AUT Journal of Modeling and Simulation

AUT J. Model. Simul., 51(2) (2019) 179-190
DOI: 10.22060/miscj.2019.15977.5149

A Hybrid Solution Method for Hub-and-Spoke Network Design under Uncertainty A Case Study to Design Optical Fiber Network in Iran

Seyed Babak Ebrahimi[1], Abdollah Sharifi[1], Maryam Nezhad-Afrasiabi[2,*], Seyed Mohammad Mirkhani[1]

[1] Department of Industrial Engineering, K.N. Toosi University of Technology, Tehran, Iran.
[2] Department of Industrial Engineering and Management Systems, Amirkabir University of Technology, Tehran, Iran.

ABSTRACT: Supply chain is an integrated system of facilities and activities. Gaining the optimum design of demand satisfaction network is one of the most important live issues in the decision making problems category. Most of previous studies considered unreal assumptions such as the lack of capacity constraints to satisfy demand in the network and in hubs. By considering the nature of the case that have been studied in this research, the assumption of unlimited capacity to satisfy the demand is justified. Another common assumption in hub location problems is the lack of direct connection between the nodes. In this research and in real world problems would be seen that the direct link between the nodes can be effective in reducing system costs and increase the efficiency of the network. The other innovations of current research is considering uncertain nature of the demand data, oscillation and changes in costs anticipation and actual hub establishing costs, fuzzy numbers are used to represent these values. Problem modeling is held in a fuzzy state and a hybrid method is represented to solve the problem. At first, defuzzification of the model is taken place. Afterwards, all possible answers are considered with the help of the Genetic Algorithm. At last, the optimum case were chosen by using VIKOR ranking method. Calculation results for the designing of optical fiber network between cities are showing good and acceptable performance of the proposed method in an acceptable solving time.

Review History:

Received: 2019-03-11
Revised: 2019-07-09
Accepted: 2019-07-30
Available Online: 2019-12-01

Keywords:

network design
hub Location
uncertain demand & setup cost
Fuzzy
VIKOR

1. Introduction

The hub-and-spoke system has been widely employed in various industrial applications, such as transportation and telecommunications system designs [1]. During the past decade the hub-and-spoke Network Design Problem has received the attention of many researchers. The aim of such problems is usually node network design, where numbers of node/spoke is selected to play the role of hubs. Hubs have the role of collecting and distributing the demand and because of the particular circumstances and the high volume of requests that are transmitted through them, have certain advantages. For example, these points are connected to all other hubs and through them they can satisfy their demands. However, due to the high volume of demand in these hubs usually satisfying the demand's costs (the connection of node/spoke to hub) will be less than satisfying the demand directly (connect two nodes together). Hub-and-spoke network design problem involves two interrelated issues: Firstly, hub location and network design and secondly, determining possible connections between hub and nodes and hub to hub. The hub location problem has various applications in the areas of transportation such as: air passenger [2] and cargo [3-6], less-than-truckload freight [7, 8], rail freight [9], urban public transportation and rapid transit [10]. Other applications areas include postal delivery [11, 12], express package and cargo delivery [13-15], telecommunications [16], and supply chains [17].

*Corresponding author's email: Mafrasiabi@aut.ac.ir

In this study the hub-and-spoke network design problem is examined, which includes two decisions of hub location and network design. This issue has been studied by an example in optic fiber communication networks. Hierarchical configuration of such networks with hubs is proven as a facilitator of communications, enhancing flexibility and costs effectiveness. Such networks have two categories: the connection of hubs with each other, which is usually possible at a higher speed, lower cost and higher efficiency and communication between nodes and hubs, that in fact, satisfies the demands. Also, the direct connection between two nodal points is possible [18]. However, due to the high capacity data transmission over optical fiber, the issue of satisfying demand is considered in an incapacitated mode. Multiple allocation is considered, means that any node can be connected to multiple hubs, and nodes, so the problem is categorized as r-Allocation P-Hub Median Problem (rAPHMP) type. Another assumption is that the hub network is a complete graph and all hubs are connected directly to each other. Yet these connections, due to higher bandwidth at hub, have lower cost than direct connection of two nodes. Also, due to special geographical, environmental and economic conditions, demand and setup costs are considered uncertain in the problem.

2. Literature review

Hub-and-spoke systems have been the subject of many studies in the past three decades [19]. O'Kelly was one of the first who investigate the positioning of hub location, and after him many researchers paid attention to investigate and development in relation to this issue [20, 21]. They have been examined different characteristics with different aims. The issue of selecting P node to convert to hub is one of the main issues in the field. In such problems, the aim is to select P node as hub and the demand is known between all nodes. Also demands should be satisfied by the minimum cost. One of the common assumptions is the completeness of the hub's network graph and demands transfer costs between hubs have the discount factor $(0 \leq \alpha \leq 1)$. Recently, Yaman and Elloumi [22] introduced a new class of P-Hub problems, which are known as the selecting r nodes from P candidate $(r \leq P)$ for converting to hubs, where a more flexible allocation from ties between nodes and hubs have been considered. In his model, if $r = P$, then the problem become the P-Hub issue, and if $r = 1$, substituted the problem can become a single allocation issue. In fact, he offered a more comprehensive model than previous models in the rAPHMP model.

One of the common assumptions in positioning of hub location is the lack of capacity constraints to satisfy demand in the network and in hubs [18, 23-25]. By considering the nature of the case that have been studied in this research, the assumption of unlimited capacity to satisfy the demand is justified. Another common assumption in hub location problems is the lack of direct connection between the nodes. This means that all paths and communication should go through hubs and two nodes can not communicate directly and independently. On the other hand, in this research and in real world problems can be seen that the direct link between the nodes can be effective in reducing system costs and increase the efficiency of the network [26, 27]. For example Aykin [26]considered that hub-and-spoke network design problem under limited hubs capacity and the direct communication between non-stop, one-stop hub and two-stop hub that is known as nonrestrictive hub policy.

Nickel, Schöbel, and Sonneborn [10] examined the positioning of hub location problem in urban traffic network, which is direct communication between nodes, and only is possible for the first and last nodes of each path. They have been investigated their research under the conditions of unlimited capacity, existence of fixed cost for establishing hubs, and cost proportional to the Euclidean distance. In their research, the significant point was the use of discount factor, and their purpose was to select the hubs location and then allocate nodes to the hubs. Yoon and Current [18] has been investigated the positioning of hub location in telecommunication networks, and in their model the relationship between nodes and hubs was allowed. Their problem was considered as unlimited capacity, certain demand, multi-commodity and it lacks of any regional restrictions on the network. Their goal was to select locations by considering fixed costs of establishing hubs, the existence of arc in the network and the variable costs associated with demands on the arcs.

Yaman [25] has been investigated the different allocation strategies and their effect on the total cost of the routing in hub networks. Also he considered the service quality considerations, flow threshold and non-stop service, as well as capacity was unlimited in his model. Setak, Karimi, and Rostami [28] proposed a comprehensive model for hub location-routing problem. In their model locating and routing is considered simultaneously and it has a multiple allocation strategy to allocate non-hub nodes to hub nodes. The objectives of their proposed model are minimizing costs of establishing a network and transferring flows. Lee and Moon [29] developed two mathematical models with realistic restrictions of Korea Post organization for the current postal logistics network by considering locations and allocations simultaneously. They proposed an Integer Linear Programming model for transportation network organization, vehicle operation and a Mixed Integer Linear Programming model that considers potential ECs for decision making while simultaneously regarding the EC location, transportation network organization, and vehicle operations. Zheng, Meng, and Sun [30] proposed a liner hub-and-spoke shipping network design problem by introducing the concept of the main port. They developed a mixed-integer programming model with nonconvex multi-linear terms for their proposed problem.

Another assumption of this research is the existence of uncertainty in the hub setup costs and uncertainty of transmitted demands between nodes. Hub setup costs are estimated before network design, but the real costs of setup may be different according to the economic and environmental conditions than earlier estimates. It is the same for demand. Some of it can be estimated, but due to the inaccuracy of the data and the time-dependency of information veracity, these data are also imprecise and uncertain. For example, research conducted in this area include: Marianov and Serra [4] that they developed an $M/D/C$ queuing model for the number of airplanes waiting to land at a hub airport. The limited capacity of the hub's airport is the reason for using this system that analyzes the queue created by the planes. They used the probability of the numbers of the customers in the system design of their model. They considered that the probability that more than b airplanes were in queue is less than or equal to a specified value and due to the complexity of the model, solve their model by using a heuristic algorithm based on tabu search algorithm.

Rahimi, Tavakkoli-Moghaddam, Mohammadi, and Sadeghi [31] presented a new bi-objective model for a multi-modal hub location problem under uncertainty and by considering congestion in the hubs. Their objective functions are an attempt to minimize the total transportation cost as well as minimizing the maximum transportation time between each pair of origin-destination nodes in the network. Yang [32] looks at stochastic air freight, incapacitated multiple allocation hub location problems where seasonal variations on demand, as well as seasonal variations on the discount factors for hub to hub Fights, are accounted for. Their model is separated into two stages: The first stage determines the number and location of hubs, regardless the impact of events and in the second stage, these coincidence and events were effective and their model has been solved considering the uncertainty of demand.

Sim, Lowe, and Thomas [33]considered the transportation centers location problem to satisfy demand at a given time. They focused on determination of a central hub between existed hubs and nodes. They modeled their problem based on the probability of demand flow from desired path and through hub. One of their main limitations was the constant level of servicing in the entire network. They also took the travel time as a random variable into account, and solved their problem with an innovative technique.

Contrevas, Cordeau, and Laporte [34] used the theory of fuzzy sets to solve the hub location problem with incapacitated capacity. In their model, dependent demand and transportation costs has been indecisive. They showed that, in the possible problems, the dependent demand and transportation costs consider can be equal with their expected values and in the stochastic problems, values can be replaced by a mathematical estimation. They study a stochastic incapacitated multiple allocation hub location problem and look at three different cases. First, they considered the demand as a stochastic variable between source and destination. Second, the uncertainty related to transportation costs was dependent and in the third case, the independent transportation costs is also considered as a stochastic variable. They used Monte Carlo simulation, and a sample average approximation method coupled with a Benders' decomposition algorithm, as a solution approach for solving the problem. Mohammadi, Tavakkoli-Moghaddam, Siadat, and Dantan [35] proposed a new hybrid meta-heuristic algorithm based on genetic and imperialist competitive algorithms. They concluded that a considerable improvement in reliability of the network can be achieved with only a little increase in the total cost.

This research is modeled based on the assumption of demand and hub's setup cost uncertainty. In addition, the possibility of nodes direct communication with each other is also considered, as well as the decision of hub network design has been considered in the modeling. The rest of this study is organized as follows: Problem modeling and its proposed solving method is presented in the section 3 and 4 example of inter-city fiber optic network is presented in section 5, and conclusions and recommendations for future research are given in section 6.

3. Mathematical models

In this research, stochastic paths from source to destination, as well as the construction of hub at desired location are used as decision variables in the formulation of the main routes. The main purpose of the model is to minimize the total cost to meet the demand, the distance between source and destination, and setup costs of the construction of hub in the desired location. In actual operations and actions, three hubs are used rarely and in most case the economic justification of the three-hub will be difficult. So in this research two hubs are considered. In practice, hub setup is a long term strategic plan and requires huge investment and its problems cannot be resolved in short term. So finding the perfect place to build the hub is very important and have a huge impact on the network. Uncertainty in demand and setup costs are assumed as triangular fuzzy numbers. In this research, in addition to the setup costs, the cost of satisfying demand per unit and costs of distance from the origin i to destination j are considered. In the case of using hub to satisfy demands, the cost of demand satisfying and distance costs are reduced based on their discount factors. One of the assumptions of this research is the existence of p-hub in model that the problem is considered as P-Hub Median, also the problem is of r-allocation type, this means that each node, at most, can use r hub to meet its demands. In this study, the X_k values determines the hub location, and Y values determines decisions about network design, routing, and allocation of appropriate flow to satisfy the demand in the network. The rest of the used variables and parameters are as follows:

N:	Nodes set
P:	Number of hubs ($P \in \mathbb{N}$)
r:	Maximum number of hubs ($r \le P$)
$\tilde{D}_{ij}$:	Demand from the origin i to destination j in state of a fuzzy number: $\tilde{D}_{ij} = \left(D_{1ij}, D_{2ij}, D_{3ij}\right)$
$\tilde{f}_k$:	Initial setup cost of kth hub in state of a fuzzy number: $\tilde{f}_k = \left(f_{1k}, f_{2k}, f_{3k}\right)$
C_{ij}:	Cost of satisfying each unit of demand from the origin i to destination j
C_{ikSj}:	Cost of satisfying each unit of demand from the origin i to destination j by S and k hub
h_{ij}:	Cost of satisfying each unit of direct distance from the origin i to destination j, without using hubs
h_{ikSj}:	Cost of satisfying each unit of distance from the origin i to destination j, by using hubs
L_{ij}:	The direct distance between node i and j
L_{ikSj}:	The distance between node i to j by using K and S hubs
M:	A positive large number which is considered as the maximum demand between nodes directly (without using hubs)

$$M = \max\left\{\sum_{j \in \mathbb{N}} \left(D_{3ij} + D_{3ji}\right)\right\} \qquad \forall i \in N \tag{1}$$

α_{kS}:	Discount factor by using K and S hubs
β_{kk}:	Discount factor by using only k as hub $(0 \le \alpha \le \beta \le 1)$ According to the above two discount factors the costs of satisfying the demand and distance costs are calculated as follows:

$$C_{ikSj} = \alpha_{kS} C_{ij} \qquad C_{ikkj} = \beta_{kk} C_{ij} \tag{2}$$

$$h_{ikSj} = \alpha_{kS} h_{ij} \qquad h_{ikkj} = \beta_{ik} h_{ij} \tag{3}$$

X_k :	Binary random variable for choosing hub in node k.
	$X_k = \begin{cases} 1 & \textit{if k has been chosen as a hub} \\ 0 & o.w \end{cases}$
Y_{ij} :	Binary random variable for demand flow between i and j $Y_{ij} = \begin{cases} 1 & \textit{if demand flow between i and j is not zero} \\ 0 & o.w \end{cases}$
Y_{iksj} :	Binary random variable for demand flow between i and j by using K and S hubs $Y_{iksj} = \begin{cases} 1 & \textit{if demand flow between i and j goes through K and S hubs} \\ 0 & o.w \end{cases}$

Thus problem model is as:

$$(\mathrm{P}) \quad \mathrm{Min}\ Z = \sum_{k\in\mathbb{N}} \tilde{f}_k X_k + \sum_{i\in\mathbb{N}}\sum_{j\in\mathbb{N}} C_{ij}\tilde{D}_{ij}Y_{ij} + \sum_{i\in\mathbb{N}}\sum_{j\in\mathbb{N}}\sum_{k\in\mathbb{N}}\sum_{S\in\mathbb{N}} C_{iksj}\tilde{D}_{iksj}Y_{iksj} + \sum_{i\in\mathbb{N}}\sum_{j\in\mathbb{N}} h_{ij}L_{ij}Y_{ij} + \sum_{i\in\mathbb{N}}\sum_{j\in\mathbb{N}}\sum_{k\in\mathbb{N}}\sum_{S\in\mathbb{N}} h_{iksj}L_{iksj}Y_{iksj} \tag{4}$$

S.t:

$$\sum_{k\in\mathbb{N}} X_k = P \tag{5}$$

$$\sum_{k\in\mathbb{N}} Y_{ik} \le r \qquad \forall_{i\in\mathbb{N}} \tag{6}$$

$$Y_{iksj} \le X_k \tag{7}$$

$$Y_{iksj} \le X_S \qquad \forall_{i,k,s,j\in\mathbb{N}} \tag{8}$$

$$Y_{iksj} \le Y_{ik} \qquad \forall_{i,k,s,j\in\mathbb{N}} \tag{9}$$

$$Y_{iksj} \le Y_{sj} \qquad \forall_{i,k,s,j\in\mathbb{N}} \tag{10}$$

$$Y_{iksj} \ge X_k + X_S - 1 \qquad \forall_{k,s\in\mathbb{N}} : k \ne S \tag{11}$$

$$Y_{iksj} \ge Y_{ik} + Y_{Sj} - 1 \qquad \forall_{k,s\in\mathbb{N}} : k \ne S \tag{12}$$

$$Y_{ij} + \sum_{i\in\mathbb{N}}\sum_{j\in\mathbb{N}} Y_{iksj} = 1 \qquad \forall_{i,j\in\mathbb{N}} : i \ne j \tag{13}$$

$$-\sum_{i\in\mathbb{N}}\sum_{j\in\mathbb{N}} Y_{ikkj} \le MX_k \qquad \forall_{k\in\mathbb{N}} : i \ne j \tag{14}$$

$$Y_{ij} \in \{0,1\} \qquad \forall_{i,j\in\mathbb{N}} : i \ne j \tag{15}$$

$$Y_{ikSj} \in \{0,1\} \qquad \forall_{i,k,S,j\in\mathbb{N}} : i \ne j \tag{16}$$

$$X_k \in \{0,1\} \qquad \forall_{k\in\mathbb{N}} \tag{17}$$

$$X_S \in \{0,1\} \qquad \forall_{S\in\mathbb{N}} \tag{18}$$

The objective function of the model (P) is minimizing the costs of setup, satisfying the demand from origin i to destination j directly or by using hubs, and distance costs from origin i to destination j either directly or by using hubs respectively. Costs of C_{ikSj} are calculated by using the formula (2) and placed in the objective function formula. The problem is designed in such a way that it should have P hub, so constraint (5) is existed and because it is a r-allocation problem, so constraint (6) was considered. Constraints (7) and (8) also guaranteed that as long as kth and Sth were not available any demand stream will not flow from it. Constraints (9) are also guaranteed that until the demand stream does not established between i and k, i-k-S-j path will not be formed. Constraint (11) ensures that flow in i-k-S-j path can be activated when hubs be present in both k and S places. Constraint (12) ensures that flow in i-k-S-j path can be activated when flows i and j, k and S be established together. Constraint (13) also ensures demand be satisfied between all i and j paths. Since the second part of the left side of constraint (14) was repeated twice in the first part, so once we subtract from it and this value when will be zero that all Y be zero. Constraints (15) to (18) also guaranteed that decision variables X are zero and one [36].

Proposed Solution method

The combined approach of Genetic Algorithm and VIKOR were used to solve the (P) model. In such a manner, first by using a defuzzing method, the fuzzy model has become to three objective scheduling problems. Then for each of these functions, the model objective has been solved by using Genetic Algorithm and the constant values obtained from it. Then each of these functions considered as a criteria in multi-criteria decision and finally, by using VIKOR ranking method, the options are ranked and the best solution is determined.

To solve the problem model according to its type, which is a fuzzy model, first a de-fuzzy method should be applied. To defuzzification, the fuzzy numbers obtained by evaluation should convert to absolute numbers, this make the decision making process easier and understandable work for managers and executive team. Taleizadeh, Niaki, and Aryanezhad [37] method is used for model defuzzification. Based on this method, if a fuzzy programming model be as follows:

$$(\mathrm{P}') \quad \mathrm{Max}\ Z = \sum_{i=1}^{n} \tilde{C}_i X_i$$

$$\mathrm{S.t:}$$

$$\sum_{i=1}^{n} \tilde{a}_{ij} X_i \le b_j \qquad j = 1, 2, \ldots, m$$

$$X_i \ge 0$$

That two triangular fuzzy numbers in the model are defined as follows:

$$\tilde{C}_i = (C_{i1}, C_{i2}, C_{i3})$$

$$\tilde{a}_{ij} = (a_{ij1}, a_{ij2}, a_{ij3})$$

So, (P') is equal to:

$$(P'')\ \text{Min } Z_1 = \sum_{i=1}^{n}(C_{i2} - C_{i1})X_i$$

$$\text{Max } Z_2 = \sum_{i=1}^{n} C_{i2} X_i$$

$$\text{Max } Z_3 = \sum_{i=1}^{n}(C_{i3} - C_{i2})X_i$$

$$\text{S.t}:$$

$$\sum_{i=1}^{n} a_{ij1} X_i \le b_j \qquad j = 1, 2, \ldots, m$$

$$\sum_{i=1}^{n} a_{ij2} X_i \le b_j \qquad j = 1, 2, \ldots, m$$

$$\sum_{i=1}^{n} a_{ij3} X_i \le b_j \qquad j = 1, 2, \ldots, m$$

$$X_i \ge 0$$

Now, the (P) model defuzzification is done using the method proposed by Taleizadeh, Niaki, and Aryanezhad [37] that its result is given below:

$$(P''')\ \text{Min} - Z_1 = \sum_{k \in \mathbb{N}}(f_{1k} - f_{2k})X_k + \sum_{i \in \mathbb{N}}\sum_{j \in \mathbb{N}} C_{ij}(D_{1ij} - D_{2ij})Y_{ij} + \sum_{i \in \mathbb{N}}\sum_{j \in \mathbb{N}}\sum_{k \in \mathbb{N}}\sum_{S \in \mathbb{N}} C_{iksj}(D_{1ij} - D_{2ij})Y_{iksj} - \sum_{i \in \mathbb{N}}\sum_{j \in \mathbb{N}} h_{ij} L_{ij} Y_{ij} - \sum_{i \in \mathbb{N}}\sum_{j \in \mathbb{N}}\sum_{k \in \mathbb{N}}\sum_{S \in \mathbb{N}} h_{iksj} L_{iksj} Y_{iksj}$$

$$\text{Min } Z_2 = \sum_{k \in \mathbb{N}} f_{2k} X_k + \sum_{i \in \mathbb{N}}\sum_{j \in \mathbb{N}} C_{ij} D_{2ij} Y_{ij} + \sum_{i \in \mathbb{N}}\sum_{j \in \mathbb{N}}\sum_{k \in \mathbb{N}}\sum_{S \in \mathbb{N}} C_{iksj} D_{2ij} Y_{iksj} + \sum_{i \in \mathbb{N}}\sum_{j \in \mathbb{N}} h_{ij} L_{ij} Y_{ij} + \sum_{i \in \mathbb{N}}\sum_{j \in \mathbb{N}}\sum_{k \in \mathbb{N}}\sum_{S \in \mathbb{N}} h_{iksj} L_{iksj} Y_{iksj}$$

$$\text{Min } Z_3 = \sum_{k \in \mathbb{N}}(f_{3k} - f_{2k})X_k + \sum_{i \in \mathbb{N}}\sum_{j \in \mathbb{N}} C_{ij}(D_{3ij} - D_{2ij})Y_{ij} + \sum_{i \in \mathbb{N}}\sum_{j \in \mathbb{N}}\sum_{k \in \mathbb{N}}\sum_{S \in \mathbb{N}} C_{iksj}(D_{3ij} - D_{2ij})Y_{iksj} + \sum_{i \in \mathbb{N}}\sum_{j \in \mathbb{N}} h_{ij} L_{ij} Y_{ij} + \sum_{i \in \mathbb{N}}\sum_{j \in \mathbb{N}}\sum_{k \in \mathbb{N}}\sum_{S \in \mathbb{N}} h_{iksj} L_{iksj} Y_{iksj}$$

Since, there is no fuzzy number in constraints, constraints do not change and constraints (5) to (18) are exactly brought in here.

4.1. Genetic Algorithm

To solve the model and to achieve an optimal solution, Genetic Algorithm is used. This algorithm, as an optimization computational algorithm, consider the set of points in solution space at each iteration computing, and effectively search the different areas of solution. In each iteration, each of the strings (chromosomes) are decoded and its objective function value Which is based on these numbers is obtained, and a fitness value is assigned to each string. Fitness value determine the probability of selection of desired string, that based on the probability of selection and applying genetic operators on selected set of strings. New strings are obtained and will be replaced by initial strings, until after a large number of iterations, the objective function value tend to a certain number and be fixed on it [4, 38, 39].

In this algorithm, strings of numbers combined with each other and constitute chromosomes and a set of chromosomes form a population (a random collection of problem answer). To make a new generation, two mutant and fusion genetic operations are done on parent's chromosomes. Certain percentage of parents chromosomes are combined as mutated. A Roulette Wheel is used to select the participating chromosomes in transplantation surgery. Roulette Wheel method, obtained the fitness value of chromosomes involved in transplantation surgery and chromosomes that have a greater fitness be involved in transplantation surgery up to offspring chromosomes be formed. Then, based on the fitness value of the new generation (the variables), the fitness function (objective function) is assessed and the new objective function value compared with the previous value and if it maintain the stop condition that was initially defined for algorithm, algorithm is finished. Otherwise, a new generation will be formed and previous steps will be repeated [39].

In general, to solve each problem using Genetic Algorithm, five important component shows genetic responses of problem, and they includes: the initial population of responses, one fitness function to determine the fitness of each response, genetic operators to manipulate the genetic structure of children during reproduction, values of parameters that are used in the algorithm (population size, number of population, etc.) are required. In this study, population size is equal to 20, the number of population equal to 150, the possibility of coupling equal to 0.7, and mutation probability is considered equal to 0.001. The algorithms flow chart is shown in Fig. (1).

4.2. VIKOR Method

VIKOR method was first introduced by [40] for complex systems of a multi-criteria decision. In this method, focusing on ranking the aim is to select options from a series of options in a problem with conflicting criteria [41].

Suppose that a group multi-criteria decision making problem have k decision maker $(K = 1, 2, \ldots, k)$, m option A_i $(i = 1, 2, \ldots, m)$, and n decision criterion C_j $(j = 1, 2, \ldots, n)$. Each option, according to all criteria, is assessed by all

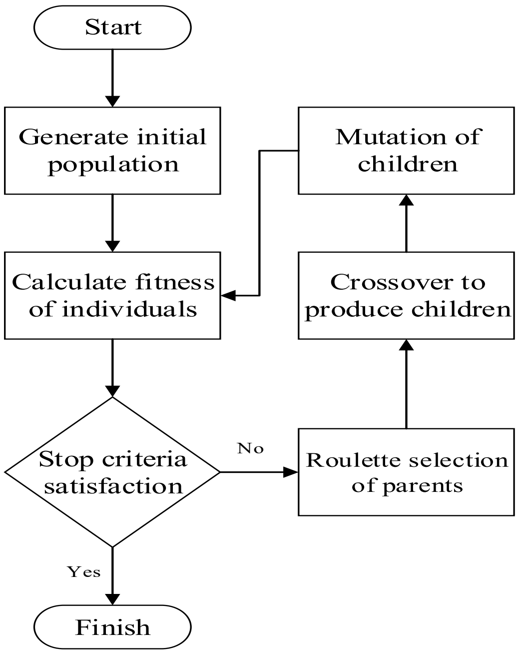

Fig 1. Genetic Algorithms flow chart

decision makers. Assessments made of all options based on n criterion by k decision maker are written in a decision matrix $D=\left[x_{ij}\right]_{m\times n}$. The process of VIKOR method process for ranking the m option is described as below [40, 41].

Step 1: Collecting decision maker's opinions to determine the weights of criterion and options, and make a decision matrix (D). The cumulative weight for all options based on each criterion is calculated as follows:

$$X_{ij}=\frac{1}{k}\sum_{k=1}^{k}X_{ijk} \quad i=1,2,\ \ldots,\ m;\ j=1,2,\ldots,n \tag{19}$$

And also the weight of each criterion is calculated as follows:

$$W_{ij}=\frac{1}{k}\sum_{k=1}^{k}W_{ijk} \quad i=1,2,\ \ldots,\ m;\ j=1,2,\ldots,n \tag{20}$$

Ultimately, the decision making matrix will be calculated as follows:

$$D=\begin{array}{c} A \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array}\begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \cdots & C_n \end{array} \\ \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ x_{m1} & x_{m2} & \cdots & x_{mn} \end{bmatrix} \end{array} \tag{21}$$

$$W=\begin{bmatrix} W_1 & W_2 & \cdots & W_n \end{bmatrix} \tag{22}$$

Step 2: formation of a normalized decision matrix (R):

$$R=\left[r_{ij}\right]_{m\times n} \quad i=1,2,\ \ldots,\ m;\ j=1,2,\ldots,n \tag{23}$$

In which r_{ij} with respect to the criteria, is calculated from the following formula:

If criterion was a loss type:

$$r_{ij}=\frac{a_j^-}{x_{ij}} \quad i=1,2,\ \ldots,\ m;\ j=1,2,\ldots,n \tag{24}$$

$$a_j^-=\min_i\left\{x_{ij}\right\} \quad i=1,2,\ \ldots,\ m;\ j=1,2,\ldots,n \tag{25}$$

If criterion was a profitable type:

$$r_{ij}=\frac{x_{ij}}{c_j^*} \quad i=1,2,\ \ldots,\ m;\ j=1,2,\ldots,n \tag{26}$$

$$c_j^*=\max_i\left\{x_{ij}\right\} \quad i=1,2,\ \ldots,\ m;\ j=1,2,\ldots,n \tag{27}$$

Step 3: Calculate the best $\left(f_j^*\right)$ and worst $\left(f_j^-\right)$ values for all criteria:

$$f_j^*=\max_j r_{ij} \quad i=1,2,\ \ldots,\ m;\ j=1,2,\ldots,n \tag{28}$$

$$f_j^-=\min_j r_{ij} \quad i=1,2,\ \ldots,\ m;\ j=1,2,\ldots,n \tag{29}$$

Step 4: Calculate the amount of S_i and R_i for all options:

$$S_i=\sum_{j=1}^{n}\frac{w_j\left(f_j^*-r_{ij}\right)}{\left(f_j^*-f_j^-\right)} \quad i=1,2,\ \ldots,\ m \tag{30}$$

$$R_i=\underset{j}{\mathrm{Max}}\left[\frac{w_j\left(f_j^*-r_{ij}\right)}{\left(f_j^*-f_j^-\right)}\right] \quad i=1,2,\ \ldots,\ m \tag{31}$$

W_j is the weight of a criteria and is a part of the input, which can be obtained by using AHP, ANP, Antropy, asking expert, etc.

Step 5: Calculate VIKOR (Q_i) value for all options:

$$Q_i = v\left[\frac{S_i - S^*}{S^- - S^*}\right] + (1-v)\left[\frac{R_i - R^*}{R^- - R^*}\right] \quad (32)$$

$$i = 1, 2, \ldots, m$$

Where: $R^* = \underset{i}{\mathrm{Min}}\, R_i$ و $R^- = \underset{i}{\mathrm{Max}}\, R_i$ و $S^* = \underset{i}{\mathrm{Min}}\, S_i$ و $S^- = \underset{i}{\mathrm{Max}}\, S_i$

V is the weight of majority strategy agreed criteria or maximum group utility; in this research, its value is considered equal to 0.5.

Step 6: Ranking the options based on the value of Q_i : with Q_i sort from low to high values, options ranking is obtained from first to last (i.e, whatever Q_i value is lower, i^{th} option has a higher priority).

5. Case study

In this section, a real case in Iran has been studied to validate the performance of the proposed model and the solution approaches. So, a special instance on hub location of Iranian aviation between 37 cities (see Fig. 2) is used. Consider a network of eight cities under investigation to design the interface fiber-optic network between them. However, the exact location of hub must be specified. In Fig. (3) the network is given in the general case (before solving and optimization). According to Fig. (3), there are three potential locations for construction of a hub, that only two of them should be selected. Fixed cost of construction of hubs in E is fuzzy number 500 million dollars, in D city fuzzy number 300 million dollars and in H is fuzzy number 400 million dollars. The cost to meet each unit of demand directly from source to destination (C_{ij}) is equal to 1,000 dollars per kilo byte (KB). The value of α_{ks} and β_{ik} is considered 0.6 and 0.8 respectively. The value of M is obtained according to the information available in the demand tables and by using formula (1) is equal to 1700. In Table (1), cities distance from each other is given and in Table (2) the anticipated demand for each city is also provided. In table (3), the cost of demand's direct meet has been provided. According to Fig. (3), cities are coded numerically as the following: 1) A 2) B 3) C 4) D 5) E 6) F 7) G 8) H. For example, when $Y_{16}=1$, it implies that the connection has been established between the A and E cities.

Now by considering the above information and after solving the defuzzified model by using Genetic Algorithm, three possible scenarios can be obtained as an answer. These scenarios are given in Table (4).

Next step is to determine the costs of relationship between cities and hubs. Each of objective function should considered as criterion and the weight of each of them should be obtained through a survey from experts, which these weights were obtained 0.2, 0.5 and 0.3 for Z_1 , Z_2 and Z_3 respectively. Now with the help of these weights and VIKOR method three possible scenarios for hubs could be ranked to determine the best solution. The final result and ranking obtained is given in Table (5).

Finally, according to the obtained ranking, D and E were

Fig 2. Iran cities in the IAD data set.

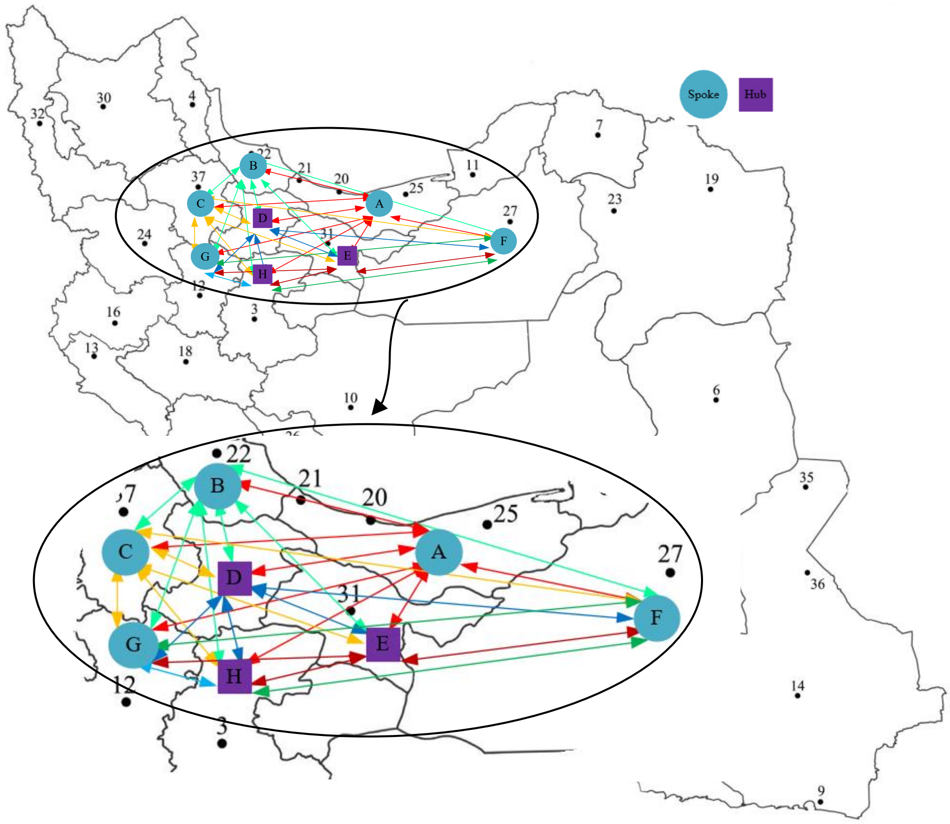

Fig 3. A network of eight cities under investigation to design the interface fiber-optic network between them.

Table 1. Cities distance from each other (L_{ij}), Kilometer (Km)

H	G	F	E	D	C	B	A	
560	586	201	250	400	580	380	0	**A**
577	407	551	323	173	253	0	380	**B**
505	330	558	430	180	0	253	580	**C**
303	243	378	150	0	180	173	400	**D**
239	336	228	0	150	430	323	250	**E**
529	564	0	228	378	558	551	201	**F**
176	0	564	336	243	330	407	586	**G**
0	176	529	239	303	505	577	560	**H**

Table 2. Anticipated demand for each city to get service in optic fiber network (Mb)

H	G	F	E	D	C	B	A	
(430,450,460)	(600,610,630)	(470,490,510)	(360,370,380)	(750,760,770)	(650,670,685)	(820,830,850)	(0,0,0)	**A**
(360,380,400)	(710,725,730)	(550,560,570)	(400,420,440)	(610,630,650)	(700,730,750)	(0,0,0)	(820,830,850)	**B**
(810,820,830)	(350,360,370)	(410,430,440)	(450,470,490)	(720,730,740)	(0,0,0)	(700,730,750)	(650,670,685)	**C**
(600,620,630)	(380,390,405)	(480,500,510)	(500,510,530)	(0,0,0)	(720,730,740)	(610,630,650)	(750,760,770)	**D**
(490,500,510)	(420,430,440)	(350,370,380)	(0,0,0)	(500,510,530)	(450,470,490)	(400,420,440)	(360,370,380)	**E**
(630,650,660)	(630,640,650)	(0,0,0)	(350,370,380)	(480,500,510)	(410,430,440)	(550,560,570)	(470,490,510)	**F**
(380,400,410)	(0,0,0)	(630,640,650)	(420,430,440)	(380,390,405)	(350,360,370)	(710,725,730)	(600,610,630)	**G**
(0,0,0)	(380,400,410)	(630,650,660)	(490,500,510)	(600,620,630)	(810,820,830)	(360,380,400)	(430,450,460)	**H**

Table 3. The cost of demand satisfaction directly and without hubs (h_{ij}), dollars/Km

H	G	F	E	D	C	B	A	
1200	1050	900	500	1250	1100	1000	0	**A**
1000	950	850	700	700	650	0	1000	**B**
980	970	700	500	900	0	650	1100	**C**
1100	750	800	600	0	900	700	1250	**D**
1000	550	650	0	600	500	700	500	**E**
900	500	0	650	800	700	850	900	**F**
1300	0	500	550	750	970	950	1050	**G**
0	1300	900	1000	1100	980	1000	1200	**H**

Table 4. Result of solving defuzzified model using Genetic Algorithm

Objective function	Objective function value	Hubs	The variables that are equal to 1 in the model
Z_1	307165106	H and F	X_1, X_2, Y_{16}, Y_{17}, Y_{26}, Y_{27}, Y_{36}, Y_{37}, Y_{46}, Y_{47}, Y_{56}, Y_{57}, Y_{67}, Y_{68}, Y_{78}, Y_{1672}, Y_{1673}, Y_{1674}, Y_{1675}, Y_{1678}, Y_{2673}, Y_{2674}, Y_{2675}, Y_{2678}, Y_{3674}, Y_{3675}, Y_{3678}, Y_{4675}, Y_{4678}
Z_2	10063000000		
Z_3	316973106		
Z_1	306228342	H and G	X_1, X_2, Y_{16}, Y_{18}, Y_{26}, Y_{28}, Y_{36}, Y_{38}, Y_{46}, Y_{48}, Y_{56}, Y_{58}, Y_{68}, Y_{76}, Y_{78}, Y_{1682}, Y_{1683}, Y_{1684}, Y_{1685}, Y_{1687}, Y_{2683}, Y_{2684}, Y_{2685}, Y_{2687}, Y_{3684}, Y_{3685}, Y_{3687}, Y_{4685}, Y_{4687}
Z_2	14767000000		
Z_3	280802552		
Z_1	300158864	G and F	X_1, X_2, Y_{17}, Y_{18}, Y_{28}, Y_{27}, Y_{38}, Y_{37}, Y_{48}, Y_{47}, Y_{58}, Y_{57}, Y_{67}, Y_{68}, Y_{78}, Y_{1782}, Y_{1783}, Y_{1784}, Y_{1785}, Y_{1786}, Y_{2783}, Y_{2784}, Y_{2785}, Y_{2786}, Y_{3784}, Y_{3785}, Y_{3786}, Y_{4785}, Y_{4786}
Z_2	9965700000		
Z_3	319680696		

Table 5. Final result and ranking obtained by VIKOR method

Option	Weight of each option by VIKOR	Rank
H-F	0	**1**
H-G	0.544352	**2**
F-G	1	**3**

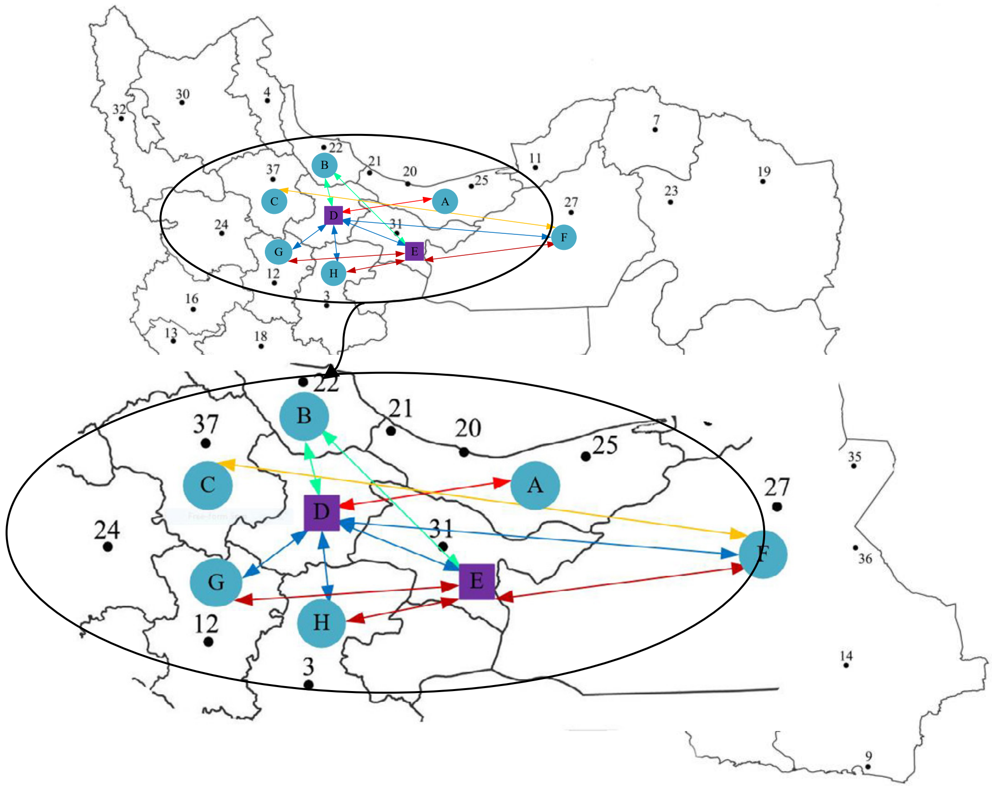

Fig 4. Optimal design of city's fiber optic communications and the determined hubs.

selected as hub and H will be removed from candidates for hub. Final and optimal design of city's fiber optic communications is shown in Fig. (4).

6. Conclusion

Network design to satisfy the demand and hub location are among the most important and difficult problems in decision-making issues. Many researchers have been investigated the matter from diverse and different viewpoints. In this research, the problem has been investigated by the assumption of uncertainty in demand and setup costs. To solve the problem a hybrid solution is used. First, the problem is modeled mathematically and then by considering fuzzy values in the model, model defuzzification was done. Then, with the help of Genetic Algorithm, all of possible scenarios for the network design and hub location have been examined and finally optimal design was selected among obtained scenarios by using VIKOR ranking method. The used approach by this study enhancing flexibility and costs effectiveness theoretically. In continues an example in moderate size with eight nodes and two hubs is used to show how the proposed solution works. The obtained answer represents the optimal network design and hub location simultaneously and was calculated in an acceptable solving time. However, according to the basic assumptions of the problem, it is suggested that the proposed model and its solution been investigated in the larger scale, to determine its performance. Also, some of the problem's assumptions can be neglected. For example, the problem can be solve in demand capacity constraints, considering more than two hub, network reliability and storage paths for communication between cities.

References

1.An, Y., Y. Zhang, and B. Zeng, *The reliable hub-and-spoke design problem: Models and algorithms.* Transportation Research Part B: Methodological, 2015. **77**(C): p. 103-122.

2. Çiftçi, M.E. and M. Şevkli, *A new hub and spoke system proposal: A case study for Turkey's aviation industry.* Journal of Air Transport Management, 2015. **47**(Complete): p. 190-198.

3. Bryan, D.L. and M.E. O'Kelly, *Hub-and-Spoke Networks in Air Transportation: An Analytical Review.* Journal of Regional Science, 1999. **39**(2): p. 275-295.

4. Marianov, V. and D. Serra, *Location models for airline hubs behaving as M/D/c queues.* Computers & Operations Research, 2003. **30**(7): p. 983-1003.

5. Martín, J.C. and C. Román, *Analyzing competition for hub location in intercontinental aviation markets.* Transportation Research Part E: Logistics and Transportation Review, 2004. **40**(2): p. 135-150.

6. Sasaki, M., A. Suzuki, and Z. Drezner, *On the selection of hub airports for an airline hub-and-spoke system.* Computers & Operations Research, 1999. **26**(14): p. 1411-1422.

7. Cunha, C.B. and M.R. Silva, *A genetic algorithm for the problem of conFiguring a hub-and-spoke network for a LTL trucking company in Brazil.* European Journal of Operational Research, 2007. **179**(3): p. 747-758.

8. Guastaroba, G., M.G. Speranza, and D. Vigo, *Intermediate Facilities in Freight Transportation Planning: A Survey.* Transportation Science, 2016. **50**(3): p. 763-789.

9. Jeong, S.-J., C.-G. Lee, and J.H. Bookbinder, *The European freight railway system as a hub-and-spoke network.* Transportation Research Part A: Policy and Practice, 2007. **41**(6): p. 523-536.

10. Nickel, S., A. Schöbel, and T. Sonneborn, *Hub Location Problems in Urban Traffic Networks*, in *Mathematical Methods on Optimization in Transportation Systems*, M. Pursula and J. Niittymäki, Editors. 2001, Springer US: Boston, MA. p. 95-107.

11. Ernst, A.T. and M. Krishnamoorthy, *Hub LocationEfficient algorithms for the uncapacitated single allocation p-hub median problem.* Location Science, 1996. **4**(3): p. 139-154.

12. Çetiner, S., C. Sepil, and H. Süral, *Hubbing and routing in postal delivery systems.* Annals of Operations Research, 2010. **181**(1): p. 109-124.

13. Kuby, M.J. and R.G. Gray, *The hub network design problem with stopovers and feeders: The case of Federal Express.* Transportation Research Part A: Policy and Practice, 1993. **27**(1): p. 1-12.

14. Yaman, H., B.Y. Kara, and B.Ç. Tansel, *The latest arrival hub location problem for cargo delivery systems with stopovers.* Transportation Research Part B: Methodological, 2007. **41**(8): p. 906-919.

15. Alumur, S. and Y.B. Kara, *A hub covering network design problem for cargo applications in Turkey.* Journal of the Operational Research Society, 2009. **60**(10): p. 1349-1359.

16. Carello G, D.C.F., Ghirardi M, Tadei R, *Solving the hub location problem in telecommunication network design: a local search approach.* Networks, 2004. **44**: p. 94-105.

17. Lapierre, S.D., A.B. Ruiz, and P. Soriano, *Designing Distribution Networks: Formulations and Solution Heuristic.* Transportation Science, 2004. **38**(2): p. 174-187.

18. Yoon, M.-G. and J. Current, *The hub location and network design problem with fixed and variable arc costs: formulation and dual-based solution heuristic.* Journal of the Operational Research Society, 2008. **59**(1): p. 80-89.

19. Azizi, N., et al., *The impact of hub failure in hub-and-spoke networks: Mathematical formulations and solution techniques.* Computers & Operations Research, 2016. **65**: p. 174-188.

20. O'Kelly, M.E., *A quadratic integer program for the location of interacting hub facilities.* European Journal of Operational Research, 1987. **32**(3): p. 393-404.

21.O'Kelly, M.E., *The Location of Interacting Hub Facilities.* Transportation Science, 1986. **20**(2): p. 92-106.

22.Yaman, H. and S. Elloumi, *Star p-hub center problem and star p-hub median problem with bounded path lengths.* Computers & Operations Research, 2012. **39**(11): p. 2725-2732.

23. Aykin, T., *Networking Policies for Hub-and-Spoke Systems with Application to the Air Transportation System.* Transportation Science, 1995. **29**(3): p. 201-221.

24.Campbell, J.F., *Hub Location and the p-Hub Median Problem.* Operations Research, 1996. **44**(6): p. 923-935

25.Yaman, H., *Allocation strategies in hub networks.* European Journal of Operational Research, 2011. **211**(3): p. 442-451.

26. Aykin, T., *Lagrangian relaxation based approaches to capacitated hub-and-spoke network design problem.* European Journal of Operational Research, 1994. **79**(3): p. 501-523.

27. Alumur, S. and B.Y. Kara, *Network hub location problems: The state of the art.* European Journal of Operational Research, 2008. **190**(1): p. 1-21.

28.Rastani, S., H. Karimi, and M. Setak, *Designing Incomplete Hub Location-routing Network in Urban Transportation Problem.* International Journal of Engineering, 2013. **26**(9): p. 997-1006

29. Lee, J.-H. and I. Moon, *A hybrid hub-and-spoke postal logistics network with realistic restrictions: A case study of Korea Post.* Expert Systems with Applications, 2014. **41**(11): p. 5509-5519.

30. Zheng, J., Q. Meng, and Z. Sun, *Liner hub-and-spoke shipping network design.* Transportation Research Part E: Logistics and Transportation Review, 2015. **75**: p. 32-48.

31. Rahimi, Y., et al., *Multi-objective hub network design under uncertainty considering congestion: An M/M/c/K queue system.* Applied Mathematical Modelling, 2016. **40**(5–6): p. 4179-4198.

32. Yang, T.-H., *Stochastic air freight hub location and flight routes planning.* Applied Mathematical Modelling, 2009. **33**(12): p. 4424-4430.

33. Sim, T., T.J. Lowe, and B.W. Thomas, *The stochastic -hub center problem with service-level constraints.* Computers & Operations Research, 2009. **36**(12): p. 3166-3177.

34. Contreras, I., J.-F. Cordeau, and G. Laporte, *Stochastic uncapacitated hub*

location. European Journal of Operational Research, 2011. **212**(3): p. 518-528.

35. Mohammadi, M., et al., *Design of a reliable logistics network with hub disruption under uncertainty.* Applied Mathematical Modelling, 2016. **40**(9–10): p. 5621-5642.
36. Farahani, R.Z., et al., *Hub location problems: A review of models, classification, solution techniques, and applications.* Computers & Industrial Engineering, 2013. **64**(4): p. 1096-1109.
37. Taleizadeh, A.A., S.T.A. Niaki, and M.-B. Aryanezhad, *A hybrid method of Pareto, TOPSIS and genetic algorithm to optimize multi-product multi-constraint inventory control systems with random fuzzy replenishments.* Mathematical and Computer Modelling, 2009. **49**(5–6): p. 1044-1057.
38. Reddy, M.J. and D.N. Kumar, *Optimal Reservoir Operation Using Multi-Objective Evolutionary Algorithm.* Water Resources Management, 2006. **20**(6): p. 861-878.
39. Deb, K., *Multi-objective Optimisation Using Evolutionary Algorithms: An Introduction*, in *Multi-objective Evolutionary Optimisation for Product Design and Manufacturing*, L. Wang, C.A.H. Ng, and K. Deb, Editors. 2011, Springer London: London. p. 3-34.
40. Opricovic, S., *Multicriteria optimization of civil engineering systems.* Faculty of Civil Engineering, Belgrade, 1998. **2**: p. 5-21.
41. Opricovic, S. and G.H. Tzeng, *Multicriteria Planning of Post-Earthquake Sustainable Reconstruction.* Computer-Aided Civil and Infrastructure Engineering, 2002. **17**(3): p. 211-220.

HOW TO CITE THIS ARTICLE

S.B. Ebrahimi, A. Sharifi, M. Nezhad-Afrasiabi, S.M. Mirkhani, A Hybrid Solution Method for Hub-and-Spoke Network Design under Uncertainty A Case Study to Design Optical Fiber Network in Iran, AUT J. Model. Simul., 51(2) (2019) 179-190.

DOI: 10.22060/miscj.2019.15977.5149