law. The plots confirm the expected behavior of the sliding variable 2 and the phase plane converging to zero in a finite time.

Fig. 30 shows the relative range in the end of engagement. It is clear that, with increasing the values of parameters the time of flight and intercepting with target is decreased. In other words, the time of zeroing relative range is decreased. Therefore, finite time stable guidance law is able to adjusting the time of intercept.

Therefore, the finite time second order sliding mode controller (35) is able to eliminate chattering in control signal and finite time stabilizing the system (31) that leads to decreasing time of flight and improving the performance of guidance law. This controller is also robust against to uncertainty (target maneuvers A_{t2}).

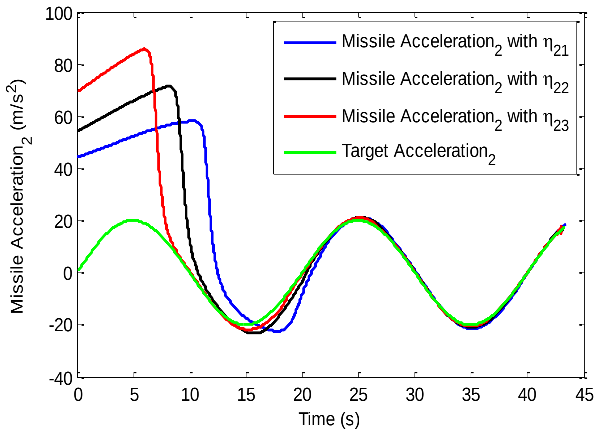

Fig. 27. missile acceleration (control input 2) and target acceleration (uncertainty) with $\eta_{21} < \eta_{22} < \eta_{23}$.

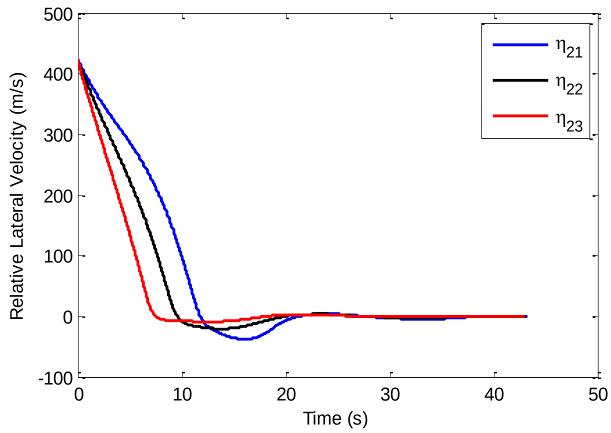

Fig. 28. relative lateral velocity (sliding variable 2) with different value for η_2 **(**$\eta_{21} < \eta_{22} < \eta_{23}$**).**

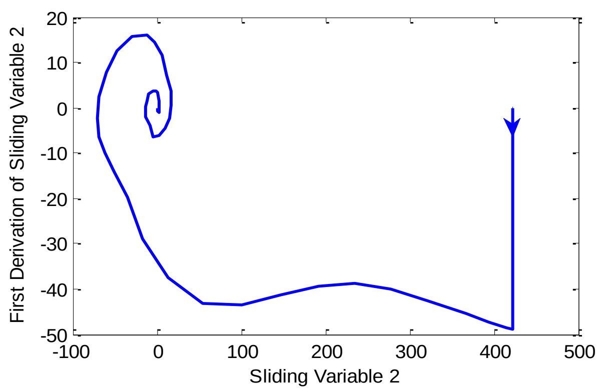

Fig. 29. phase plane of sliding variable 2.

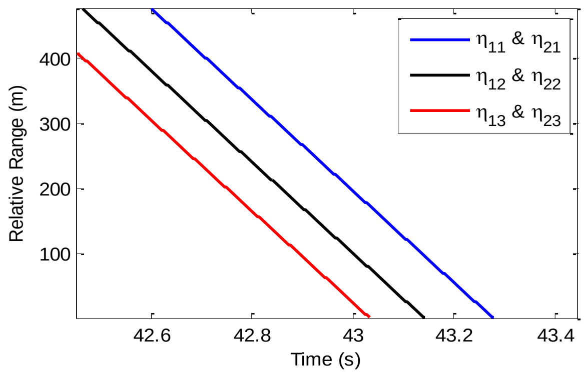

Fig. 30. relative range with different values for η_1 and η_2 ($\eta_{11} < \eta_{12} < \eta_{13}$ and $\eta_{21} < \eta_{22} < \eta_{23}$)

5. Conclusions

In this paper, a new finite time second order sliding mode control is proposed. The finite time stability of this algorithm is proved using Lyapunov method. Applying this controller in nonlinear uncertain systems leads to producing a smooth control signal and improving the overall performance of system without estimation of uncertainties. It is demonstrated via simulation that the proposed algorithm has better performance than boundary layer method with high precision.

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