Torsion Analysis of High-Rise Buildings using Quadrilateral Panel Elements with Drilling D.O.F.s

Afshin Meshkat-Dini[i]; Mohsen Tehranizadeh[ii]

ABSTRACT

Generally, the finite element method is a powerful procedure for analysis of tall buildings. Yet, it should be noted that there are some problems in the application of many finite elements to the analysis of tall building structures. The presence of artificial flexure and parasitic shear effects in many lower order plane stress and membrane elements, cause the numerical procedure to converge in a low rate. Nevertheless, very large hardware memory storage is needed because of using fine meshes. Hence, it should be better to develop and use elements which can model the structural system of tall buildings in coarse finite element meshes and converge fast. The panel type finite elements presented in this study, have vertical and horizontal degrees of freedom similar to those of wide column analogy in the frame method. There are two rotational degrees of freedom to be defined at the two end of the panel element, which denote the rotational freedom equal to the first derivative of lateral displacement. The proposed elements can simply be used in tall building analysis. The application of the proposed elements can be performed without using a fine mesh. Examples are given to denote the accuracy and efficiency of the presented panel elements.

KEYWORDS

Tall Building, Quadrilateral Element, In Plane Rotation Degree of Freedom, Strain Based Panel Element.

1. INTRODUCTION

The advantage of application of simple and complex shear core walls in tall buildings to resist against forces produced by lateral loads, has been recognized quite well. Obviously under asymmetrical arrangement in the case of eccentric lateral loads or in the case of structural plan, torsional effects are produced in the overall behavior of a tall building. On the other hand, it is much simpler to study the effects of torsion if the thin walled structures such as non-planar coupled shear walls, allowed to warp freely. However, this is *St. Venant's torsion* effect. Non-planar shear walls and shear cores especially those which have closed section by strong connecting beams, resist against lateral loads to warp. As a result of the restraint to warping, vertical and longitudinal stress develops in the wall panels. This action is well known and referred to as *Vlasov's torsion* [1]-[7].

There are many computational methods and analytical procedures to analyze the overall behavior of tall buildings which contain lateral load resistant systems such as shear cores, shear walls and strong façade frames. These analytical procedures can be categorized into the continuum method, the frame method which is classified into the solid wall and the wide column analogies, the finite element method and the finite strip method [8]-[15].

Application of the finite element method to analyze shear core walls in tall buildings would be dated back to the 1960s. Generally, the finite element method can be used as a powerful tool for analyzing any type of building structures. There are many types of developed finite elements which have been used in modeling and analyzing of tall buildings with various types of structural systems such as single or complex shear cores, coupled wall-frame structures and bundled frame tube systems. However, the application of the thin walled beam elements, the strain based and displacement based finite elements, can be noted in this branch of structural engineering researches [16]-[19].

Based on the researches in the field of tall buildings

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analysis, application of all developed elements are not straightforward. Some major problems are involved. The exact definition of in-plane rotational freedom at each node, the probable existence of artificial flexure and parasitic shear effects in some so-called lower order plane stress elements (such as finite element Q_4) and the inefficiency of using lower order plane stress elements to model the coupling beams in the structural system of tall buildings, are of the most well-known problems. Many of the existing definitions of wall panel joint and coupling beam rotations would lead to incompatibility in the analytical model. According to the researches, the correct definition for compatible coupling rotation between the beam elements and the wall panels must be defined as the rotations of vertical fibers at the joint. The vertical fiber rotations ω_1 and ω_2 are shown in Figure 1. This is the only exact definition of rotational degrees of freedom in the tall building analysis [20]-[25].

Two developed displacement based and strain based panel elements are presented in this paper. The presented panel elements are developed based on the beam type displacement functions and the corresponding strain functions which have the ability of the illustration of pure bending. Both of the aforementioned displacement-based and strain-based functions can represent the internal shear–flexure interaction of shear wall panels in high-rise buildings. There are nine degrees of freedom to be defined for each of the presented elements. The degrees of freedom include external and internal horizontal translations (i.e., the degrees of freedom u_1, u_2 and the degree of freedom u_3 respectively), as well as four vertical translations and two correct defined in-plane rotations. The analytical concepts of the presented elements are similar to those of a solid wall element with in-plane rotational degrees of freedom which is defined as the rotation of the vertical fibers at the node (i.e., the degrees of freedom ω_1 and ω_2 which are shown in Figure 1). The presented panel elements can model the complex system of tall shear core walls in a coarse mesh while converging fast.

2. THE PROPOSED PANEL ELEMENT PD

The panel element PD, which is presented in this paper, has been developed according to definition of beam behavior. The nodal degrees of freedom are $u_1, \omega_1, v_1, v_2, u_2, \omega_2, v_3, v_4, u_3$. Furthermore, both rotational freedoms ω_1 and ω_2 are defined as $\omega = -\partial u/\partial y$, which is the vertical fiber rotation. It should be noted that both panel elements PS and PD have the same analytical structure and shape. Figure 1 shows the definition of the nine degrees of freedom to be assigned to both the displacement based panel element PD and the strain based panel element PS.

The panel element PD was formulated as a displacement-based element. The lateral displacement function u(y) is of order of four in the y direction and v(x,y) is linear in both directions. This element can obviously represent the strain state of pure bending and is therefore free of parasitic shear effects. Furthermore, as the lateral deflection u(y) is of order of four in the longitudinal direction, the panel element can deform laterally like an extended Euler-Bernoulli beam. This type of flexural behavior is often referred to as beam-type action. The displacement functions u(y) and v(x,y) of the element PD are as defined in Equations (1) and (2).

$$[u(y)]_{PD} = \alpha_1 + \alpha_2 y + \alpha_3 y^2 + \alpha_4 y^3 + \alpha_9 y^4 \tag{1}$$

$$[v(x,y)]_{PD} = \alpha_5 + \alpha_6 x + \alpha_7 y + \alpha_8 xy \tag{2}$$

Solving the nine α coefficients by equating all nodal translations and rotations to the nine degrees of freedom of the panel element shown in Figure 1 and substituting their values back into (1) and (2), the displacement functions u(y) and v(x,y) are obtained as noted in Equations (3) and (4).

$$\begin{aligned}[u(y)]_{PD} = &\left[-\frac{y^4}{2b^4}+\frac{y^3}{4b^3}+\frac{y^2}{b^2}-\frac{3y}{4b}\right]u_1 + \\ &\left[\frac{y^4}{4b^3}-\frac{y^3}{4b^2}-\frac{y^2}{4b}+\frac{y}{4}\right]\omega_1 + \\ &\left[-\frac{y^4}{2b^4}-\frac{y^3}{4b^3}+\frac{y^2}{b^2}+\frac{3y}{4b}\right]u_2 + \\ &\left[-\frac{y^4}{4b^3}-\frac{y^3}{4b^2}+\frac{y^2}{4b}+\frac{y}{4}\right]\omega_2 + \\ &\left[\frac{y^4}{b^4}-\frac{2y^2}{b^2}+1\right]u_3\end{aligned} \tag{3}$$

$$\begin{aligned}[v(x,y)]_{PD} = &\frac{1}{4}\left(1-\frac{x}{a}\right)\left(1-\frac{y}{b}\right)v_1 + \\ &\frac{1}{4}\left(1+\frac{x}{a}\right)\left(1-\frac{y}{b}\right)v_2 + \\ &\frac{1}{4}\left(1-\frac{x}{a}\right)\left(1+\frac{y}{b}\right)v_3 + \\ &\frac{1}{4}\left(1+\frac{x}{a}\right)\left(1+\frac{y}{b}\right)v_4\end{aligned} \tag{4}$$

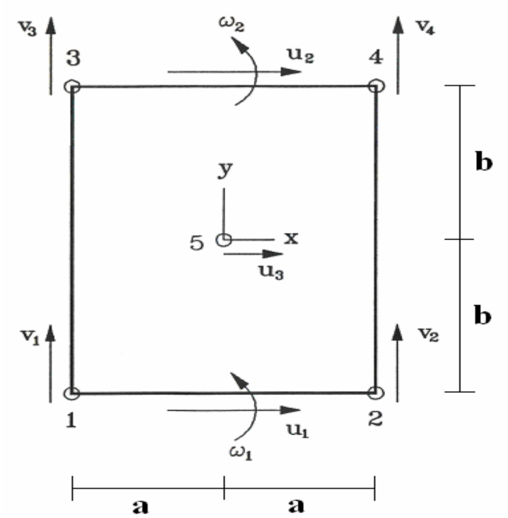

Figure 1: The proposed panel element with nine degrees of freedom. The shape of both panel elements PD and PS is identical.

As previously noted in many researches, the computational efficiency of the tall building model can be properly improved without significant loss of accuracy by making the assumption that the lateral strains in the wall panels and wide column elements are negligible. This means $\varepsilon_x = 0$ which is commonly accepted for tall building analysis. The stiffness matrix of the panel element PD is evaluated according to the formulation procedure following that of the standard finite element method. The stiffness matrix so derived, has two parts corresponded to the flexural and the shear effects, given in the Appendix.

3. THE PROPOSED PANEL ELEMENT PS

The panel element PS has also in-plane rotational degrees of freedom defined as $\omega = -\partial u/\partial y$ which is equal to the rotation of vertical axes of the beam-wall joint. According to the relating researches, this is the only correct definition that can ensure rotational compatibility in tall buildings modeling. The parameter u is the lateral displacement function of the element, shown in Figure 1. The assumed strain field of the panel element PS is given in Equations (5) – (7).

$$(\varepsilon_x)_{PS} = 0 \tag{5}$$

$$(\varepsilon_y)_{PS} = \beta_1 + \beta_2 x + \beta_3 xy \tag{6}$$

$$(\gamma_{xy})_{PS} = \beta_4 + \beta_5 y + \beta_6 y^3 \tag{7}$$

As noted in Equation (5), the horizontal strain is assumed to be negligible in tall building modeling. This simplification is based on the fact that wide column elements in tall framed tube buildings and shear wall panels in high rise wall-frame structures, generally behave like cantilever deep beam elements. Hence, the lateral strains and stresses are much smaller than the shear-flexure stress resultants. Research results in the field of tall buildings analysis show that using exact stress function analysis for a rectangular panel with an aspect ratio greater than three, neglecting lateral stress resultants in the overall behavior causes an error less than 3% in the shear-flexure stress resultants. This is an applicable assumption which can be used in modeling of tall buildings, especially at the floor levels [26]-[29].

The axial strain along the y axis is shown by the coefficient β_1, and given Equation (8). The notification of bending stress resultant ε_y is formulated by assuming a linear variation with the height of the panel element as shown in Equation (6). The coefficients β_1, β_2 and β_3 were used to achieve this aim. This assumption is based on the linearization of the variation of bending moments with the height of stories of a tall building. The coefficients β_4, β_5 and β_6 in Equation (7) also denote a cubic variation for shear strain along axis y. This formulation is based on the definition of a higher order function for the variation of γ_{xy} with height of the proposed panel element. This is similar to the phenomena which is to be denoted for shear strains in Timoshenko beam theory [30]-[32].

$$\beta_1 = \frac{1}{4b}\left[(v_3 + v_4) - (v_1 + v_2)\right] \tag{8}$$

The shear strain function defined in Equation (7), is a similar expression of γ_{xy} which can be used in wide column and solid wall analogy for tall building analysis. The four parameters θ_1, θ_2 and ω_1, ω_2 describe horizontal and vertical fiber in-plane rotations at the upper and lower chords of both panel elements PS and PD.

$$\gamma_{xy} = (\theta_1 - \omega_1)\left[\frac{1}{2} - \frac{3}{2}\left(\frac{y}{2b}\right) + 2\left(\frac{y}{2b}\right)^3\right] + (\theta_2 - \omega_2)\left[\frac{1}{2} + \frac{3}{2}\left(\frac{y}{2b}\right) - 2\left(\frac{y}{2b}\right)^3\right] \tag{9}$$

The three coefficients β_7, β_8 and β_9 are needed to represent the rigid body displacement functions. Both rigid body displacement functions are added to those obtained by integrating the strain functions ε_x and ε_y given in Equations (5) and (6). Hence, the full displacement functions u(y) and v(x,y) are defined as follows,

$$[u(y)]_{PS} = \beta_7 - \beta_9 y + f_1(y) \tag{10}$$

$$[v(x,y)]_{PS} = \beta_8 + \beta_9 x + \int \varepsilon_y dy + f_2(x) \tag{11}$$

Two complementary functions $f_1(y)$ and $f_2(x)$ are obtained according to accomplishing a few algebraic computations upon the analytical expression for γ_{xy}. As a result, the higher order polynomial for $f_1(y)$ is finally obtained as follows;

$$f_1(y) = -\frac{1}{2}\beta_2 y^2 - \frac{1}{6}\beta_3 y^3 + \beta_4 y + \frac{1}{2}\beta_5 y^2 + \frac{1}{4}\beta_6 y^4 \tag{12}$$

The nine coefficients β_1 to β_9, are determined by equating the nodal translations and rotations to the nine defined degrees of freedom of the panel element and solving the system of equations thus obtained. Following this approach and using Equations (10) and (11) will lead to the strain–displacement matrix [B], given in the appendix.

It should be noted that according to the results of applying the standard finite element method, the lateral displacement function u(y) for both panel elements PS and PD is identical. Therefore, both panel elements PS and PD are formulated as strain-based and displacement-based finite elements respectively, with horizontal displacement function u(y) which is constant in the x direction and of order of four in the y direction. It is noteworthy to indicate, the stiffness matrix of the panel element PS has also two parts which are corresponding to both the flexural and the shear effects. The part due to the flexural effects of this stiffness matrix is essentially equal to that of obtained for the panel element PD.

$$\left[K\right]_{PS} = t\int_A [B]_{PS}^T [D_m][B]_{PS}\, dA \tag{13}$$

The parameter t is the thickness of the panel element shown in Figure 1. The strain-displacement matrix [B] for the element PS is obtained according to the standard finite element method. It should be noted that because of using the assumption $\varepsilon_x = 0$, the material matrix $[D_m]$ would be a diagonal matrix of rank three. The diagonal components are E_x, E_y and G_{xy} respectively. In the case of isotropic material, both E_x and E_y are equal to E as Young's modulus of elasticity. Hence the parameter G_{xy} should be also equal to G, named as shear modulus of elasticity.

4. Application of The elements PS and PD

As noted before, the presence of parasitic shear effects in many of the finite elements causes the elements to be deformed too stiff in bending mode. This problem has not been improved by using the finite element Q_4 unless the mesh of the elements is very fine. Since the structural system of tall buildings is subjected principally to bending actions, this is indeed an obvious problem. The source of the trouble is the inability of the some lower order elements to curve themselves to follow the deformed shape of the structure, especially when subjected to bending [20,23,29].

However, the best way of dealing with parasitic shear effects is to avoid them by using finite elements that can represent the strain state of pure bending. Nevertheless, it is noted that a good element for tall building modeling should be one with rotational in-plane degrees of freedom to ensure direct connection to the lintel beams, and can represent the strain state of pure bending so as to avoid parasitic shear effects, and spans at most only one storey so that there will be no excess continuity problem.

The panel elements PS and PD are not subjected to parasitic shear effects. These beam-type panel elements are a rectangular four-node element with four degrees of freedom at the upper and lower chords and an internal degree of freedom u_3 at the center of the element. The nodal freedoms at each chord include a horizontal translation, two vertical translations, and an in-plane rotation defined as the rotation of the vertical fiber at the node. Both elements PS and PD satisfy all the aforementioned criteria for a good element in order to model and analyze tall buildings [27,28,30,36].

A. Analysis of a Tall Coupled Shear Core

A 20 storey coupled shear core structure shown in Figure 2, is analyzed while using PS and PD panel elements. This structure has been already analyzed based on the continuum method [33],[34]. This symmetric core wall structure is formed of two channel shaped non-planar wall units coupled together by lintel beams at floor levels. In the analysis, each channel shaped wall unit is treated as an assembly of multiple planar wall units. It is noted that only one layer of elements per story is used. The wall panels which are connected to the lintel beams, have been modeled by five elements. The other panels were also modeled by four elements.

The analytical results in conjunction with the roof rotation ϕ_{Top} and the vertical stresses at the base of the core structure σ_A, σ_B are obtained and compared with those of Coull's study using laminar analogy [34] and Macleod's study based on the application of solid wall analogy [21]. These results are given in Table 1. It can be seen from the results obtained from the panel elements PS and PD and those calculated by the continuum method and the frame method that they agree approximately with each other.

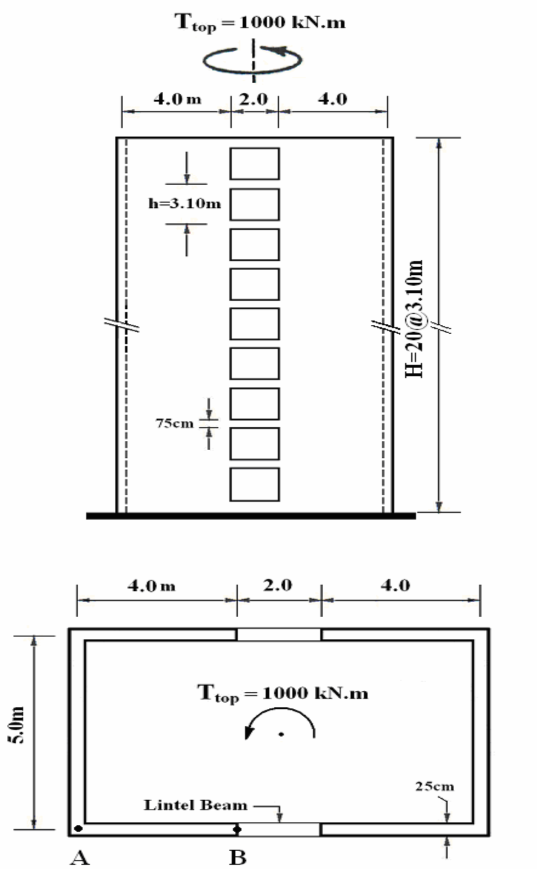

Figure 2: The structural system and applied torsional load for the analyzed Tall Coupled Shear Core.

The differences are due to the fact that in the laminar analogy, the wall panels are connected at the corners continuously with the height of tall building. Meanwhile, in the frame methodology and the finite element procedure, the wall panels are usually connected only at floor levels, especially when using panel type elements. Furthermore, as noted by Haji-Kazemi [10] and Rutenberg [35] the assumption of neglecting shear deformations effect which considered in the general continuum method, may cause considerable errors in the tall building response parameters. This conceptual remark has already been explained in the other researches [2,5,11,12,14].

TABLE 1
ANALYZED RESULTS FOR THE TALL COUPLED SHEAR CORE

	ϕ_{Top} 10^{-4} rad	σ_A kg/cm^2	σ_B kg/cm^2
Panel Element PS	1.795	-0.847	-1.861
Panel Element PD	1.727	-0.835	-1.429
Macleod [21]	1.950	-0.637	-1.394
Coull [34]	1.870	-0.743	-1.338

B. Analysis of a Tall Core Supported Building

An asymmetric tall core supported structure is considered in this study. This structure has been also analyzed by Stafford Smith et al [1], Pekau et al [19], Ha et al [16],[20] and Macleod et al [21], based on using various analytical methodologies. The dimensions and loading of the structural system are shown in Figure 3. The storey height is 3.81 m, lintel beam depth is 0.457m, Young's modulus E = 2.76×10^4 MPa and shear modulus G = 1.20×10^4 MPa. Obviously because of the structural asymmetry introduced by the access opening will induce significant torsional and warping stresses under the effects of lateral loads. It should be noted that only one layer of elements per story is used for the analysis. The two wall panels in this example structure which are directly connected to the lintel beams, have been modeled by five elements. The other panels were modeled by four elements.

Table 2 shows the structural rotation ϕ_{Top} with respect to the shear center of the core wall as well as the vertical stresses $\sigma_A, \sigma_B, \sigma_C$ at the foundation level, calculated by the finite element analysis using PS and PD elements and those obtained by aforementioned references. The agreement between the results obtained according to analysis using panel Elements PS and PD and those obtained by applying the other methodologies is almost exact. These results have been obtained based on three dimensional analysis of the structure shown in Figure 3. All of the parameters $\sigma_A, \sigma_B, \sigma_C$ can be used for illustration of the distribution of vertical stresses as shown in Figure 4.

Nevertheless, small differences among the results for corresponding parameters shown in Table 2 can be seen with the exception of σ_B. The main source of differences is explained based on the conceptual principles and analytical assumptions to be applied to each method of analysis. Smith's closed form analysis [1] is based on the continuum method with the assumption of neglecting shear deformation effects and Macleod's frame analogy

[21] is also based on using solid wall elements without defined in-plane rotational degrees of freedom. The results of Pekau's finite storey method [19] are obtained by the application of thin walled beam elements for modeling of core wall and replacing the band of coupling beams by an elastically equivalent continuum. This model for coupling beams has already been applied to the macro-element model developed by Ha [16] for tall building analysis.

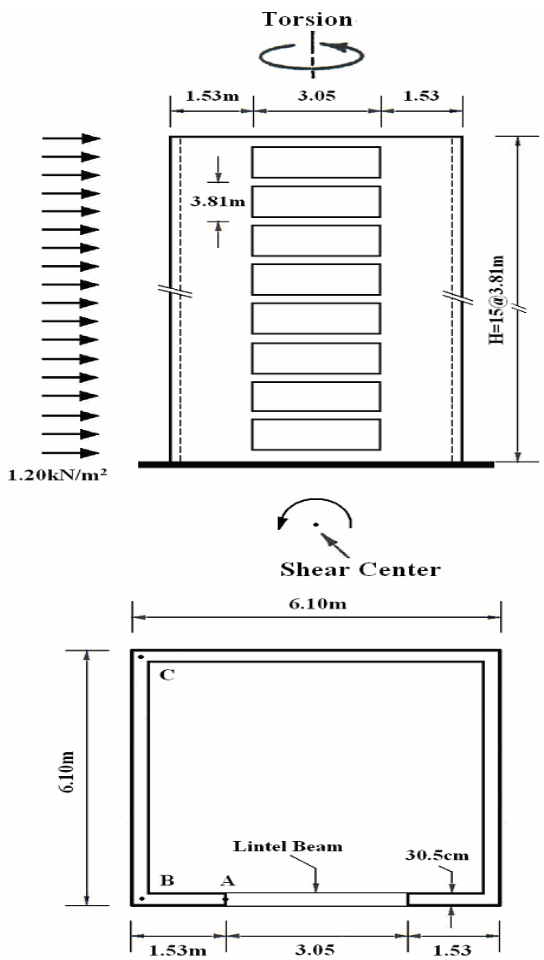

Figure 3: The structural system and applied lateral load for the analyzed Tall Core Supported Structure.

Furthermore, Ha's macro-element methodology [20] uses lower order strain based and displacement based finite elements without in-plane rotational degrees of freedom. However, the results show that the application of strain based panel element PS and displacement panel element PD together with the simple beam elements, can reliably model the stiffness characteristics and the overall behavior of tall building structures. This applicable conclusion has already been described in the authors' other researches [26 to 28, 30, 36].

Generally, in high-rise buildings, the required lateral stiffness is typically provided by the use of complex or simple system of planar shear walls, shear cores, framed tubes with fully or semi-rigid connections. Braced or rigid frames are not always used as lateral load resistant system in tall buildings. However, under action of lateral loads, tall building systems deform in a combination of both the cantilever bending and the shear-racking modes. The shear mode is associated with framing systems, whereas the cantilever bending mode is generally associated with tall shear cores, shear walls and tubular structures. The analysis of such high-rise structures is often complex. This is because of the large number of connecting joints and structural elements which can interact together, in which the flexibility of finite-sized joints as well as panel zone effects, and also the need for three-dimensional modeling where torsion of asymmetrical shape is involved [2,3,5, 24, 25,37].

TABLE 2
ANALYZED RESULTS FOR THE TALL CORE SUPPORTED STRUCTURE

	ϕ_{Top} 10^{-3} rad	σ_A kg/cm^2	σ_B kg/cm^2	σ_C kg/cm^2
Panel Element PD	3.45	-37.2	7.9	45.4
Panel Element PS	3.49	-37.2	8.4	45.2
Smith [1]	3.00	-36.7	4.3	45.3
Pekau [19]	3.70	-37.3	5.0	47.4
Ha [20] (6DOFD)	2.85	-35.4	6.9	39.7
Ha [20] (6DOFS)	3.38	-35.2	7.7	42.2
Macleod [21]	2.67	-44.3	6.3	40.7

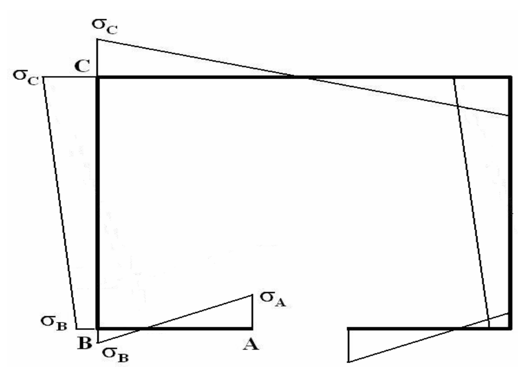

Figure 4: The schematic distribution of the vertical stresses produced by combined shear-flexure and torsional-warping overall interaction of the tall structure.

5. CONCLUSION

It should be noted that the most simple and lower order displacement based elements may be affected by the problem of parasitic shear, especially when used in coarse meshes. The appearance of this effect renders the elements too stiff under a bending mode. These effects are obviously observed when using the bilinear finite element Q_4. It is noteworthy to say that the strain based finite elements perform remarkably well under both shear and bending modes. The proposed panel elements PD and PS are found to be efficient and can be used to model high-rise buildings. The efficiency and versatility of the strain based panel element PS and the displacement based panel element PD are illustrated by applying them to the analysis of core wall structures in single and coupled shapes.

The nodal degrees of freedom include three horizontal and four vertical translations, and two in-plane rotations. Both in-plane drilling degrees of freedom are defined as the rotation of the vertical fibers at both upper and lower chords of the element. The results are in good agreement when compared to solutions obtained based on the other analytical methods. The application of the presented panel elements PD and PS is simple and flexible in tall buildings analyzes. The author's studies show that applying these panel elements needs only one layer of elements per storey which would suffice for the high-rise building analysis.

6. APPENDIX

The stiffness matrix due to displacement-based panel element PD is given as follows;

$$[K]_{PD} = [K_F] + [K_S]$$

$$[K_F] = E.t\begin{bmatrix} \frac{a^3}{b^3} & & & & & & & & \\ -\frac{a^3}{b^2} & \frac{a^3}{b} & & & & & & & \\ 0 & 0 & \frac{a}{3b} & & & & & & \\ 0 & 0 & \frac{a}{6b} & \frac{a}{3b} & & & & & \\ -\frac{a^3}{b^3} & \frac{a^3}{b^2} & 0 & 0 & \frac{a^3}{b^3} & & & & \\ -\frac{a^3}{b^2} & \frac{a^3}{b} & 0 & 0 & \frac{a^3}{b^2} & \frac{a^3}{b} & & & \\ 0 & 0 & -\frac{a}{3b} & -\frac{a}{6b} & 0 & 0 & \frac{a}{3b} & & \\ 0 & 0 & -\frac{a}{6b} & -\frac{a}{3b} & 0 & 0 & \frac{a}{6b} & \frac{a}{3b} & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$[K_S] = G.t\begin{bmatrix} \frac{254a}{105b} & & & & & & & & \\ \frac{-29a}{105} & \frac{32ab}{105} & & & & & & & \\ \frac{23}{30} & \frac{b}{30} & \frac{b}{3a} & & & & & & \\ \frac{-23}{30} & \frac{-b}{30} & \frac{-b}{3a} & \frac{b}{3a} & & & & & \\ \frac{2a}{105b} & \frac{13a}{105} & \frac{-7}{30} & \frac{7}{30} & \frac{254a}{105b} & & & & \\ \frac{-13a}{105} & \frac{2ab}{21} & \frac{-b}{30} & \frac{b}{30} & \frac{29a}{105} & \frac{32ab}{105} & & & \\ \frac{7}{30} & \frac{-b}{30} & \frac{b}{6a} & \frac{-b}{6a} & \frac{-23}{30} & \frac{b}{30} & \frac{b}{3a} & & \\ \frac{-7}{30} & \frac{b}{30} & \frac{-b}{6a} & \frac{b}{6a} & \frac{23}{30} & \frac{-b}{30} & \frac{-b}{3a} & \frac{b}{3a} & \\ \frac{-256a}{105b} & \frac{16a}{105} & \frac{-8}{15} & \frac{8}{15} & \frac{-256a}{105b} & \frac{-16a}{105} & \frac{8}{15} & \frac{-8}{15} & \frac{512a}{105b} \end{bmatrix}$$

The strain-displacement matrix [B] of the panel element PS is also defined in the formulations given as follows;

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} = [B]_{PS}\,.\{D\}$$

$$[B]_{PS} = \begin{bmatrix} 0 & \frac{-3}{2b^3}xy & \frac{2y}{b^2}\left(1-\left(\frac{y}{b}\right)^2\right) \\ 0 & \frac{3}{2b^2}xy & \frac{2y^3-b^2y-b^3}{2b^3} \\ 0 & \frac{x-a}{4ab} & \frac{y-b}{4ab} \\ 0 & \frac{x+a}{-4ab} & \frac{y-b}{-4ab} \\ 0 & \frac{3}{2b^3}xy & \frac{2y}{b^2}\left(1-\left(\frac{y}{b}\right)^2\right) \\ 0 & \frac{3}{2b^2}xy & \frac{-2y^3+b^2y-b^3}{2b^3} \\ 0 & \frac{x-a}{-4ab} & \frac{y+b}{-4ab} \\ 0 & \frac{x+a}{4ab} & \frac{y+b}{4ab} \\ 0 & 0 & \frac{4y}{b^2}\left(\left(\frac{y}{b}\right)^2-1\right) \end{bmatrix}^T$$

The displacement vector {D} includes all the nine degrees of freedom, defined in the Figure 1. This vector is identical for both panel elements PD and PS.

$$\{D\} = [u_1 \quad \omega_1 \quad v_1 \quad v_2 \quad u_2 \quad \omega_2 \quad v_3 \quad v_4 \quad u_3]^T$$

It is worth mentioning that in tall building modeling, the parameter θ is equal to $\partial v/\partial x$ and the parameter ω is equal to $-\partial u/\partial y$, too. This is a general assumption which is defined to the wide column, the solid wall and the panel element analogies. Furthermore, all degrees of freedom which are defined to both panel elements PD and PS (with the exception of u_3) are similar to those which have already been denoted to the developed elements in the other mentioned references.

The flexural part of the stiffness matrix due to strain-based panel element PS is exactly the same as that of panel element PD. There are also obvious differences between two shear stiffness matrices of both presented elements.

$$[K]_{PS} = [K_F] + G.t \begin{bmatrix} \frac{128a}{105b} & & & & & & & & \\ \frac{-8a}{105} & \frac{116ab}{105} & & & & & & & \\ \frac{4}{15} & \frac{8b}{15} & \frac{b}{3a} & & & & & & \\ \frac{-4}{15} & \frac{-8b}{15} & \frac{-b}{3a} & \frac{b}{3a} & & & & & \\ \frac{128a}{105b} & \frac{-8a}{105} & \frac{4}{15} & \frac{-4}{15} & \frac{128a}{105b} & & & & \\ \frac{8a}{105} & \frac{94ab}{105} & \frac{7b}{15} & \frac{-7b}{15} & \frac{8a}{105} & \frac{116ab}{105} & & & \\ \frac{-4}{15} & \frac{7b}{15} & \frac{b}{6a} & \frac{-b}{6a} & \frac{-4}{15} & \frac{8b}{15} & \frac{b}{3a} & & \\ \frac{4}{15} & \frac{-7b}{15} & \frac{-b}{6a} & \frac{b}{6a} & \frac{4}{15} & \frac{-8b}{15} & \frac{-b}{3a} & \frac{b}{3a} & \\ \frac{-256a}{105b} & \frac{16a}{105} & \frac{-8}{15} & \frac{8}{15} & \frac{-256a}{105b} & \frac{-16a}{105} & \frac{8}{15} & \frac{-8}{15} & \frac{512a}{105b} \end{bmatrix}$$

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