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Eigenvalue Assignment Of Discrete-Time Linear Systems With State And Input Time-Delays

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ABSTRACT

Time-delays are important components of many dynamical systems that describe coupling or interconnection between dynamics, propagation or transport phenomena, and heredity and competition in population dynamics. The stabilization with time delay in observation or control represents difficult mathematical challenges in the control of distributed parameter systems. It is well-known that the stability of closed-loop system achieved by some stabilizing output feedback laws may be destroyed by whatever small time delay there exists in observation. In this paper a new method for eigenvalue assignment of discrete-time linear systems with state and input time-delays by static output feedback matrix is presented. The main result is an iterative method that only requires linear equations to be solved at each iteration. In this scheme, first a linear delayed system by defining an augmented vector is changed to standard form, then output feedback matrix K is calculated by inverse eigenvalue problem. We investigate all types of delays in the states, inputs or both for discrete – time linear systems. A simple algorithm and an illustrative example are presented to show the advantages of this new technique.

KEYWORDS

Eigenvalue assignment; Inverse eigenvalue problem; Output feedback; Time-delay system;

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1. INTRODUCTION

In many physical and biological phenomena, the rate of variation in the system state depends on past states. This characteristic is called a delay or a time delay, and a system with a time delay is called a time-delay system. Time delay phenomena were first discovered in biological systems and were later found in many engineering systems, such as mechanical transmissions, fluid transmissions, metallurgical processes, and networked control systems. They are often a source of instability and poor control performance.

Time delays commonly occur in many mechanical and electrical systems in the path between system inputs and system output and are considered as a major source of instability and poor performance of a system. Time delays can include measurement delays, transmission delays and calculation delays [4].

Time-delay systems have attracted the attention of many researchers because of their importance and widespread occurrence. Basic theories describing such systems were established in the 1950s and 1960s; they covered topics such as the existence and uniqueness of solutions to dynamic equations, stability theory for trivial solutions, etc. That work laid the foundation for the later analysis and design of time-delay systems.

The robust control of time-delay systems has been a very active field for the last 20 years and has spawned many branches, for example, stability analysis, stabilization design, H_∞ control, passive and dissipative control, reliable control, guaranteed-cost control, H_∞ filtering, Kalman filtering, and stochastic control. Regardless of the branch, stability is the foundation. So, important developments in the field of time-delay systems that explore new directions have generally been launched from a consideration of stability as the starting point.

Stability is a very basic issue in control theory and has been extensively discussed in many monographs. Research on the stability of time-delay systems began in the 1950s, first using frequency-domain methods and later also using time-domain methods. Frequency-domain methods determine the stability of a system from the distribution of the roots of its characteristic equation or from the solutions of a complex Lyapunov matrix function equation.

Time-delay in the feedback loop of control systems often leads to instability or poor performance of the system, therefore, eigenvalue placing of delayed systems is a crucial problem in modern control theory. Classical eigenvalue placement techniques of ordinary differential equations cannot be applied for delayed systems, since the number of eigenvalues to be controlled is much larger than the degrees of freedom in the controller. Although, complete eigenvalue placement is usually not possible for delayed systems, finding the optimal control parameters that result in the smallest spectral radius is still a difficult task.

Since the time-delay systems have important rule in static sciences, many researchers have studied and proposed various methods of eigenvalue assignment for this systems. Initial works for input time-delay was done by Kurzweil in 1963. Koepcke, later introduced the method of augmention of the state vector for time-delayed systems in 1965 and after him, a number of researchers have used Koepcke's technique for the control of these systems. As example, a survave of robustness of the eigenvalue assignment with output feedback matrix was done by Li (2001). Dong and Wei (2012) investigated stabilizatin and Determined feedback matrix in linear and nonlinear discrete time-delay systems. Li (2000) converted the output feedback eigenvalue assignment problem to a more general matrix inverse eigenvalue problem and then obtained solutions by solving a system of bilinear equations using Newton type algoritms. Xia, Liu, Shi, Rees, and Thomas (2007) investigated the stability of discrete-time systems with a constant delay by using a lifting method. Based on the scaled small gain theorem, new stability criteria were proposed in the paper (Li & Gao, 2011) in terms of linear matrix inequalities in combination with an approximation on the state delay.

Linear multivariable discrete-time systems with time-delays fall into three categories. The first category comprises systems in which the states are delayed by the same or different amounts, and are referred to as state-delayed systems; the second category comprises systems in which all the inputs are delayed by the same or different amount to an integer sub-multiple of the time-delay, and are referred to as input-delayed systems. The third category comprises systems in which the states and inputs are delayed by the same or different amounts. in this paper, we introduce a new method of eigenvalue assignment for all tree cases of the time delay system and describe them in three items, separately.

The aim of this research is to construct output feedback matrix for augmented system so that the closed-loop system has the desirable and prescribed eigenvalues.

Karbassi and Tehrani (2002) have extended the method of Karbassi and Bell (1993) and introduced a new technique for the parameterizations of state feedback controllers in eigenvalue assignment. It has been shown that a set of non-linear system of equations can be generated from considering the characteristic polynomial of the corresponding vector companion form of the system matrix. Modarres and Karbassi (2009) describe a method for control of discrete- time linear systems with state and input time-delays.

Recently, Ahsani Tehrani obtained a method for localization of eigenvalues in small specified regions of complex plane by state feedback matrix.

In this paper, an efficient and novel technique is presented that is completely different from the existing methodologies. The approach is based on the matrix inverse eigenvalue problem that it dose not need to solve non-linear equations. Finally, the design technique is described in an algorithm and illustrated with an example.

2. Problem Statement

Three distinct cases will be considered.

(a)- State-delayed system. Consider a linear state-delayed multivariable controllable and observable system defined by the state and output equations

$$x(\mathrm{i}+1) = A_0 x(\mathrm{i}) + \sum_{j=1}^{r} A_j x(i-j) + Bu(\mathrm{i}) \tag{1}$$

$$\mathrm{y(i)} = C_0 x(\mathrm{i}) + \sum_{j=1}^{r} C_j x(i-j) \tag{2}$$

where $x \in R^n$ is state vector, $u \in R^m$ is input vector and $y \in R^l$ is output vector. It is assume that $1 \le m \le n$, A_0, $A_j \in R^{n\times n}$ for $j = 1,2,\dots,r$, $B \in R^{n\times m}$ and $C_0, C_j \in R^{l\times n}$ are open-loop, input and output matrices respectively.

Now, if we take the augmented state vector such as

$$x_1(i+1) = \begin{bmatrix} x(i+1) \\ x(i) \\ \vdots \\ x(i-(r-1)) \end{bmatrix} \tag{3}$$

then equation (1) may be expressed in the non-delayed form

$$x_1(\mathrm{i}+1) = \bar{A} x_1(\mathrm{i}) + \bar{B} u(\mathrm{i})$$

$$\mathrm{y(i)} = \bar{C} x_1(\mathrm{i}) \tag{4}$$

In which

$$\bar{A} = \begin{bmatrix} A_0 & A_1 & \cdots & A_r \\ I_n & 0 & \cdots & 0 \\ \vdots & \ddots & \cdots & \vdots \\ 0 & \cdots & I_n & 0 \end{bmatrix} \quad \bar{B} = \begin{bmatrix} B \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

$$\bar{C} = [C_0 \quad C_1 \quad \cdots \quad C_r] \tag{5}$$

(b)- input-delayed system. Consider a linear state-delayed multivariable controllable and observable system defined by the state and output equations

$$x(i+1) = A_0 x(i) + B_0 u(i) + \sum_{\vartheta=1}^{l} B_\vartheta u(i-\vartheta) \tag{6}$$

$$y(i) = \mathrm{Cx(i)} \tag{7}$$

where $A \in R^{n\times n}$, $B_0, B_\vartheta \in R^{n\times m}$ for $\vartheta = 1,2,\dots,l$ and $C \in R^{n\times r}$ are state, input and output constant matrices, respectively.

if we take the augmented state vector such as

$$x_1(i+1) = \begin{bmatrix} x(i+1) \\ u(i) \\ u(i-1) \\ \vdots \\ u(i-(l-2)) \\ u(i-(l-1)) \end{bmatrix} \tag{8}$$

then equation (6) may be expressed in the non-delayed form

$$x_1(\mathrm{i}+1) = \bar{A} x_1(\mathrm{i}) + \bar{B} u(\mathrm{i}) \tag{9}$$

$$\mathrm{y(i)} = \bar{C} x_1(\mathrm{i}) \tag{10}$$

wich that

$$\bar{A} = \begin{bmatrix} A_0 & B_1 & B_2 & \cdots & B_{l-1} & B_l \\ 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & I_m & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots I_m & 0 & 0 \\ 0 & 0 & 0 & \cdots & I_m & 0 \end{bmatrix} \qquad \bar{B} = \begin{bmatrix} B_0 \\ I_m \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix} \tag{11}$$

$$\bar{C} = [C \quad 0 \quad 0 \quad \cdots \quad 0 \quad 0]$$

(c)- state and input delayed system. Consider a linear state-delayed multivariable controllable and observable system defined by the state and output equations

$$x(i+1) = A_0 x(i) + \sum_{j=1}^{r} A_j x(i-j) + B_0 u(i) + \sum_{\vartheta=1}^{l} B_\vartheta u(i-\vartheta) \tag{12}$$

$$\mathrm{y(i)} = C_0 x(\mathrm{i}) + \sum_{j=1}^{r} C_j x(i-j) \tag{13}$$

if we take the augmented state vector such as

$$x_1(i+1) = \begin{bmatrix} x(i+1) \\ x(i) \\ x(i-1) \\ \vdots \\ x(i-(r-1)) \\ u(i) \\ u(i-1) \\ u(i-2) \\ \vdots \\ u(i-(l-1)) \end{bmatrix} \tag{14}$$

then equation (12) may be expressed in the non-delayed form

$$x_1(\mathrm{i+1}) = \bar{A}x_1(\mathrm{i}) + \bar{B}u(\mathrm{i}) \quad (15)$$

$$\mathrm{y(i)} = \bar{C}x_1(\mathrm{i}) \quad (16)$$

which that

$$\bar{A} = \begin{bmatrix} A_0 & A_1 & \cdots & A_{r-1} & A_r & B_1 & B_2 & \cdots & B_{l-1} & B_l \\ I_n & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & I_n & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & I_n & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 & I_m & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 & 0 & I_m & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & I_m & 0 \end{bmatrix} \quad \bar{B} = \begin{bmatrix} B_0 \\ 0 \\ 0 \\ \vdots \\ 0 \\ I_m \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \quad (17)$$

$$\bar{C} = [C_0 \quad C_1 \quad C_2 \quad \cdots \quad C_r \quad 0 \quad 0 \quad 0 \quad \cdots \quad 0] \quad (18)$$

The aim of eigenvalue assignment for the system given in (4),(9) and (15) is to design an output feedback controller matrix, K, producing a closed-loop system with a satisfactory response by shifting suitable eigenvalues from undesirable to desirable locations. We define control low as

$$u(i) = Ky(i) = K\bar{C}x_1(i) \quad (19)$$

The program is to obtain an output feedback K, such that eigenvalues of the closed-loop system $\Gamma = \bar{A} + \bar{B}K\bar{C}$ are in the desired spectrum $L = \{\lambda_1, \lambda_2, ..., \lambda_n\}$, where $\lambda_i \in \not\subset$ for i = 1,2, ..., n, such that this closed-loop systems presents a suitable performance.

A method to solve eigenvalue assignment can then be found for the system given in (4),(9) and (15) by any standard techniques such as that of Karbassi and saadatjou [6], who have developed a method for obtaining parameterized output feedback controllers with linear parameters for time optimal control of discrete-time systems.

3. Inverse Eigenvalue Problem

Definition 1. The matrix inverse eigenvalue problem is that given four linearly independent sets of real n-vectors

$$\{x_1, x_2, \ldots, x_p\}, \{x_{p+1}, x_{p+2}, \ldots, x_{p+q}\},$$

$$\{y_1, y_2, \ldots, y_p\}, \{y_{p+1}, y_{p+2}, \ldots, y_{p+q}\}.$$

With $p + q \le n$ and a set of complex numbers $L = \{\lambda_1, \lambda_2, ..., \lambda_n\}$, find a real n × n matrix Γ such that

$$\Gamma x_i = y_i, \quad \mathrm{i = 1,2,...,p} \quad (20)$$

$$\Gamma^t x_j = y_j, \quad \mathrm{j=p+1,...,p+q} \quad (21)$$

and the spectrum of Γ is L. where we assume that the set L is closed under complex conjugation, i.e.

$$\lambda \in L \Leftrightarrow \bar{\lambda} \in L. \quad (22)$$

Let

$$X_r = \left[x_1, x_2, \ldots, x_p\right], X_l = \left[x_{p+1}, x_{p+2}, \ldots, x_{p+q}\right],$$

$$Y_r = \left[y_1, y_2, \ldots, y_p\right], Y_l = \left[y_{p+1}, y_{p+2}, \ldots, y_{p+q}\right].$$

Clear that if the matrix Γ of the problem exists, the following consistency condition must be satisfied

$$X_l^t Y_r = Y_l^t X_r. \quad (23)$$

We have following theorem. [2]

Theorem 1. If the matrix inverse eigenvalue problem satisfies the consistency condition Equation (9), then the necessary and sufficient condition for the existence of the matrix Γ is that there are vectors $u_i \in \int_u^i$ and $v_i \in \int_v^i$, i=1,2,…,n, such that

$$u_i^t v_j = \sigma_{ij}, \quad i, j = 1, 2, \ldots, \mathrm{n} \quad (24)$$

Where $\int_u^i$ and $\int_v^i$ are the null spaces $(\lambda_i X_l^t - Y_l^t)$ and $(\lambda_i X_r^t - Y_r^t)$ respectively. If such u_i exist, then Γ can be obtained using the equation

$$\Gamma = T^{-1} diag\{\lambda_1, \lambda_2, \ldots, \lambda_n\} T \quad (25)$$

$$T = \begin{bmatrix} u_1 & u_2 & \cdots & u_n \end{bmatrix}$$

$$T^{-1} = \begin{bmatrix} v_1 & v_2 & \cdots & v_n \end{bmatrix} \quad (26)$$

Let the base vectors of $\int_u^i$ and $\int_v^i$ form the matrices S_u^i and S_v^i respectively. Then the vectors u_i and v_i can be expressed as

$$u_i = S_u^i z_i \quad (27)$$

$$v_i = S_v^i w_i \quad (28)$$

where z_i and w_i are column vectors with the same dimensions as the spaces $\int_u^i$ and $\int_v^i$ respectively.

Then from Equation (24) we have

$$z_i^t (S_u^i)^t S_v^j w_j = \sigma_{ij} \quad i, j = 1, 2, \ldots, \mathrm{n} \quad (29)$$

This Equation can be solve with the iterative method. Briefly, at first we assign some initial values to all w_i, then the n-systems of linear equations can be solve easily [5].

In this paper, we consider $\Gamma = \bar{A} + \bar{B}K\bar{C}$ and V_1 and U_1 matrices formed by the base vectors of the null spaces of $\bar{B}^t$ and $\bar{C}$ respectively. Then we have

$$\Gamma V_1 = (\bar{A} + \bar{B}K\bar{C})V_1 = \bar{A}V_1 \tag{30}$$

$$U_1^t\Gamma = U_1^t(\bar{A} + \bar{B}K\bar{C}) = U_1^t\bar{A} \tag{31}$$

Let $X_l = U_1$, $X_r = V_1$, $Y_l = \bar{A}^t U_1$ and $Y_r = \bar{A}V_1$,

Obviously the problem satisfies the consistency condition as

$$X_l^t Y_r = U_1^t \Gamma V_1 = Y_l^t X_r \tag{32}$$

The matrices U_1 and V_1 can be obtained through QR decompositions for $\bar{B}$ and $\bar{C}$:

$$\bar{B} = [U_0 \;\; U_1]\begin{bmatrix} R \\ 0 \end{bmatrix},\; \bar{C} = \begin{bmatrix} S & 0 \end{bmatrix}\begin{bmatrix} V_0^t \\ V_1^t \end{bmatrix} \tag{33}$$

where $[U_0 \;\; U_1]$ and $[V_0 \;\; V_1]$ are orthogonal matrices.

According to Theorem(1) we can find Γ. If such Γ exists, the matrix K can be computed through the equation

$$K = \bar{B}^+(\Gamma - \bar{A})\bar{C}^+ \tag{34}$$

where $\bar{B}^+$ and $\bar{C}^+$ are the Moor-penrose generalized inverse of $\bar{B}$ and $\bar{C}$ respectively. From the equations $\bar{B}^+ = R^{-1}U_0^t$ and $\bar{C}^+ = V_0 S^{-1}$ we can show that K is the required matrix. i.e.

$$\begin{aligned} \bar{A} + \bar{B}K\bar{C} &= \bar{A} + \bar{B}\bar{B}^+(\Gamma - \bar{A})\bar{C}^+\bar{C} \\ &= \bar{A} + U_0U_0^t(\Gamma - \bar{A})V_0V_0^t \\ &= \bar{A} + (I - U_1U_1^t)(\Gamma - \bar{A})(I - V_1V_1^t) = \Gamma \end{aligned}$$

This method is generally solved and when $\bar{B}$ and $\bar{C}$ are of full rank and rank($\bar{B}$) + rank($\bar{C}$) ≥ rank($\bar{A}$), we can expect a solution with probability 1 for a given set L.

4. Algorithm

Object: To obtain output feedback matrix K, for which the eigenvalues of the closed loop systems are located in a prescribed spectrum.

Input: the matrices $A_0, A_j \in R^{n\times n}$ and $C_0, C_j \in R^{l\times n}$ for $j = 1,2,\ldots,r$ and $B_0, B_\vartheta \in R^{n\times m}$ for $\vartheta = 1,2,\ldots,l$ and the eigenvalue spectrum $L = \{\lambda_1, \lambda_2, \ldots, \lambda_n\}$

Output: The output feedback matrix K , such that the eigenvalues of closed-loop system fall into the prescribed spectrum.

Step 1: define the augmented state vector $x_1(i+1)$ and then Calculate $\bar{A}$, $\bar{B}$ and $\bar{C}$, in order to a linear delayed system by defining an augmented vector is changed to standard form.

Step 2: Obtain X_l and X_r that are null space of B_n^t and C respectively, then calculate $Y_l = \bar{A}^t U_1$, $Y_r = \bar{A}X_r$ and then null space of $(\lambda_i X_l - Y_l)$ and $(\lambda_i X_r - Y_r)$.

Step 3: Obtain $Z = [z_1 \; z_2 \; \cdots \; z_n]$ and $W = [w_1 \; w_2 \; \cdots \; w_n]$.

Step 4: Calculate matrices T, Γ then obtain K.

5. Illustrative Examples

Example 1: Consider a discrete-time system with a single state delay, the problem is to find the output feedback controller matrix K for assigning the eigenvalue spectrum L={−0.3, -0.1, 0, 0.1, 0.3, 0.5} to the delay system.

$$x(i+1) = \begin{bmatrix} 1 & 0 & 1 \\ 0 & -1 & 1 \\ 0 & 0 & 2 \end{bmatrix} x(i) + \begin{bmatrix} 2 & 0 & 1 \\ -1 & 2 & -3 \\ 0 & 3 & 4 \end{bmatrix} x(i-1) + \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 0 & 2 \end{bmatrix} u(i)$$

$$y(i) = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 2 & -3 \end{bmatrix} x(i) + \begin{bmatrix} 1 & -4 & 0 \\ 1 & 0 & 5 \end{bmatrix} x(i-1)$$

Step 1: Inter matrices A, B, C and eigenvalues and Calculate $\bar{A}$, $\bar{B}$ and $\bar{C}$.

$$\bar{A} = \begin{bmatrix} 1 & 0 & 1 & 2 & 0 & 1 \\ 0 & -1 & 1 & -1 & 2 & -3 \\ 0 & 0 & 2 & 0 & 3 & 4 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \quad \bar{B} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 0 & 2 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\bar{C} = \begin{bmatrix} 1 & 0 & -1 & 1 & -4 & 0 \\ 0 & 2 & -3 & 1 & 0 & 5 \end{bmatrix}$$

Step 2: Obtain X_l and X_r that are null space of B_n^t and C respectively, then calculate $Y_l = \bar{A}^t U_1$, $Y_r = \bar{A}X_r$ and then null space of $(\lambda_i X_l - Y_l)$ and $(\lambda_i X_r - Y_r)$.

$$X_l = \begin{bmatrix} 0.8944 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -0.4472 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$X_r = \begin{bmatrix} -0.2517 & 0.2353 & -0.9087 & 0.0408 \\ -0.4414 & 0.1177 & 0.1769 & 0.8094 \\ 0.6369 & 0.1469 & -0.1551 & 0.5405 \\ 0.1517 & 0.9095 & 0.2398 & -0.1530 \\ -0.1842 & 0.2495 & -0.1285 & -0.1632 \\ 0.5284 & -0.1408 & -0.2118 & 0.0311 \end{bmatrix}$$

$$Y_l = \begin{bmatrix} 0.8944 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1.7889 & 0 & 0 & 0 \\ -1.3416 & 0 & 0 & 0 \\ -0.8944 & 0 & 0 & 0 \end{bmatrix}$$

$$Y_r = \begin{bmatrix} 1.2171 & 2.0603 & -0.7960 & 0.3065 \\ -0.0269 & 0.0412 & -0.1934 & -0.5357 \\ 2.8347 & 0.4789 & -0.15426 & 0.7161 \\ -0.2517 & 0.2353 & -0.9087 & 0.0408 \\ -0.4414 & 0.1177 & 0.1769 & 0.8094 \\ 0.6369 & 0.1469 & -0.1551 & 0.5405 \end{bmatrix}$$

Step 3: Obtain $Z = [z_1\ z_2\ \cdots\ z_n]$ and $W = [w_1\ w_2\ \cdots\ w_n]$. [12]

$$Z = \begin{bmatrix} -0.0448 & 0 & 0 & 0 & 4.3395 & -5.1400 \\ -2.8456 & 0 & 0 & 0 & -2.5907 & 6.1160 \end{bmatrix}$$

$$W = \begin{bmatrix} 0.3667 & 0 & 0 & 0 & 0.1556 & 0.1188 \\ -0.4071 & 0 & 0 & 0 & -0.2186 & 0.0644 \end{bmatrix}$$

Step 4: Calculate matrices T, Γ then obtain K.

$$\Gamma = \begin{bmatrix} -1.5702 & 1.4998 & 0.7110 & 0.4248 & -0.3229 & -0.2087 \\ 0.3347 & 0.6482 & 0.1228 & 1.9264 & -1.4518 & -0.9571 \\ -5.1404 & 2.9995 & 1.4220 & -3.1505 & 2.3542 & 1.5827 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

$$K = \begin{bmatrix} 0.8601 & 0.4007 \\ -0.1173 & -0.0842 \end{bmatrix}$$

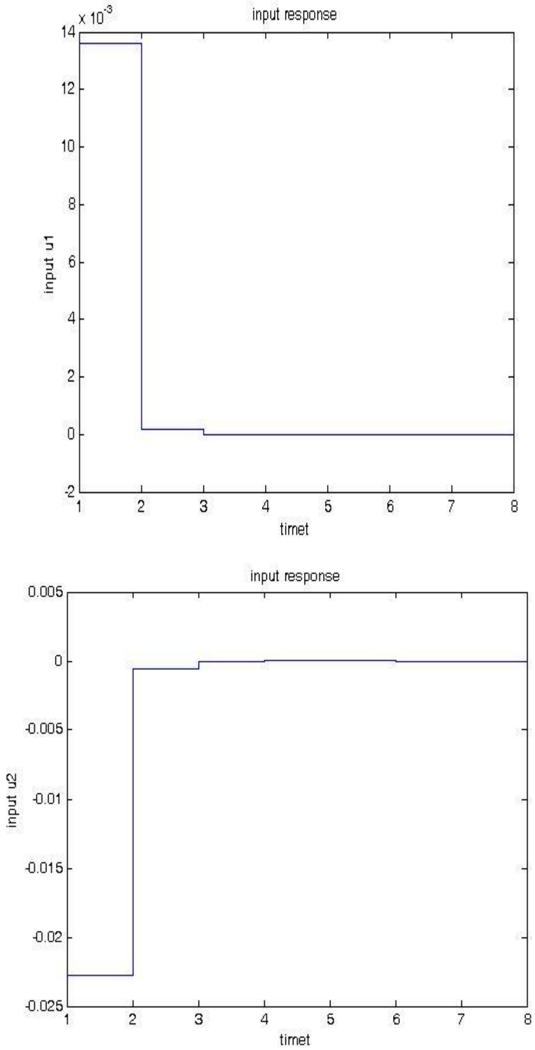

Fig. 1. The input vector Elements u(i) converge to zero.

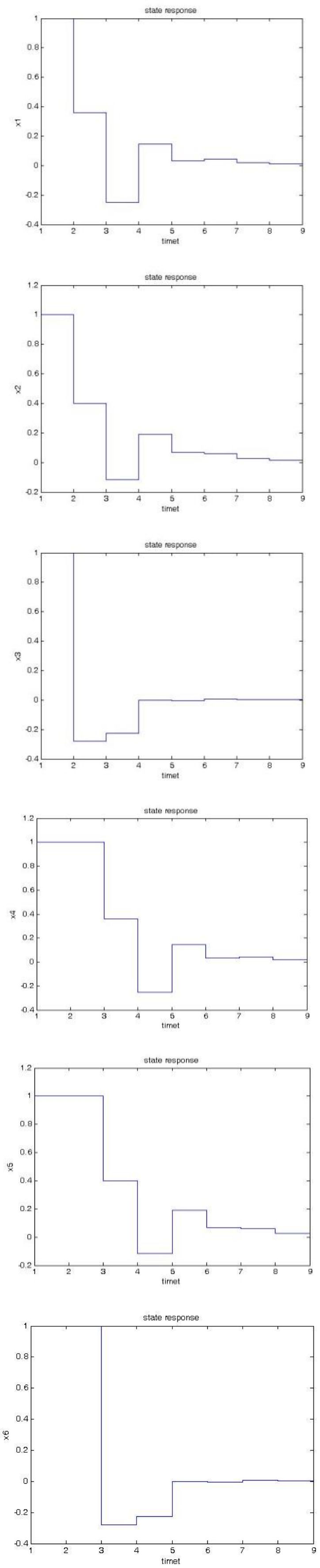

Fig. 2. The state vector Elements x(i) converge to zero.

Example 2: Consider a discrete-time system with a single input delay, the problem is to find the output feedback controller matrix K for assigning the eigenvalue spectrum L={−0.6, -0.4, -0.2, 0.3, 0.5} to the delay system.

$$x(i+1) = A_0\, x(i) + B_0 u(i) + B_1 u(i-1)$$

$$y(i) = \mathrm{Cx(i)}$$

$$x(i+1) = \begin{bmatrix} 2 & -1 & -3 \\ 5 & 1 & 3 \\ 7 & 2 & 1 \end{bmatrix} x(i) + \begin{bmatrix} 3 & 2 \\ 0 & 1 \\ -2 & 2 \end{bmatrix} u(i) + \begin{bmatrix} 2 & 1 \\ 1 & -3 \\ 5 & 2 \end{bmatrix} u(i-1)$$

$$y(i) = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 2 & -3 \end{bmatrix} x(i)$$

Step 1: Inter matrices A, B, C and eigenvalues and Calculate $\overline{A}$, $\overline{B}$ and $\overline{C}$.

$$\overline{A} = \begin{bmatrix} A_0 & B_1 \\ 0 & 0 \end{bmatrix} \qquad \overline{B} = \begin{bmatrix} B_0 \\ I_2 \end{bmatrix}$$

$$\overline{C} = \begin{bmatrix} C & 0 \end{bmatrix}$$

$$\overline{A} = \begin{bmatrix} 2 & -1 & -3 & 2 & 1 \\ 5 & 1 & 3 & 1 & -3 \\ 7 & 2 & 1 & 5 & 2 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad \overline{B} = \begin{bmatrix} 3 & 2 \\ 0 & 1 \\ -2 & 2 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\overline{C} = \begin{bmatrix} 1 & 0 & -1 & 0 & 0 \\ 0 & 2 & -3 & 0 & 0 \end{bmatrix}$$

Step 2: Obtain X_l and X_r that are null space of B_n^t and C respectively, then calculate $Y_l = \overline{A}^t U_1$, $Y_r = \overline{A} X_r$ and then null space of $(\lambda_i X_l - Y_l)$ and $(\lambda_i X_r - Y_r)$.

$$X_l = \begin{bmatrix} 0.1975 & -0.2212 & -0.1225 \\ -0.8829 & 0.1206 & -0.3208 \\ 0.3512 & 0.1463 & -0.1781 \\ 0.1099 & 0.9562 & 0.0111 \\ -0.2145 & 0.0293 & 0.9221 \end{bmatrix}$$

$$X_r = \begin{bmatrix} 0.4851 & 0 & 0 \\ 0.7276 & 0 & 0 \\ 0.4851 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$Y_l = \begin{bmatrix} -1.5612 & 1.1845 & -3.0961 \\ -0.3780 & 0.6344 & -0.5547 \\ -2.8901 & 1.1718 & -0.7733 \\ 1.2681 & 0.4095 & -1.4565 \\ 3.5487 & -0.2906 & 0.4838 \end{bmatrix}$$

$$Y_r = \begin{bmatrix} -1.2127 & 2 & 1 \\ 4.6082 & 1 & -3 \\ 5.3358 & 5 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Step 3: Obtain $Z = [z_1 \; z_2 \; \cdots \; z_n]$ and $W = [w_1 \; w_2 \; \cdots \; w_n]$. [12]

$$Z = \begin{bmatrix} 0.0012 & -0.0019 & -0.0087 & -0.0260 & 0.0051 \\ -0.0029 & 0.0102 & 0.0036 & -0.0195 & -0.0073 \end{bmatrix}$$

$$W = \begin{bmatrix} 23.5422 & -75.9008 & 29.3578 & -59.4249 & 64.0943 \\ -13.8739 & 62.9751 & 85.5516 & -20.8806 & -95.6316 \end{bmatrix}$$

Step 4: Calculate matrices T, Γ then obtain K.

$$T = \begin{bmatrix} -0.0013 & 0.0053 & -0.0008 & 0.0049 & 0.0010 \\ -0.0008 & 0.0030 & -0.0027 & -0.0213 & 0.0069 \\ -0.0009 & 0.0023 & -0.0086 & -0.0230 & -0.0018 \\ 0.0013 & -0.0029 & -0.0025 & -0.0032 & 0.0029 \\ -0.0022 & 0.0075 & 0.0004 & -0.0059 & -0.0043 \end{bmatrix}$$

$$\Gamma = \begin{bmatrix} 10.7791 & -25.7131 & -43.8291 & -19.1860 & 34.5415 \\ 3.0565 & -7.6128 & -11.9313 & -4.8958 & 8.6541 \\ 1.1100 & -3.6305 & -4.9537 & 10.3408 & 2.5442 \\ 0.0495 & -0.3664 & 0.4690 & -1.4396 & 0.1528 \\ 0.8680 & -3.4352 & 0.3584 & 7.0846 & -0.6063 \end{bmatrix}$$

$$K = \begin{bmatrix} 4.8878 & -0.1774 \\ 4.4936 & 0.1494 \end{bmatrix}$$

6. CONCLUSIONS

In this research, we investigate a new method with framework for explicit formulas for output feedback controllers with linear equations in arbitrary eigenvalue assignment for linear multivariable controllable and observable systems; which is based on the matrix inverse eigenvalue problem. There are many approaches for this problem. For example, [3,11,12] state that output feedback matrix can obtain from state feedback matrix under certain conditions. This method requires solving non-linear equations therefore is very costly in gain matrices. The method proposed in this paper does not require prior knowledge of the open-loop eigenvalues and the controller does not impose any restrictions on the position of the desired eigenvalues or their nature and multiplicity. so we can use it for discrete and continues linear systems. The error of method will be zero when $\overline{B}$ and $\overline{C}$ are invertible.

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