Delay-Dependent Robust Asymptotically Stable for Linear Time Variant Systems

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ABSTRACT

In this paper, the problem of delay dependent robust asymptotically stable for uncertain linear time-variant system with multiple delays is investigated. A new delay-dependent stability sufficient condition is given by using the Lyapunov method, linear matrix inequality (LMI), parameterized first-order model transformation technique and transformation of the interval uncertainty in to the norm bounded uncertainty.

A numerical example is presented to illustrate our present stability criterion allows an upper bound which is bigger on the size of the delay in comparison with those in the literature.

KEYWORDS

Lyapunov-Krasovskii functional, Linear matrix inequality, Parameterized first-order model transformation, Time-delay systems.

1. INTRODUCTION

Time delay is frequently a source of instability and it is often encountered in various areas of control systems, such as economical systems, biology [1, 10], engineering, neural network [12], transport phenomena and population dynamics [3, 7].

A system is said to be stable independent of delay (delay-independent stable) if it is stable when delay parameter assumes all nonnegative values. The stability of a system is delay-dependent if it is stable in some domain of delays [9]. Delay independent or delay-dependent stability can be easily derived by an appropriate choice of the terms involved in the Lyapunov-Krasovskii functional. Many criteria for checking the stability of time delay systems have been given so far.

In this paper, the linear matrix inequality (LMI) method with the parameterized first order model transformation technique is employed to derive a new delay-dependent robust asymptotically stable condition for the linear time-variant systems with multiple delays.

2. NOTATIONS

The following standard notation will be used throughout the paper. Let $R^{n\times m}$ denotes the set of all $n\times m$ real matrices, A^T be the transpose of matrix, $\tau>0$, and $A<B$ (resp., $A\leq B$) means that the matrix $B-A$ is positive definite (resp., positive semi-definite) for any two symmetric matrices *alongwith* Let $N=[n_{ij}],\ M=[m_{ij}]\in \mathbb{R}^{n\times m}$ and $n_{ij}\leq m_{ij}$, We define the set matrices

$[N,M]=\{A=[a_{ij}]_{n\times m}:\ n_{ij}\leq a_{ij}\leq m_{ij}\}$.

In addition $PC^0_{t_0}$ (resp., $PC^1_{t_0}$) denotes the space of all uniformly bounded piecewise continuous (resp., piecewise continuous differentiable) real matrix-valued functions defined on $[t_0,\infty)$. The Banach space

$C_n([-\tau,0])\triangleq C([-\tau,0],\mathbb{R}^n)$

of continuous vector functions mapping the delay interval into $\mathbb{R}^n$ with uniform convergence topology, where $\tau>0$ with standard supremum norm,

$\|\phi\|=Sup_{\theta\in[-\tau,0]}\|\phi(\theta)\|$

for given $\phi\in C_n([-\tau,0])$ and $\|.\|$ refers to either the Euclidean vector norm or the induced matrix 2-norm.

3. STABILITY CRITERIA FOR TIME-VARIANT SYSTEMS WITH MULTIPLE DELAYS

Let us consider the linear time-variant system with multiple delays as follows,

$$\dot{x}(t)=A_0(t)x(t)+\sum_{k=1}^{m}A_k(t)x(t-\tau_k) \tag{1}$$

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with initial conditions of time instant t_0

$$x_{t_0}(\theta) = x(t_0+\theta) = \phi(\theta), \quad \theta\in[-\bar{\tau},0] \tag{2}$$

where $x(t)=(x_1(t),...,x_n(t))^T$ is state vector at time t in usual sense. $A_k(t) \in PC_{t_0}^0$, are $n\times n$ state matrixes such that their components are not known precisely but satisfying $A_k(t) \in [N_k(t), M_k(t)]$, for $k=0,1,...,m$ where $N_k(t) = [n_{ij}^k(t)]$, $M_k(t) = [m_{ij}^k(t)] \in PC_{t_0}^0$ with $n_{ij}^k(t) \le m_{ij}^k(t)$ for all $t \in [t_0, \infty]$. The vector function $\phi(0)$ is an element of Banach space $C_n([-\tau,0])$ and $\tau_k \le \bar{\tau}_k \le \bar{\tau} < \infty$, $k=0,1,...,m$ are uncertain time-invariant delays where $\bar{\tau} = \max\{\bar{\tau}_k : k=0,1,...,m\}$.

The time-invariant system associated with system (1) is of the form

$$\dot{x}(t) = A_0 x(t) + \sum_{k=1}^{m} A_k x(t-\tau_k) \tag{3}$$

where $A_k \in [N_k, M_k]$, for $k=0,1,...,m$.

Now define $B_k(t) = (N_k(t) + M_k(t))/2 = [b_{ij}^k]$, $H_k(t) = (M_k(t) - N_k(t))/2 = [h_{ij}^k]$ where k = 0,1,...,m.

and $E_k(t) = [E_1^k, ..., E_n^k] \in \mathbb{R}^{n\times n^2}$ such that each E_l^k, $l = 1,...n$ is an n × n array with entry $e_{ij}^k = \sqrt{h_{lj}^k}$ for $l = i$ and $e_{ij}^k = 0$ for $l \ne i$ where $i,j = 1,...n$. Also define $F_k(t) = [F_1^k, ..., F_n^k]^T \in \mathbb{R}^{n^2\times n}$ such that each F_l^k, $l = 1,...n$ is an n × n array with entry $f_{ij}^k = \sqrt{h_{il}^k}$ for $l = j$ and $e_{ij}^k = 0$ for $l \ne j$ where $i,j = 1,...n$. It is easy to verify that

$$E_k(t)E_k^T(t) = diag(\sum_{j=1}^{n} h_{1j}^k, ..., \sum_{j=1}^{n} h_{nj}^k) \tag{4}$$

and

$$F_k^T(t)F_k(t) = diag(\sum_{i=1}^{n} h_{i1}^k, ..., \sum_{i=1}^{n} h_{in}^k). \tag{5}$$

Let $\Sigma_k \in [-I_{n^2}, I_{n^2}]$, k=0,…m where I_{n^2} is $n^2\times n^2$ identity matrix. It is obvious that $\Sigma_k \in \mathrm{diag}(\varepsilon_{11}^k,...,\varepsilon_{1n}^k,...,\varepsilon_{n1}^k,...,\varepsilon_{nn}^k)$, such that $|\varepsilon_{ij}^k| \le 1$, $i,j = 1,...,n$ and furthermore $\Sigma_k^T \Sigma_k = \Sigma_k \Sigma_k^T \le I_{n^2}$, $k=0,1,...m$.

Let $\mathcal{N}[N_k(t), M_k(t)] = \{A_k(t) = B_k(t) + E_k(t)\Sigma_k F_k(t): \Sigma_k \in [-I_{n^2}, I_{n^2}]\}$, then we have the following lemma, [4, 12].

Lemma 3.1. [4] For $k=0,1,...,m$ the equalities $[N_k(t), M_k(t)] = \mathcal{N}[N_k(t), M_k(t)]$ always hold.

This lemma shows that the linear time-variant interval system (1) is equivalent to the following linear system subject to norm bounded structured uncertainties described by the equation

$$\dot{x}(t) = (B_0(t) + E_0(t)\Sigma_0 F_0(t))x(t) + \sum_{k=1}^{m}(B_k(t)+E_k(t)\Sigma_k F_k(t))x(t-\tau_k). \tag{6}$$

Correspondingly, associated with system (3) we have

$$\dot{x}(t) = (B_0 + E_0\Sigma_0 F_0)x(t) + \sum_{k=1}^{m}(B_k + E_k\Sigma_k F_k)x(t-\tau_k), \tag{7}$$

where $\Sigma_k \in [-I_{n^2}, I_{n^2}]$, k=0,…m.

Therefore, when one is looking for stability condition which depends on delay, the standard step is to replace the original systems (1) and (3) by the systems (6) and (7), [6, 5, 9].

Also, the following lemmas are essential for the proof of the main theorem, [2].

Lemma 3.2. [2] Let $\omega(t) = \int_{a(t)}^{b(t)} \int_{t-\theta}^{t} f(s)ds d\theta$ Then the following is satisfied,

$$\frac{d}{dt}\omega(t) = (b-a)f(t) - (1-\dot{b})\int_{t-b}^{t-a} f(s)ds + (\dot{b}-\dot{a})\int_{t-a}^{t} f(s)ds \tag{8}$$

Since $x(t)$ is continuously differentiable for $t \ge 0$, by using the Leibnitz-Newton formula we have

$$x(t-\tau) = x(t) - \int_{-\tau}^{0}\dot{x}(t+\theta)d\theta = x(t) - \int_{-\tau}^{0} Ax(t+\theta)+A_d x(t+\theta-\tau)d\theta$$

which is used in reference [9] to transfer the system

$$\dot{x}(t) = Ax(t) + A_d x(t-\theta), \tag{9}$$

into the distributed delay system,

$$\dot{x} = (A+C)x(t) + (A_d - C)x(t-\tau)$$

$$\int_{-\tau}^{0} Ax(t+\theta)+A_d x(t+\theta-\tau)d\theta, \tag{10}$$

where C is a parametric matrix which derives the stability less restrictive to some degree. Since in this process only one integration over one delay interval is used, the process is called parameterized first-order model transformation.

The stability of (10) implies the stability of the system (9) for all $\tau\in[0,\bar{\tau}]$, see [9] and references therein.

By applying the above model transformation to the system (6), we have

$$\dot{x}(t) = [B_0(t) + E_0(t)\Sigma_0 F_0(t) + \sum_{k=1}^{m} C_k]x(t) +$$
$$\sum_{k=1}^{m} [B_k(t) + E_k(t)\Sigma_k F_k(t) - C_k]x(t-\tau_k) -$$
$$\sum_{k=1}^{m} C_k \int_{t-\tau_k}^{t} [(B_0(\theta) + E_0(\theta)\Sigma_0 F_0(\theta))x(\theta) + \qquad (11)$$
$$\sum_{k=1}^{m} (B_k(\theta) + E_k(\theta)\Sigma_k F_1(\theta))x(\theta - \tau_k)]d\theta.$$

The stability of this system implies the stability of the system (6). Therefore we focus on stability of the last one.

Theorem 3.3. The interval system (6) is robust asymptotically stable for any $\tau_i \in [0, \bar{\tau}_i]$, $i = 1,...,m$ with $A_k(t) \in [N_k(t), M_k(t)]$, $k = 0,1,...,m$, if there exist positive constant scalars λ_j, α_{ij}, positive definite matrices $P = P^T > 0$, $R_i = R_i^T > 0$, $Q_{ij} = Q_{ij}^T > 0$, and constant matrix $W_i \in \mathbb{R}^{n\times n}$, for $i, j = 1,...m$ such that

$$\Omega_1 = PB_0(t) + B_0^T(t)P + \lambda_0 F_0^T(t)F_0(t) +$$
$$\sum_{k=1}^{m} W_k + W_k^T + b_k S_k(t) f_k \tau_k S_{k0}(t) +$$
$$\sum_{i=1}^{m}\sum_{j=1}^{m} \frac{\bar{\tau}_j}{L_{ij}} W_i Q_{ij}^{-1} W_i^T + \bar{\tau}_j L_{ij} S_{ij}(t) +$$
$$\sum_{i=1}^{m}\sum_{j=1}^{m} \frac{1}{\alpha_{ij} L_{ij}} \int_{t-\bar{\tau}_i}^{t} W_i E_j(\lambda) E_j^T(\lambda) W_i^T d\lambda +$$
$$\sum_{i=1}^{m} (\frac{1}{b_i}[(PB_i(t) - W_i)R_j^{-1}(PB_i(t) - W_i)^T +$$
$$\frac{1}{\lambda_i} PE_i(t)E_i^T(t)P] + \frac{\bar{\tau}_j}{f_i} W_i Q_{i0}^{-1} W_i^T) +$$
$$\sum_{i=1}^{m} \frac{1}{\alpha_{i0} f_i} \int_{t-\bar{\tau}_i}^{t} W_i E_0^T(\lambda) W_i^T d\lambda + \frac{1}{\lambda_0} PE_0(t)P < 0$$

for $i, j = 1,...m$ where

$$S_{i0}(t) = B_0^T(t)Q_{i0}B_0(t) + \alpha_{i0}F_0^T(t)F_0(t) \qquad (12)$$
$$S_{ij}(t) = B_j^T(t+\bar{\tau}_i)Q_{ij}B_j(t+\bar{\tau}_j) + \alpha_{ij}F_j^T(t+\bar{\tau}_j)F_j(t+\bar{\tau}_j) \qquad (13)$$
$$S_i(t) = R_i + \lambda_i F_i^T(t+\bar{\tau}_i)F_i(t+\bar{\tau}_i), \qquad (14)$$

b_0, b_1, b_2 are any positive constant, d is any real constant and the corresponding model transformation matrices in (11) is given by $C_i = P^{-1}W_i$.

Proof. Consider the Lyapunov-Krasovskii functional defined as follows

$$V(x(t)) = x^T(t)Px(t) + \sum_{i=1}^{m} b_i \int_{t-\bar{\tau}_i}^{t} x^T(\theta)S_i(\theta)x(\theta)d\theta +$$
$$\sum_{i=1}^{m} f_i \int_{-\tau_i}^{t} \int_{t+\lambda}^{t} x^T(\theta)S_{i0}(\theta)x(\theta)d\theta + \qquad (15)$$
$$\sum_{i=1}^{m}\sum_{j=1}^{m} L_{ij} \int_{-\tau_i-\tau_j}^{\tau_j} \int_{t+\lambda}^{t} x(\theta)S_{ij}(\theta)x^T(\theta)d\theta d\lambda,$$

where b_i, f_i, L_{ij} for $i, j = 1,...,m$ are arbitrary constant coefficients.

The time-derivative of this functional along with the positive half trajectories of the systems (11), can be expressed as follows:

$$\dot{V}(x(t)) = x^T(t)(PB_0(t) + B_0^T(t)P + \sum_{i=1}^{m}(W_i + W_i^T))x(t) +$$
$$2x^T(t)PE_0(t)\Sigma_0 F_0(t)x(t) +$$
$$2\sum_{i=1}^{m} x^T(t)(PB_i(t) - W_i)x(t-\tau_i) +$$
$$2\sum_{i=1}^{m} x^T(t)(PE_i(t)\Sigma_i F_i(t))x(t-\tau_i) -$$
$$2\sum_{i=1}^{m} \int_{t-\tau_i}^{t} x^T(t)W_i B_0(\lambda)x(\lambda)d\lambda -$$
$$2\sum_{i=1}^{m} \int_{t-\tau_i}^{t} x^T(t)W_i E_0(\lambda)\Sigma_0 F_0(\lambda)x(\lambda)d\lambda -$$
$$2\sum_{i=1}^{m}\sum_{j=1}^{m} \int_{t-\tau_i}^{t} x^T(t)W_i B_j(\lambda)x(\lambda-\tau_j)d\lambda -$$
$$2\sum_{i=1}^{m}\sum_{j=1}^{m} \int_{t-\tau_i}^{t} x^T(t)W_i E_j(\lambda)\Sigma_j F_j(\lambda)x(\lambda-\tau_j)d\lambda +$$
$$x^T(t)(\sum_{i=1}^{m} b_i S_i(t))x(t) - \qquad (16)$$
$$\sum_{i=1}^{m} b_i x^T(t-\tau_i)S_i(t-\tau_i)x(t-\tau_i) -$$
$$\sum_{i=1}^{m} f_i \tau_i x^T(t)S_{i0}x(t) - \sum_{i=1}^{m} f_i \int_{t-\tau_i}^{t} x^T(\lambda)S_{i0}(\lambda)x(\lambda)d\lambda -$$
$$\sum_{i=1}^{m}\sum_{j=1}^{m} L_{ij} \int_{t-\tau_i-\tau_j}^{t-\tau_j} x^T(\lambda)S_{ij}(\lambda)x(\lambda)d\lambda +$$
$$\sum_{i=1}^{m}\sum_{j=1}^{m} \tau_i L_{ij} x^T(t)S_{ij}(t)x(t).$$

Using the following inequalities for any positive real number $\beta > 0$ and any positive definite matrix D,

$$-2u^T v \le 2u^T v \le \beta u^T D^{-1} u + \beta^{-1} v^T Dv,$$

where $u, v \in R^n$, [8, 11]. We have

$$2\sum_{i=1}^{m} x^T(t)(PB_i(t)-W_i)x(t-\tau_i) \le$$
$$\sum_{i=1}^{m} b_i^{-1}x^T(t)(PB_i(t)-W_i)R_i^{-1}(PB_i(t)-W_i)^T x(t)+ \quad (17)$$
$$\sum_{i=1}^{m} b_i x^T(t-\tau_i)R_i^{-1}x(t-\tau_i).$$

$$2\sum_{i=1}^{m} x^T(t)PE_i(t)\Sigma_i F_i(t)x(t-\tau_i) \le$$
$$\sum_{i=1}^{m} \lambda_i^{-1}b_i^{-1}x^T(t)PE_i(t)E_i^T(t)Px(t)+ \quad (18)$$
$$\sum_{i=1}^{m} \lambda_i b_i x^T(t-\tau_i)F_i^T(t)F_i(t)x(t-\tau_i).$$

$$-2\sum_{i=1}^{m}\int_{t-\tau_i}^{t} x^T(t)W_i B_0(\lambda)x(\lambda)d\lambda \le$$
$$2\sum_{i=1}^{m} \tau_i f_i^{-1}x^T(t)W_i Q_{i0}^{-1}W_i^T x(t)+ \quad (19)$$
$$\sum_{i=1}^{m} f_i\int_{t-\tau_i}^{t} x^T(\lambda)B_0^T(\lambda)Q_{i0}B_0(\lambda)x(\lambda)d\lambda$$

$$-2\sum_{i=1}^{m}\int_{t-\tau_i}^{t} x^T(t)W_i E_0(\lambda)\Sigma_0 F_0(\lambda)x(\lambda)d\lambda \le$$
$$\sum_{i=1}^{m}\frac{1}{\alpha_{i0}f_0}\int_{t-\tau_i}^{t} x^T(t)W_i E_0(\lambda)E_0^T(\lambda)W_i^T x(t)d\lambda+ \quad (20)$$
$$\sum_{i=1}^{m} \alpha_{i0}f_i\int_{t-\tau_i}^{t} x^T(\lambda)F_0^T(\lambda)F_0(\lambda)x(\lambda)d\lambda.$$

$$-2\sum_{i=1}^{m}\sum_{j=1}^{m}\int_{t-\tau_i}^{t} x^T(t)W_i B_j(\lambda)x(\lambda-\tau_j)d\lambda \le$$
$$\sum_{i=1}^{m}\sum_{j=1}^{m}\frac{1}{L_{ij}}\tau_i x^T(t)Q_{ij}^{-1}W_i^T x(t)+$$
$$\sum_{i=1}^{m}\sum_{j=1}^{m} L_{ij}\int_{t-\tau_i}^{t} x^T(\lambda-\tau_j)B_j^T(\lambda)Q_{ij}B_j(\lambda)x(\lambda-\tau_j)d\lambda.$$
$$(21)$$

$$-2\sum_{i=1}^{m}\sum_{j=1}^{m}\int_{t-\tau_i}^{t} x^T(t)W_i E_j(\lambda)\Sigma_j F_j(\lambda)x(\lambda-\tau_j)d\lambda \le$$
$$\sum_{i=1}^{m}\sum_{j=1}^{m}\frac{1}{\alpha_{ij}L_{ij}}\int_{t-\tau_i}^{t} x^T(t)W_i E_j(\lambda)E_j^T(\lambda)W_i^T x(t)d\lambda+$$
$$\sum_{i=1}^{m}\sum_{j=1}^{m} \alpha_{ij}L_{ij}\int_{t-\tau_j}^{t} x^T(\lambda-\tau_j)F_j^T(\lambda)F_j(\lambda)x(t-\tau_j)d\lambda.$$
$$(22)$$

Substituting (17-22) into (16), we get,

$\dot V(x(t)) \le x^T(t)\Omega_1 x(t)$, where

$$\Omega_1 = PB_0(t)+B_0^T(t)+\lambda_0 F_0^T(t)F_0(t)+$$
$$\sum_{k=1}^{m} W_k + W_k^T + b_k S_k(t) f_k \tau_k S_k 0(t)+$$
$$\sum_{i=1}^{m}\sum_{j=1}^{m}\frac{\bar\tau_j}{L_{ij}}W_i Q_{ij}^{-1}W_i^T + \bar\tau_j L_{ij}S_{ij}(t)+$$
$$\sum_{i=1}^{m}\sum_{j=1}^{m}\frac{1}{\alpha_{ij}L_{ij}}\int_{t-\bar\tau_i}^{t} W_i E_j(\lambda)E_j^T(\lambda)W_i^T d\lambda+$$
$$\sum_{i=1}^{m}(\frac{1}{b_i}[(PB_i(t)-W_i)R_i^{-1}(PB_i(t)-W_i)^T+$$
$$\frac{1}{\lambda_i}PE_i(t)E_i^T(t)P]+\frac{\bar\tau_j}{f_i}W_i Q_{i0}^{-1}W_i^T)+$$
$$\sum_{j=1}^{m}\frac{1}{\alpha_{i0}f_i}\int_{t-\bar\tau_i}^{t} W_i E_0(\lambda)E_0^T(\lambda)W_i^T d\lambda+$$
$$\frac{1}{\lambda_0}PE_0(t)E_0(t)P.$$

Since $\Omega_1 < 0$, it is easy to show that $\dot V(x(t)) < 0$ if $x(t) \ne 0$ and $\dot V(x(t)) = 0$, if and only if x(t) = 0. Therefore by Lyapunov-Krasovskii stability theorem, the origin of the system (11) is robust asymptotically stable for $A_k(t) \in [N_k(t), M_k(t)]$, $k = 0,1,...,m$, and $\tau_i \in [0, \bar\tau_i]$, consequently the origin of the system (6) is robust asymptotically stable which completes the proof.

In the above theorem, if we let $m = 1, W = W1$, $S_0 = \bar\tau Q_{10}$, $S_1 = \bar\tau Q_{11}$, $R = R_1$, $\tau = \tau_1$, $\alpha_0 = \alpha_{10}$, $\alpha_1 = \tau\alpha_{11}$ then the following result is immediate.

Corollary 3.4. System (1) with m = 1 is robust asymptotically stable for any $A_k(t) \in [N_k(t), M_k(t)]$, $k = 0,1$ if there exist constant scalers $\lambda_i > 0, \alpha_i > 0$, symmetric and positive definite matrices $P = P^T > 0$, $R_i = R_i^T > 0$, $S_i = S_i^T > 0$ for $i = 0,1$ and constant matrix $W \in \mathbb{R}^{n\times n}$ such that

$$\Omega_{11} = PB_0(t)+B_0^T(t)P+W+W^T+b_1R+$$
$$(\lambda_0+f_1\alpha_0)F_0^T(t)F_0(t)+$$
$$(b_1\lambda_1+L_{11}\alpha_1)F_1^T(t+\bar\tau)F_1(t+\bar\tau)+$$
$$L_{11}B_1^T(t+\bar\tau)S_1B_1(t+\bar\tau)+f_1B_0^T(t)S_0B_0(t)+$$
$$\bar\tau^2 W(\frac{S_1^{-1}}{L_{11}}+\frac{S_0^{-1}}{f_1})W^T+\frac{1}{\lambda_i}PE_1(t)E_1^T(t)P+$$

$$\frac{1}{\lambda_0} P E_0(t) E_0^T P + \frac{1}{b_1}(PB_1(t) - W) R^{-1} (PB_1(t) - W)^T +$$
$$\bar{\tau}\int_{t-\bar{\tau}}^{t} W (f_1^{-1}\alpha_0^{-1} E_0(\lambda) E_0^T(\lambda) +$$
$$L_{11}^{-1}\alpha_1^{-1} E_1(\lambda) E_1^T(\lambda)) W^T d\lambda < 0.$$

The corresponding model transformation matrix is given by $C = P^{-1} W$.

4. Example

Consider the same interval system as given in [12],

$$\dot{x}(t) = A_0(t) x(t) + A_1(t) x(t - \tau),$$

where $A_0(t) = \Lambda_0 + \varepsilon_0 \sin^2 t \cdot I_2$, $A_1(t) = \Lambda_1 + \varepsilon_1 \cos^2 t \cdot I_2$, such that Λ_0, Λ_1 are known 2×2 matrices, $\varepsilon_0, \varepsilon_1$ are uncertain but bounded as $|\varepsilon_0| \le 1, |\varepsilon_1| \le 1$.

It is easy to see that, $Ak(t) \in [N_k(t), M_k(t)]$, k = 0, 1 where

$$N_0(t) = \Lambda_0 - \sin^2 t \cdot I_2 \qquad M_0(t) = \Lambda_0 + \sin^2 t \cdot I_2$$
$$N_1(t) = \Lambda_1 - \cos^2 t \cdot I_2 \qquad M_1(t) = \Lambda_1 + \cos^2 t \cdot I_2 .$$

Hence, by assuming $B_0(t) = \Lambda_0,\ B_1(t) = \Lambda_1$, $H_0(t) = \sin^2 t \cdot I_2,\ H1(t) = \cos^2 t \cdot I_2$, and

$$E_0(t) E_0^T(t) = F_0(t) F_0^T(t) = \sin^2 t \cdot I_2$$
$$E_1(t) E_1^T(t) = F_1(t) F_1^T(t) = \cos^2 t \cdot I_2$$

We have

$$\Omega_{11} = P\Lambda_0 + \Lambda_0^T P + W + W^T + b_1 R +$$
$$(\lambda_0 + f_1\alpha_0)\sin^2 t \cdot I_2 + (b_1\lambda_1 + L_{11}\alpha_1)\cos^2 t \cdot I_2 +$$
$$L_{11}\Lambda_1^T S_1 \Lambda_1 + f_1 \Lambda_0^T S_0 \Lambda_0 + \bar{\tau}^2 W (S_1^{-1} L_{11}^{-1} + S_0^{-1} f_1^{-1}) W^T +$$
$$\frac{1}{\lambda_0} P \sin^2 t \cdot I_2 P + \frac{1}{\lambda_1} P \cos^2 t \cdot I_2 P +$$
$$\frac{1}{b_1}(P\Lambda_1 - W) R^{-1} (P\Lambda_1 - W)^T +$$
$$\tau^2 W \left(\frac{1}{f_1\alpha_0}\sin^2 \lambda . I_2 + \frac{1}{L_{11}\alpha_1}\cos^2 \lambda . I_2\right) W^T .$$

Also if we assume $b = b_1, f = f_1, L = L_{11}, 2\alpha = \alpha_0 = \alpha_1, 2\lambda = \lambda_0 = \lambda_1$, then we have

$$\Omega_{11} = P\Lambda_0 + \Lambda_0^T P + W + W^T + bR + (2\lambda + 2f\alpha) I_2 +$$
$$(2b\lambda + 2L\alpha) I_2 + L\Lambda_1^T S_1 \Lambda_1 + f\Lambda_0^T S_0 \Lambda_0 +$$
$$\bar{\tau}^2 W (S_1^{-1} L^{-1} + S_0^{-1} f^{-1}) W^T + \frac{1}{\lambda} P^2 +$$
$$\frac{1}{b}(P\Lambda_1 - W) R^{-1} (P\Lambda_1 - W)^T + \frac{\tau^2}{2\alpha}\left(\frac{1}{f} + \frac{1}{L}\right) W W^T .$$

For example let $b = 1/6$, $f = L = 1/2$ and assume

$$\Lambda_0 = \begin{bmatrix} -2 & 0 \\ 0 & -1.9 \end{bmatrix}, \quad \Lambda_1 = \begin{bmatrix} -0.5 & 0 \\ 0.1 & -0.5 \end{bmatrix}$$

Then one feasible solution for associated linear matrix inequality (LMI) is

$$p = diag(172.2344,\ 159.9684) \qquad R = diag(37.0500,\ 0.1602)$$
$$S_0 = diag(8.8125,\ 8.8647) \qquad S_1 = diag(34.9834,\ 30.8433)$$
$$W = diag(-83.7071, -79.8313) \qquad \lambda = 114.6665, \quad \alpha = 56.8986$$

Therefore, by substituting into the right hand side of inequality , Ω_{11} we get

$$\Omega_{11} \le \begin{bmatrix} -187.027570 + 2052.041992\tau^2 & 7.014642528 \\ 7.014642528 & -100.822578 + 1907.101854\tau^2 \end{bmatrix} < 0.$$

Therefore $\tau < 0.301897627$ and $\tau < 0.229928049$. Consequently the system is robust asymptotically stable for $\tau \in [0,\ 0.229928049]$ which is a larger domain for delay with respect to example 1 of reference [12].

5. CONCLUSION

In this paper, we have investigated the robust asymptotical stability issue of linear interval time variant systems with uncertain delays. A new delay dependent stability condition is derived by using the Lyapunov method, (LMI), parameterized first-order model transformation technique and introducing ingeniously real constants. Based on a present criterion, a new upper bound on the size of delays is presented. A numerical example is also provided to demonstrate the effectiveness of the new result.

6. References

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