



A recursive numerical algorithm to computing Fourier series coefficients to find cylinder potential in electrodynamics

Mostafa Ijavi^{1*}, Mohamad Moradi², Rasoul Riahi¹

¹ Department of physics, Faculty of science, Lorstan University, Khoram Abad, Iran

² Department of Physics, Karaj Branch, Islamic Azad University, Karaj, Iran

ABSTRACT: In this article, a new approach to find the Fourier expansion coefficients have been carried out by a recursive algorithm without computing their correspondent integral. Finally, in virtue of this new method, some Partial differential equations have been solved and compared with their exact solutions. After deriving the recursive relation, some differential equations have been solved with the partial differentiation and compared with the numerical answers in addition. The Fourier series coefficients were computed more accurate and fast in this method as compared to others then we calculate cylinder potential in electrodynamics by this method. The results show that the algorithm proposed in this paper has achieved better results.

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1- INTRODUCTION

For the first time the well-known mathematician, Joseph Fourier, devised of new series method respect to trigonometric functions in terms of sines and cosines while he was studying the heat conduction. It was just one of the many applications of Fourier series [1]-[21]. The Fourier series and transform has many applications, in fact any field of physical such as engineering, physics, applied mathematics, and chemistry. It could be given some examples of various branches of sciences where the Fourier transform is involved in finding their solutions. They are used in the analysis of current flow in electrical engineering, analysis of sound waves and signal processing. In physical issues, it could be applied to solve posed problems in Astronomy, optics and trapping. They are also used in mathematics to solve ordinary and partial differential equations and integral equations. Fourier's ideas can also be found in electronically synthesized music, chemistry and talking computer chips. These are just a few of examples of the many uses of Fourier series in the world of science and technology. In physics and engineering, expanding functions in terms of sines and cosines is very useful duo to the fact that it allows someone to manipulate periodic functions more easily, for instance, discontinuous or difficult functions to

represent their analytically form as a rigorous mathematical analysis techniques. In particular, the fields of electronics, quantum mechanics, and electrodynamics all of them make enormous uses of the Fourier series. In addition, other methods based on the Fourier Series, such as the Fast Fourier Transform (FFT) [1]-[3], a form of a Discrete and finite Fourier Transform (DFT), are especial useful for the areas of Digital Signal Processing (DSP) and Spectral Analysis.

The FFT strategy has been applied by Borak et al. [16]. They have demonstrated its efficacy by comparison with Monte-Carlo simulation for a variety of models. In another investigation, it has been showed how Fourier's convolution theorem can be used to price certain exotic options in combination with the FFT [17] and [18]. A different approach has been done by using the characteristic function to approximate the marginal transitional probability density of returns by Fang et al. [19]. Zhang et al. [20] have showed that this methodology can be extended to the pricing of early-exercise commodity options under the Ornstein-Uhlenbeck process.

In comparison with Fourier series expansion coefficients, we introduced a new algorithm based on Taylor expansion function that the corresponding ones in this method are computed based on recursive equations without necessity to any integral computing. Therefore, not only finding

*Corresponding author's email: m.ijavi@aut.ac.ir



coefficients are simple respect to direct method, but also speed and error of computations are definitely better than the traditional method. This paper is organized as follow. In next section, we describe details of this method and write the related procedures correspond to each stage. Afterwards, we express some applicable examples to realize and emphasizing this method. As an example, with the comparison of this method with Simpson integration method, we realized that in the Simpson method more 3996000 processes are needed to reach the same accuracy. As the conclusion, one can say that this method is far accurate than Simpson method since computes the Fourier coefficients analytically.

2- FINDING FOURIER COEFFICIENTS WITHOUT INTEGRATION

For any periodic real and bounded function F, defined on closed interval, with period length 2l, and with a finite number of discontinuous points, its Fourier series expansion is indicated as follow:

$$F(u) = a_0 + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi}{l}u\right) + b_n \sin\left(\frac{n\pi}{l}u\right), -l < u < l \tag{1}$$

Such that its coefficients are defined so

$$a_0 = \frac{1}{2l} \int_{-l}^l F(x) dx \tag{2a}$$

$$a_n = \frac{1}{l} \int_{-l}^l F(x) \cos\left(\frac{n\pi}{l}x\right) dx, n = 1, 2, \dots \tag{2b}$$

$$b_n = \frac{1}{l} \int_{-l}^l F(x) \sin\left(\frac{n\pi}{l}x\right) dx, n = 1, 2, \dots \tag{2c}$$

By applying change of variable $u = x - x_0$, it could be inferred

$$F(x - x_0) = a_0 + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi}{l}(x - x_0)\right) + b_n \sin\left(\frac{n\pi}{l}(x - x_0)\right) \tag{3}$$

That $(-l + x_0 < x < l + x_0)$ and its correspond coefficients are defined so

$$a_0 = \frac{1}{2l} \int_{x_0-l}^{x_0+l} F(x - x_0) dx \tag{4a}$$

$$a_n = \frac{1}{l} \int_{x_0-l}^{x_0+l} F(x - x_0) \cos\left(\frac{n\pi}{l}(x - x_0)\right) dx \tag{4b}$$

$$b_n = \frac{1}{l} \int_{x_0-l}^{x_0+l} F(x - x_0) \sin\left(\frac{n\pi}{l}(x - x_0)\right) dx \tag{4c}$$

On the other hand, we can rewrite this differentiable function based on Taylor expansion around the point $x = x_0$.

$$F(u) = \sum_{n=0}^{\infty} c_n u^n = \sum_{n=0}^{\infty} c_n (x - x_0)^n \tag{5}$$

Afterwards, we express the computed Fourier expansion coefficients (FEC) of simple polynomials from different orders which are the approximations from the same polynomials order.

One-point FEC:

Applying Fourier extension Equations (3) and (4) for $F(u) = F_0(u) = 1, (-l < u < l)$, we get

$$a_{0,0} = 1, a_{n,0} = 0, b_{n,0} = 0 \tag{7}$$

Two-point FEC:

Applying Fourier extension Equations (3) and (4) for $F(u) = F_1(u) = u, (-l < u < l)$, we get

$$a_{0,1} = 0, a_{n,1} = 0, b_{n,1} = -\frac{2l}{n\pi} (-1)^n \tag{8}$$

Three-point FEC:

Applying Fourier extension Equations (3) and (4) for

$F(u) = F_2(u) = u^3, (-l < u < l)$, we get

$$a_{0,2} = \frac{l^2}{3}, \quad a_{n,2} = \frac{4l^2}{(n\pi)^2}(-1)^n, \quad b_{n,2} = 0 \quad (9)$$

M-point FEC:

Applying Fourier extension Equations (3) and (4) for $F(u) = F_m(u) = u^m, (-l < u < l, m \geq 4)$, we get

$$a_{n,m} = \frac{1}{l} \int_{-l}^l u^m \cos\left(\frac{n\pi}{l}u\right) du \quad (10)$$

With integration by part we could get:

$$a_{n,m} = \frac{1}{l} \left[\begin{aligned} & \frac{l}{n\pi} u^m \sin\left(\frac{n\pi}{l}u\right) \Big|_{-l}^l \\ & - \frac{lm}{n\pi} \int_{-l}^l u^{m-1} \sin\left(\frac{n\pi}{l}u\right) du \end{aligned} \right] \quad (11)$$

After a little simplification we reach to these conclusions:

$$a_{n,m} = -\frac{lm}{n\pi} b_{n,m-1} \quad (12)$$

Such that $b_{n,m-1} = \frac{1}{l} \int_{-l}^l u^{m-1} \sin\left(\frac{n\pi}{l}u\right) du$ again with integration by part, we get the following equality:

$$b_{n,m} = \frac{1}{l} \left[\begin{aligned} & \frac{-l}{n\pi} u^m \cos\left(\frac{n\pi}{l}u\right) \Big|_{-l}^l \\ & + \frac{lm}{n\pi} \int_{-l}^l u^{m-1} \cos\left(\frac{n\pi}{l}u\right) du \end{aligned} \right] \quad (13)$$

Again after a little simplification we reach to these conclusions:

$$b_{n,m} = -\frac{(-1)^n l^m}{n\pi} [1 - (-1)^m] + \frac{lm}{n\pi} a_{n,m-1} \quad (14)$$

Also with attention to Equations (12), and (14), we get recursive equality for coefficients $a_{n,m}, b_{n,m}$:

$$a_{n,m} = \frac{(-1)^n m l^m}{(n\pi)^2} [1 - (-1)^{m-1}] - \left(\frac{l}{n\pi}\right)^2 m(m-1) a_{n,m-2} \quad (15a)$$

$$b_{n,m} = -\frac{(-1)^n l^m}{n\pi} [1 - (-1)^m] - \left(\frac{l}{n\pi}\right)^2 m(m-1) b_{n,m-2} \quad (15b)$$

$$a_{0,m} = \frac{l^m}{2(m+1)} [1 - (-1)^{m-1}] \quad (15c)$$

In other words, we get the following equalities for even and odd indexes of coefficients;

$$a_{n,2m} = \frac{4(-1)^n m l^{2m}}{(n\pi)^2} - 2\left(\frac{l}{n\pi}\right)^2 m(2m-1) a_{n,2m-2}, a_{n,2m+1} = 0 \quad (16a)$$

$$b_{n,2m+1} = -\frac{2(-1)^n l^{2m+1}}{n\pi} - 2\left(\frac{l}{n\pi}\right)^2 m(2m+1) b_{n,2m-1}, b_{n,2m} = 0 \quad (16b)$$

Finally, for first coefficient we have

$$a_{0,2m} = \frac{l^{2m}}{2(2m+1)}, \quad a_{0,2m+1} = 0 \quad (16c)$$

Therefore, by these recursive equations we could introduce a procedure for computing the coefficients of Fourier extension related to polynomials $F(u) = u^m$. We recall this as first procedure. All these codes have been written by Matlab language.

Now by this algorithm the Fourier extension in Equation (1), for real function $F(u)$, with attention to truncated Taylor extension from m -th order could be computed as follow:

$$F(u) = \sum_{k=0}^M c_k \left[\begin{array}{l} a_{0,k} + \sum_{n=1}^{\infty} a_{n,k} \cos\left(\frac{n\pi}{l}u\right) \\ + b_{n,k} \sin\left(\frac{n\pi}{l}u\right) \end{array} \right] \quad (17a)$$

$$F(u) = c_0 a_{0,0} + \sum_{k=1}^M c_k \left[\begin{array}{l} a_{0,k} + \sum_{n=1}^{\infty} a_{n,k} \cos\left(\frac{n\pi}{l}u\right) \\ + b_{n,k} \sin\left(\frac{n\pi}{l}u\right) \end{array} \right] \quad (17b)$$

$$F(u) = c_0 + \sum_{k=1}^M c_k \left[\begin{array}{l} a_{0,k} + \sum_{n=1}^{\infty} a_{n,k} \cos\left(\frac{n\pi}{l}u\right) \\ + b_{n,k} \sin\left(\frac{n\pi}{l}u\right) \end{array} \right] \quad (17c)$$

$$F(u) = c_0 + c_1 \left[\begin{array}{l} a_{0,1} + \sum_{n=1}^{\infty} a_{n,1} \cos\left(\frac{n\pi}{l}u\right) \\ + b_{n,1} \sin\left(\frac{n\pi}{l}u\right) \end{array} \right] \quad (17d)$$

$$+ \sum_{k=2}^M c_k \left[\begin{array}{l} a_{0,k} + \sum_{n=1}^{\infty} a_{n,k} \cos\left(\frac{n\pi}{l}u\right) \\ + b_{n,k} \sin\left(\frac{n\pi}{l}u\right) \end{array} \right]$$

$$F(u) = c_0 + c_1$$

$$\left[\begin{array}{l} \sum_{n=1}^{\infty} \frac{2l}{n\pi} (-1)^{n+1} \\ \sin\left(\frac{n\pi}{l}u\right) \end{array} \right] + \sum_{k=2}^M c_k \left[\begin{array}{l} a_{0,k} + \sum_{n=1}^{\infty} a_{n,k} \\ \cos\left(\frac{n\pi}{l}u\right) \\ + b_{n,k} \sin\left(\frac{n\pi}{l}u\right) \end{array} \right] \quad (17e)$$

In the following, we introduce other equations to have a more simplicity

$$g(u) = \sum_{n=1}^{\infty} \frac{2l}{n\pi} (-1)^{n+1} \sin\left(\frac{n\pi}{l}u\right) \quad (18a)$$

$$F_k(u) = a_{0,k} + \sum_{n=1}^{\infty} a_{n,k} \cos\left(\frac{n\pi}{l}u\right) \quad (18b)$$

$$+ b_{n,k} \sin\left(\frac{n\pi}{l}u\right)$$

In conclusion:

$$F(u) = c_0 + c_1 g(u) + \sum_{k=2}^M c_k F_k(u) \quad (19)$$

Although, we continue to test the algorithms, we will study a few examples that we have their exact answer, because of we want to study the applied problems. As we know, in applied problems, the functions are generally unknown and always, examine we examine such a problems by interpolation method.

Tip 1) since this analytical method, exactly without any error solves the integrals of Fourier coefficients, thus for a power series with few terms (finite series) is suitable. But, for a power series of $\ln(u)$, e^{-x^2} and ... which are infinite series, we want to calculate the coefficients using this method; there is an error of truncate in the series. Therefore, Fourier coefficients also will confront errors as a result.

Tip 2) According to the tip 1, in the following if we want to solve a function by the Fourier series, there is another error, due to calculating $\sin\left(\frac{n\pi}{l}u\right)$ and $\cos\left(\frac{n\pi}{l}u\right)$ in the series. But this is unavoidable in the physical issues. These problems will front of errors, but we will calculate the coefficients in a more accurate way as it follows. In the realistic situations, the problems never will be solved by an infinite series, but also it will be express experimentally by a finite power series.

First, we use (present) a simple problem.

First problem: Here, we want to check the value of the function $F(x) = x^4$ in the interval of $a < x < b$ by Fourier series and finally compare with its main answer.

Solution: We study the problem in two methods.

First method:

At first, we must to expand the function $F(x)$ around u_0 in a form that $-l < (x-x_0) < l$ will be satisfied.

$$-l = a - x_0 < x - x_0 = u < l = b - x_0 \quad (18)$$

Therefore:

$$x_0 = \frac{b+a}{2}, l = \frac{b-a}{2} \tag{19}$$

According to the Equation (19), the function $F(u)$ will be in the form:

$$F(u) = x^4 = \sum_{k=0}^M c_k (x-x_0)^k = \sum_{k=0}^M c_k u^k, -l < u < l \tag{20}$$

And

$$c_0 = x_0^4, c_1 = 4x_0^3, c_2 = 6x_0^2, c_3 = 4x_0, c_4 = 1, M = 4 \tag{21}$$

Here, the function $F(u)$ is that standard function which we studied it and find the algorithms. Thus, according to the second algorithms, we can study this function in a Fourier series from around the point x_0 . For showing the numerical calculations in a better way, we firstly give the algorithms, and then we write its code by MATLAB.

The absolute error of the graph is as follows:

The second method:

We increase the period interval in such a way that consists of specified or defined interval. We increase the period until it consist the contended interval and there is no need to expand it around a special point.

If x be in the $-l < x < l$ interval, then: $-l \leq a < x < b \leq l$.

$$F(x) = x^4, -l \leq a < x < b \leq l \tag{22}$$

If $|a| < |b|$, then $l = b$ in here. Otherwise $l = a$. To show the numerical calculations in a better way, we firstly give the algorithms, and then we write its code by MATLAB. At this stage, we investigate the algorithm for $l = b = 2, a = 1$, and $M = 4$.

For different N value (N is number of Furier series summation), the absolute errors are presented below (figure 2)

By comparing the two methods, we can find that the MATLAB software is sensitive to the loops number and by increasing this number will result in the accuracy, whereas it must not be. Thus, the second method is more effective. Figures 2, shows increase in accuracy by increasing N value.

In the next step, we study a nonlinear function and we change it to a linear problem using Taylor expansion and then we compare it with Simpson method. The Fourier coefficients has been computed with Simpson method first and then the out-put has been compared with the current method. It is so important to notice that these two methods has been compared in the same process numbers first and then the numbers of processes to reach the same accuracy.

In the previous method, there were two “For” rings inside each other, so the number of process repeating is $N(M-1)$ that is $M = 4$ for this problem. If we want to solve this problem by Simpson method with the same process numbers, we should choose the integral pitch as $(h = \frac{2l}{M} = 1, l = \frac{b-a}{2})$ wherein h is the integral pitch and M is the number of pitches. Since the value of M is low, we expect to have a high pitch and so the accuracy of our calculations will be decreased.

Now we investigate the integration pitch in Simpson method for $h = 0.01, 0.001$, and 0.0001 . The number of processes in two methods are $(K-M)N$ with attention to their algorithms. For the $h = 0.01$ and $N = 1000$, we have the graph below ($K = \frac{2l}{h} = 400, (K-M)N = 396000$).

The graph indicates that 39996000 processes are needed in Simpson method to gain the same accuracy of our method. So we can say that the represented method in this paper is more accurate than Simpson method.

Problem 2: In this section, we study the function $F(x) = \ln(2+x)$ using Fourier expansion between $-0.8 < x < 0.8$.

Solution:

Taylor series of this function will be in the form:

$$F(x) = \ln(2+x) = \ln(2) + \sum_{k=1}^M \frac{(-1)^{k+1}}{k \times 2^k} x^k, -0.8 < x < 0.8 \tag{23}$$

Using the Equation (23), the function to order of M and more M value will result in more calculations accuracy of the Taylor series.

$$\text{Error} = \frac{u_{max}^M}{M \times 2^M} = \frac{0.8^M}{M \times 2^M} \tag{24}$$

If we suppose to expand this series up to the first twenty terms, then the Error is:

$$\text{Error} = \frac{0.8^{20}}{20 \times 2^{20}} = \frac{(5.7646)}{2^{20}} \times 10^{-4} \ll 1 \tag{25}$$

We suppose that we have a 20 sentences series that

its answer is logarithmic. In the other hand, we do not need to solve the logarithmic sentence and a presentation of a logarithmic series is enough. Now, we introduce the algorithms of this problem and show its accuracy by a graph.

Which, its absolute error is presented below(Figures 4):

As the method is sensitive to the kind of program used and also to the recuration loop, we can improve it with some changes.

The point in here is that the logarithmic function has been put in MATLAB software by default and the value of those functions has been already determined. The main goal of this stage in our research is to compare the accuracy of two methods, therefore we suppose that we have the first 21 sentences and want to analyze them. Since we know the exact value of them, we can do it now.

Since the error in integration in the Simpson method is very high, we only plot one case of those values. This technique is not suitable for the functions with multi series sentences. For the $h = 0.0001$ and $N = 3000$, we have the graph below ($K = \frac{2l}{h} = 4000, (K - M)N = 3996 \times 3001$).

So the represented method is more accurate for multi series and is suggested for evaluation of analytical problems. Therefore we can use the proposed method for solving the physical problems with reasonable results. As the method is sensitive to the kind of program used and also to the recuration loop, we can improve it with some changes.

Thus, according to the advantages we apply the proposed method to the applied problems in electromagnetic.

3- THE INTERNAL POTENTIAL OF A LONG HOLLOW CYLINDER

We study the internal potential of a long hollow cylinder with radius R . By changing the boundary conditions, the

algorithms will change. In this paper, one of the possible kind of the boundary conditions will be studied:

$$v_{(R,\varphi)} = \sum_{k=0}^M c_k \varphi^k \quad -\pi < \varphi < \pi \tag{26}$$

For calculating the potential inside a long hollow cylinder, we must solve the Laplace equation of $\nabla^2 \ddot{\Phi}(\tilde{r}, \ddot{\phi}) = 0$.

It will results the potential for inside the cylinder in the form:

$$\Phi(\rho, \varphi) = a_0 + \sum_{n=1}^N \rho^n (a_n \cos(n\varphi) + b_n \sin(n\varphi)) \tag{27}$$

According to the potential equation and boundary conditions we have:

$$\Phi(R, \varphi) = v_{(R,\varphi)} = a_0 + \sum_{n=1}^N R^n (a_n \cos(n\varphi) + b_n \sin(n\varphi)) \tag{28}$$

Using Equation (28), we find that it is a form of Fourier series and by the proposed method presented here, we try to solve it to define series coefficients

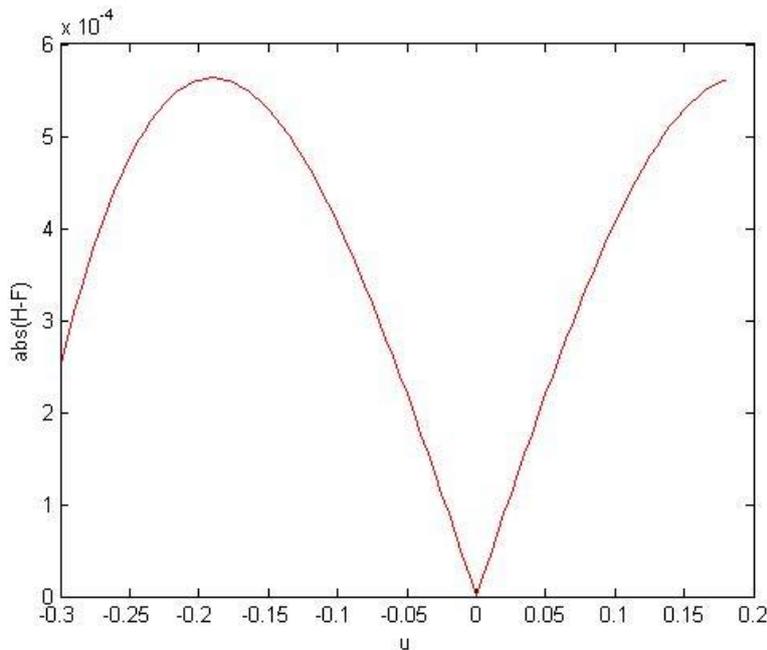


Fig. 1. Where u is function variable, h is the actual value of the function and F is the Fourier value of the function. $H-F$ is the difference between the actual value and the value of the Fourier function series. The limit of Fourier series summation is for $N=201$.

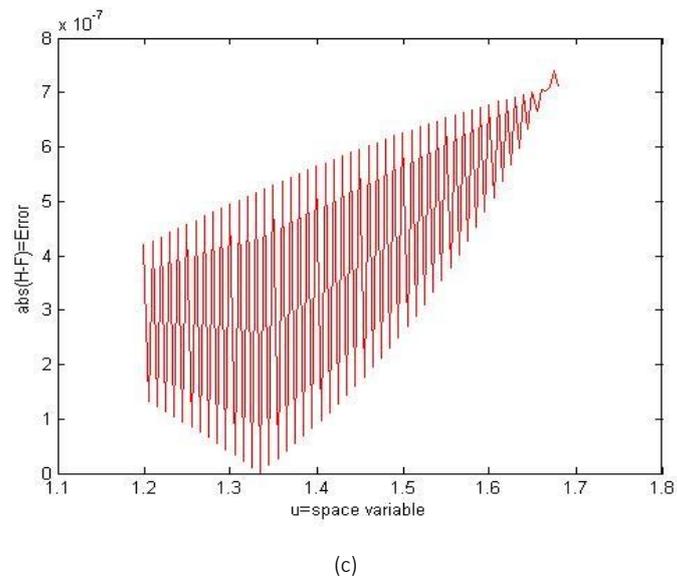
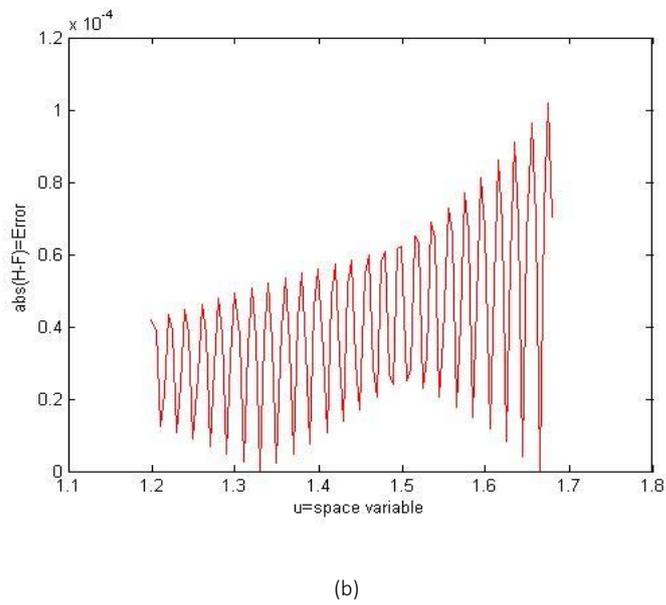
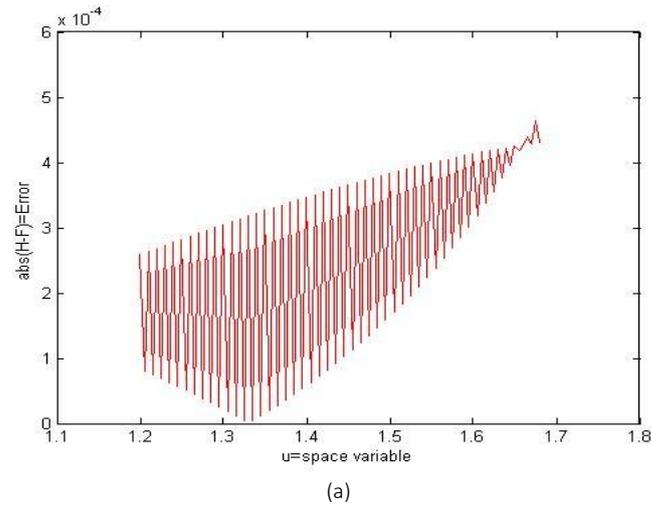
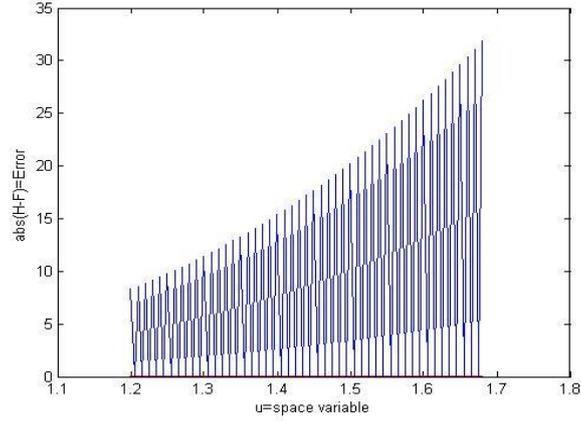
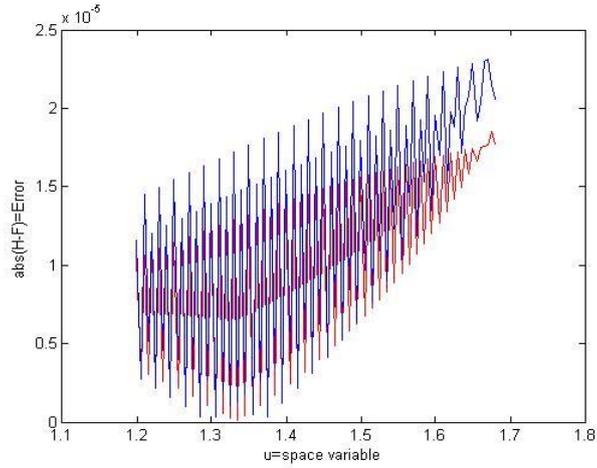


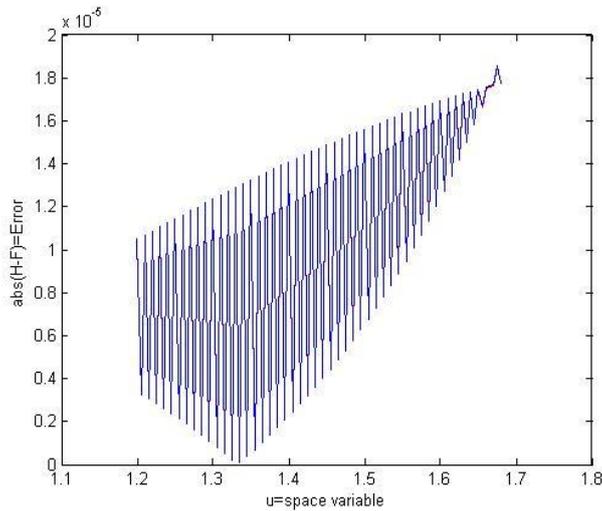
Fig. 2. (a) The graph for $N=201$, (b) The graph for $N=501$ and (c) The graph for $N=5501$. Where N is number of Furier series summation.



(a)



(b)



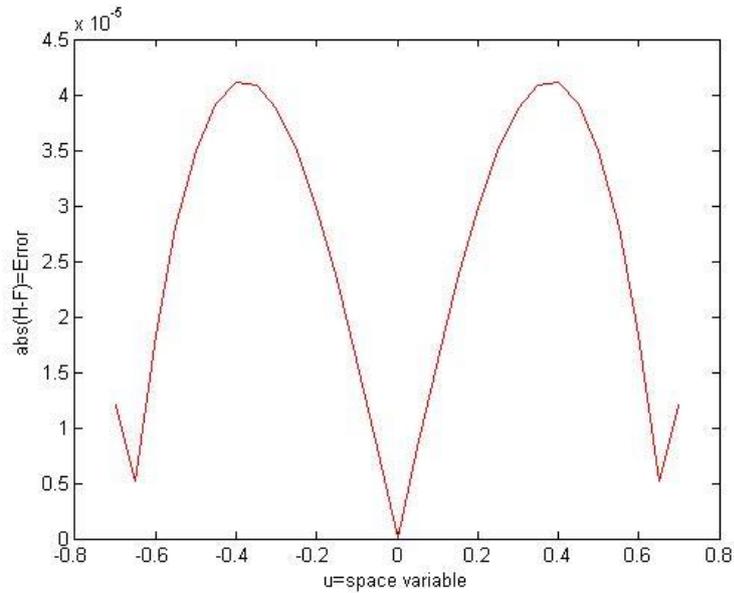
(c)

Fig. 3. (a) The graph for and the red graph is error of our method and the blue is Simpson error. For the and , we have the graph

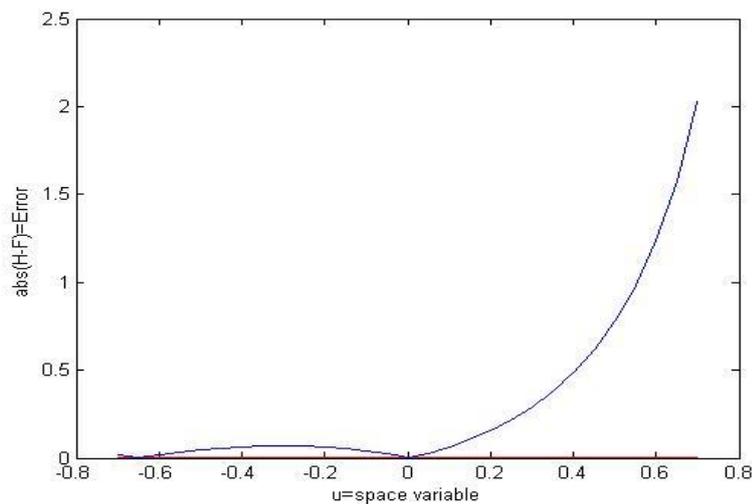
in this Figure ($K = \frac{2l}{h} = 4000, (K - M)N = 3996000$), (b) The graph for and the red graph is error of our method and the blue is Simpson

error. For the $h=0.0001$ and , $N=1000$ we have the graph in this Figure ($K = \frac{2l}{h} = 40000, (K - M)N = 39996000$) and (c) The graph for

$N=1000$ and the red graph is error of our method and the blue is Simpson error.



(a)



(b)

Fig. 4. (a) The graph for $N=5001$ and $M=20$. These graphs show a good approximation. Therefore we can use the proposed method for solving the physical problems with reasonable results.

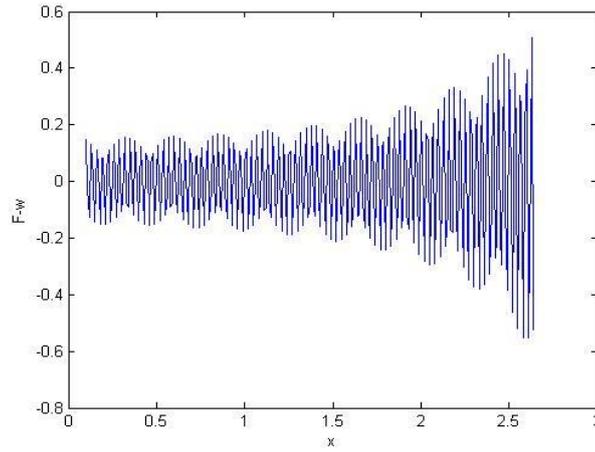
(b) The graph for $N=3001$ and $M=20$ and $h=0.0001$. We want to compare this method with the Simpson method; therefore we plot the comparison graphs with the (b).

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} v_{(R,\varphi)} d\varphi = \sum_{k=0}^M \frac{c_k (\pi)^k (1 + (-1)^k)}{K+1 \cdot 2} = \sum_{k=0}^M a_{0,k} \tag{29a}$$

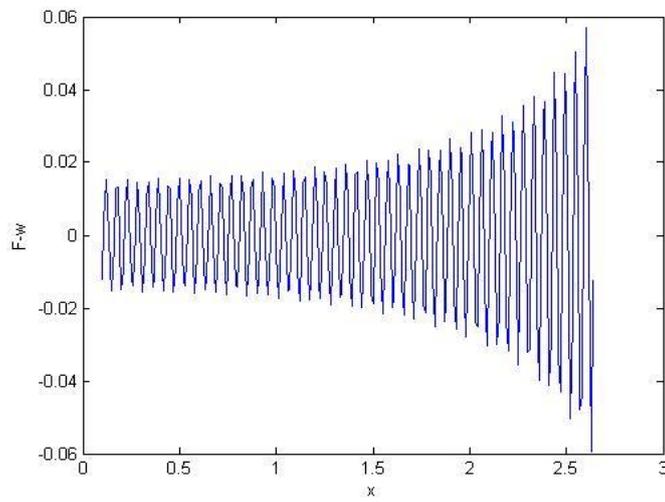
$$a_n = \frac{1}{\pi R^n} \int_{-\pi}^{\pi} v_{(R,\varphi)} \cos(n\varphi) d\varphi, \quad b_n = \frac{1}{\pi R^n} \int_{-\pi}^{\pi} v_{(R,\varphi)} \sin(n\varphi) d\varphi \tag{29b}$$

According to the Equation (26) we have:

$$a_n = \frac{1}{\pi R^n} \sum_{k=0}^M \left[c_k \int_{-\pi}^{\pi} \varphi^k \cos(n\varphi) d\varphi \right] = \frac{1}{\pi R^n} \sum_{k=0}^M c_k y_{n,k}, \quad n = 1, 2, \dots, N \tag{30a}$$



(a)



(b)

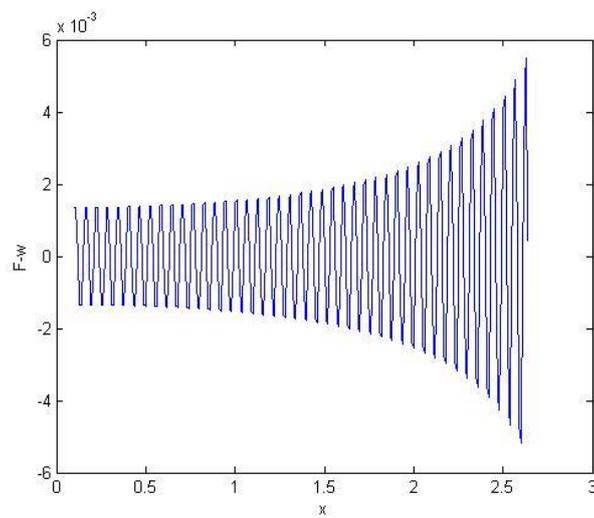
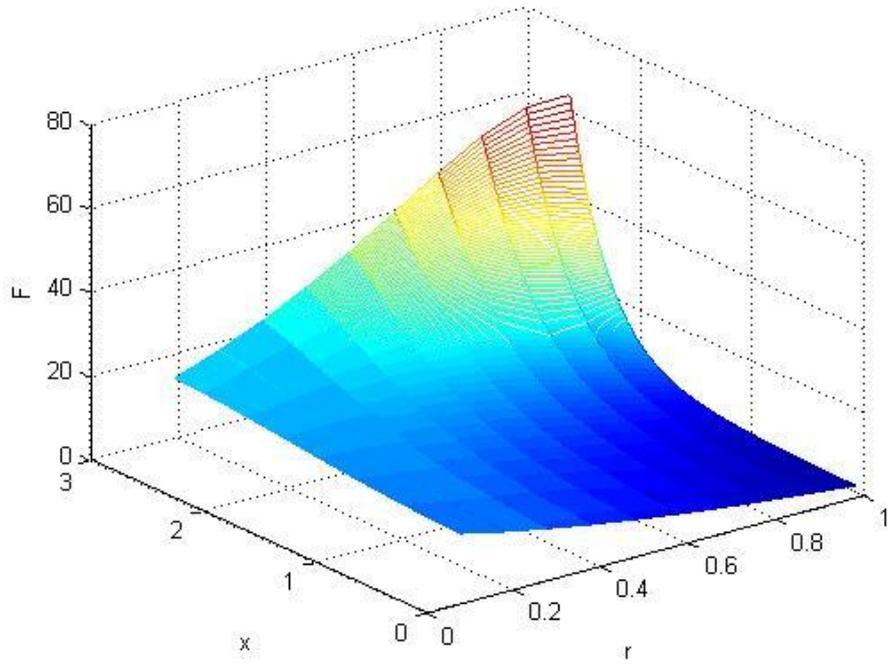
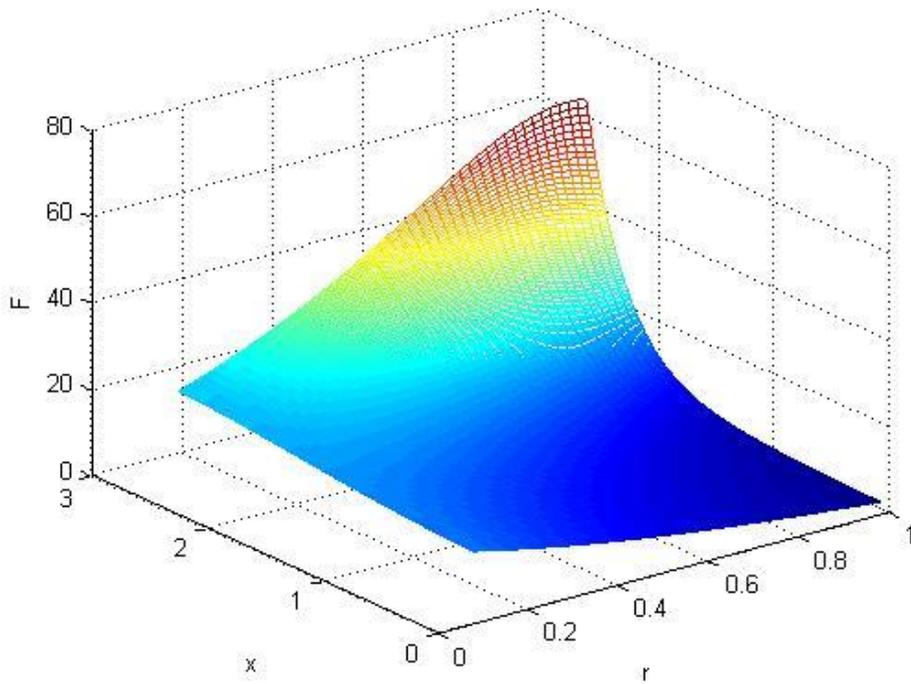


Fig. 5. Here θ is angle in cylindrical coordinate, ϕ is the potential value at the boundary and ϕ_{FS} is resulted Fourier value of potential at the boundary. e_1 is the difference between the actual value and the value of the potential function Fourier series at the boundary. (a) Absolute error for ϕ . (b) Absolute error for ϕ_{FS} . (c) Absolute error for e_1 .



(a)



(b)

Fig. 6. Where θ is the angle in cylindrical coordinate, r is radial variable and F is potential everywhere inside the cylinder. (a) Potential chart for $N=20001$ with 0.1 radial step. (b) Potential chart for $N=20001$ with 0.02 radial step.

$$b_n = \frac{1}{\pi R^n} \sum_{k=0}^M \left[c_k \int_{-\pi}^{\pi} \varphi^k \sin(n\varphi) d\varphi \right] = \tag{30b}$$

$$\frac{1}{\pi R^n} \sum_{k=0}^M c_k z_{n,k}, n=1,2,\dots,N$$

$$y_{n,k} = -\frac{k}{n} z_{n,k-1}, y_{n,0} = 0, y_{n,1} = 0 \tag{30c}$$

$$z_{n,k} = -\frac{(-1)^n (1 - (-1)^k)}{n} \pi^k + \frac{k}{n} y_{n,k-1}, z_{n,0} = 0, z_{n,1} = -\frac{2(-1)^n}{n} \pi \tag{30d}$$

As a result for the potential inside the cylinder we have:

$$\Phi(\rho, \varphi) = a_0 + \frac{1}{\pi} \sum_{k=0}^M \sum_{n=1}^N \frac{\rho^n}{R^n} (c_k y_{n,k} \cos(n\varphi) + c_k z_{n,k} \sin(n\varphi)) \tag{31}$$

We study the solution of this problem in the same way of the previous problem and we present it in a more simple way:

$$\Phi(\rho, \varphi) = a_0 - 2c_1 \sum_{n=1}^N \frac{\rho^n (-1)^n}{R^n} \sin(n\varphi) + \frac{1}{\pi} \sum_{k=2}^M \sum_{n=1}^N \frac{\rho^n}{R^n} (c_k y_{n,k} \cos(n\varphi) + c_k z_{n,k} \sin(n\varphi)) \tag{32}$$

Using the Equation (32), we express the algorithms for calculating the potential value.

Now, we study the algorithms for the equations:

$$c_k = \frac{1}{2^k}, M = 10, R = 1, \quad 0 < \varphi < 2\pi \tag{33}$$

This program will give the potential in every point and is related to two parameters of angle and also to the internal radius.

If we want more information about the accuracy of the program, we must compare its value according to the boundary conditions and the value which is presented by the algorithms in the boundary for different N values.

Also, calling the Equation (32) in programing at the boundary $R = r$ is figures 5:

As we can see, by increasing the number of power series terms, the accuracy in the boundary has been increased. Here, the accuracy is the point which is very important, because of using this technique, the series coefficient has been solved analytically and the errors are the result of computer calculations and not an integral error of the method.

If we want to call the potential for few points in the cylinder, the chart will be in the form below:

With a 0.1 radial step, we have the graph which is presented below (figures 6)

As seen in the figure 6, in the radius near to zero, the change in the angle would not change the potential but it would increase the relation of them.

According to the chart of Figure 6, one can conclude the potential variations in the cylinder and also the effect of boundary conditions on the internal potential behavior. In another paper, we will study different boundary conditions to have a more insight on the applications of the proposed method. This method can be used and will be extended to the electron and ion capture in the cylindrical situations.

4- CONCLUSIONS

This method is very convenient from the point of accuracy and time efficiency for the calculation of Fourier's coefficients and the dependent problems. This method has capability to solve the problems with limit sentences analytically, so its accuracy is very high. As seen in the figure 6, in the radius near to zero, the change in the angle would not change the potential but it would increase the relation of them..

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