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# *Robust Fuzzy Gain-Scheduled Control of the 3-Phase IPMSM*

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**ABSTRACT**

This article presents a fuzzy robust Mixed - Sensitivity Gain - Scheduled $H_\infty$ controller based on the Loop - Shaping methodology for a class of MIMO uncertain nonlinear Time - Varying systems. In order to design this controller, the nonlinear parameter - dependent plant is first modeled as a set of linear subsystems by Takagi and Sugeno's (T - S) fuzzy approach. Both Loop - Shaping methodology and Mixed - Sensitivity problem are then introduced to formulate the frequency - domain specifications. Furthermore, a Regular Weights Selection Method (RWSM) is used to devise a systematic design for choosing properly the weighting matrices. Afterwards, for each linear subsystem, an $H_\infty$ controller is designed via linear matrix inequality (LMI) approach. Such controllers are said to be scheduled by the Time - Varying parameter measurements in real time. The Parallel Distributed Compensation (PDC) is then used to design the controller for the overall system and the total linear system is also obtained through using the weighted sum of the local linear subsystems. Several results show that the proposed method can effectively meet the performance requirements like robustness, good load disturbance rejection and tracking responses, and fast transient responses for the 3 - phase interior permanent magnet synchronous motor (IPMSM). Finally, the superiority of the proposed control scheme is approved in comparison with the feedback linearization controller, the $H_2/H_\infty$ Controller and the $H_\infty$ Mixed - Sensitivity controller methods.

**KEYWORDS**

Robust control, T- S Fuzzy Model, Gain - Scheduled Controller, Mixed- Sensitivity Problem, Time- Varying System, 3- Phase Interior Permanent Magnet Synchronous Motor (IPMSM).

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## 1- INTRODUCTION

In the recent years, IPMSM has been progressively replacing DC and induction motors in a wide range of drive many industrial applications such as: robotic actuators, rolling mills, integrated starters/alternators. The reason why an IPMSM has become so well - liked is essentially due to its many pleasant characteristics such as high efficiency, exceptional power density and excellent torque generating and high torque - to - current ratio. Numerous nonlinear and linear controllers have been developed for the IPMSM. In recent years, for getting acceptable drive performance, good position and speed tracking, disturbances attenuation and torque control many issues should be appropriately considered, C. - K. Lin et al. [1] used nonlinear position controller design with input - output feedback linearization technique for an IPMSM control system. S.S. Yang et al. [2] implemented a robust speed tracking of PMSM servo systems by equivalent disturbance attenuation. Y. X. Suet al.[3] employed the automatic disturbances rejection controller for precise motion control for a PMSM drive. Ming - Chang Chou et al.[4] proposed a development of robust current 2 - DOF controllers for a PMSM drive with reaction wheel load. Cheng - Kai Lin et al. [5] designed the adaptive back stepping PI sliding - mode control for interior permanent magnet synchronous motor drive systems. V. Azimi et al. [6] designed robust multi - objective $H_2/H_\infty$ tracking control based on T - S fuzzy model for a class of nonlinear uncertain drive systems. V. Azimi et al. [7] deal with position and current control of an interior Permanent - Magnet Synchronous Motor by using Loop - Shaping methodology: blending of $H_\infty$ Mixed - Sensitivity problem and T - S fuzzy model scheme. The Takagi - Sugeno (T - S) fuzzy model, which is often used in the literature, can approximate a wide class of nonlinear dynamic systems. The published papers employed the T - S fuzzy model technique to different drive systems [8, 12].

The main contribution of this research is to introduce fuzzy robust Mixed - Sensitivity Gain - Scheduled $H_\infty$ control for a class of MIMO uncertain nonlinear Time - Varying PM synchronous motor via T - S fuzzy model. The problem of Gain - Scheduled $H_\infty$ control for an IPMSM system which possesses not only parameter uncertainties but also external disturbances is first considered. A brief review over several robust $H_\infty$, Loop - Shaping and Mixed - Sensitivity problem schemes based on the employment of LMIs theory proposed in [13 - 15] is given. In the proposed method, the nonlinear plant is modeled by a properly developed Takagi - Sugeno (T - S) fuzzy model. The fuzzy model is described by fuzzy IF - THEN rules which represent local input - output relations of a nonlinear system. In this research, tracking of rotor angular position and electromagnetic torque are selected as the main objectives, in the event that reluctance effects and torque ripple are also reduced. The PDC approach is utilized to design the Gain - Scheduled $H_\infty$ controller for the overall system, as well as the overall fuzzy model of the system is achieved by fuzzy blending of the local linear system models. Eventually, several numerical results prove that the proposed method has the striking superiority against the former controller methods such as: feedback linearization, $H_2/H_\infty$ and $H_\infty$ Mixed - Sensitivity.

The rest of the paper is organized as follows. Problem statement and mathematical model of IPMSM are proposed in Section 2. In Section 3, design of robust Mixed - Sensitivity Gain - Scheduled $H_\infty$ controller is introduced. Simulation results of the closed - loop system with the proposed technique are presented in Section 4. Finally, the paper is concluded in Section 5.

## 2 - PROBLEM STATEMENT AND MATHEMATICAL MODEL OF IPMSM

### Problem Statement

The dynamics of IPMSM are nonlinear and may also contain uncertain and time - varying parameters such as viscous damping coefficient and stator windings resistance. Consequently, the control performance of IPM synchronous motors in various applications is highly touchy to the variations of external loads and system parameters. Whereas these variations are noxious factors in the IPMSM control system, accordingly in this article, an innovative robust angular position and electromagnetic torque control will be presented.

The major contributions of this study are:

- successful employment of a convenient T - S fuzzy model on behalf of the original nonlinear plant,
- successful design of feasible robust position and electromagnetic torque controller, based on a suitable T - S fuzzy model, in the presence of time - varying parameters and unknown load disturbances,
- successful development of the transient responses and disturbance attenuation of position and torque tracking, when the parameters in the system are varied in a wide range in the real time,
- successful robustness of the designed system when the parameters in the system dynamic are varied in a wide range over the time to ensure that all closed - loop performance specifications are satisfied in the presence of unavoidable model uncertainty,
- successful superiority of the proposed strategy in comparison with the former design procedures,
- successful responses of the electromagnetic torque control for various torque commands in order to overcome the different unknown load torque disturbances.

### B. Mathematical Model of IPMSM

The nonlinear time - varying electrical and mechanical equations for the 3 - phase interior permanent magnet synchronous motor in the d - q reference frame can be written as [1, 2, 6, 7] :

$$\frac{\partial\theta_r}{\partial t}=\omega_r$$
$$\frac{\partial\omega_r}{\partial t}=\frac{3}{2}\frac{\rho_0}{J_m}\left[\left(L_d-L_q\right)i_d+\varphi_f\right]i_q-\frac{B_m(t)}{J_m}\omega_r-\frac{C_l}{J_m}$$
$$\frac{\partial i_d}{\partial t}=-\frac{R_s(t)}{L_d}i_d+\rho_0\frac{L_q}{L_d}\omega_r i_q+\frac{1}{L_d}v_d$$
$$\frac{\partial i_q}{\partial t}=-\rho_0\frac{\varphi_f}{L_q}\omega_r-\rho_0\frac{L_d}{L_q}\omega_r i_d-\frac{R_s(t)}{L_q}i_q+\frac{1}{L_q}v_q \quad (1)$$

where

$$\underline{x}=\begin{bmatrix}\theta_r & \omega_r & i_d & i_q\end{bmatrix}^T$$
$$\underline{u}=\begin{bmatrix}v_d & v_q\end{bmatrix}^T$$

In this equation, $\theta_r$ is the angular position of the motor shaft, $\omega_r$ is the angular velocity of the motor shaft, $i_d$ is the direct current and $i_q$ is the quadrature current. $\varphi_f$ is the flux linkage of the permanent magnet, $\rho_0$ is the number of pole pairs, $R_s(t)$ is the stator windings resistance, $Ld$ and $Lq$ are the direct and quadrature stator inductances, respectively. $J_m$ is the rotor moment of inertia, $B_m(t)$ the viscous damping coefficient and $C_l$ is the load torque. $v_d$ is the direct voltage and $v_q$ is the the quadrature voltage. The electromagnetic torque of the motor can be described as

$$\tau_e=\frac{3}{2}\rho_0\left[\left(L_d-L_q\right)i_d i_q+\varphi_f i_q\right] \quad (2)$$

The time - varying electrical parameters $R_s(t)$ and $B_m(t)$ are supposed to differ from their nominal values $R_{so}$ and $B_{mo}$ [2] by

$$R_s(t)=R_{s0}\left(1+0.1\sin(2\pi t)\right)$$
$$B_m(t)=B_{m0}\left(1+0.1\sin(10\pi t)\right) \quad (3)$$

### C.T - S Fuzzy Model of IPMSM

In this section, the T - S fuzzy dynamic model is described by fuzzy IF - THEN rules, which represent the local linear input - output relations of the nonlinear systems [16, 17]. The fuzzy dynamic model is proposed by Takagi and Sugeno. The ith rule of the T - S fuzzy dynamic model with parametric uncertainties can be described as [18, 19]:

$$IF\ v_1(t) is\ M_{i1}\ and...and\ v_p(t) is\ M_{ip}\ THEN$$
$$\dot{x}(t)=\left[A_i+\Delta A_i\right]x(t)+\left[B_{1i}+\Delta B_{1i}\right]w(t)$$
$$+\left[B_{2i}+\Delta B_{2i}\right]u(t)$$
$$z(t)=\left[C_{1i}+\Delta C_{1i}\right]x(t)+\left[D_{11i}+\Delta D_{11i}\right]w(t)$$
$$+\left[D_{12i}+\Delta D_{12i}\right]u(t)$$
$$y(t)=\left[C_{2i}+\Delta C_{2i}\right]x(t)+\left[D_{21i}+\Delta D_{21i}\right]w(t)$$
$$+\left[D_{2\ i}+\Delta D_{2\ i}\right]u(t)$$
$$i=1,\dots,r \quad (4)$$

where, $M_{ip}$ is the fuzzy set, $r$ is the number of IF - THEN rules and $v_1\rightarrow v_p(t)$ are the premise variables,

$x(t) \in R^n$ is the state vector, $u(t) \in R^m$ is the control input vector, $w(t) \in R^q$ is the disturbance input vector, $y(t) \in R^p$ is the output vector. The matrices: $\Delta A_i$, $\Delta B_{1i}$, $\Delta B_{2i}$, $\Delta C_{1i}$, $\Delta C_{2i}$, $D_{11i}$, $D_{12i}$, $D_{21i}$ and $D_{2\,i}$ repre - sent the uncertainties in the system defined as:

$$\begin{aligned}
&\Delta A_i = G(x(t),t)H_{1i}\,;\Delta B_{1i} = G(x(t),t)H_{2i}\,;\\
&\Delta B_{2i} = G(x(t),t)H_{3i}\,;\Delta C_{1i} = G(x(t),t)H_{4i}\,;\\
&\Delta C_{2i} = G(x(t),t)H_{5i}\,;\Delta D_{11i} = G(x(t),t)H_{6i}\,;\\
&\Delta D_{12i} = G(x(t),t)H_{7i}\,;\Delta D_{21i} = G(x(t),t)H_{8i}\,;\\
&\Delta D_{21\,i} = G(x(t),t)H_{9i}
\end{aligned} \tag{5}$$

where $H_{ji}\,; j = 1,\ldots,9$ are known matrix functions which characterize the structure of the uncertainties. Furthermore, the following inequality holds

$$\left\|G(x(t),t)\right\| \le \sigma, \sigma > 0 .$$

Define an affine parameter - dependent system as:

$$\begin{aligned}
&E(\rho)\dot{x} = A(\rho)x + B_1(\rho)w + B_2(\rho)u\\
&z = C_1(\rho)x + D_1\ (\rho)w + D_{12}\,(\rho)u\\
&y = C_2(\rho)x + D_{21}\,(\rho)w + D_{22}\,(\rho)u\\
&S(\rho) = S_0 + \rho_1 S_1 + \ldots + \rho_n S_n\\
&\begin{bmatrix} A(\rho)+jE\,(\rho) & B(\rho)\\ C(\rho) & D(\rho)\end{bmatrix} = \begin{bmatrix} A_0 + jE_0 & B_0\\ C_0 & D_0\end{bmatrix} +\\
&\rho_1\begin{bmatrix} A_{\rho_1} + jE_{\rho_1} & B_{\rho_1}\\ C_{\rho_1} & D_{\rho_1}\end{bmatrix} + \ldots + \rho_n\begin{bmatrix} A_{\rho_n} + jE_{\rho_n} & B_{\rho_n}\\ C_{\rho_n} & D_{\rho_n}\end{bmatrix}\\
&B(\rho_i) = \begin{bmatrix} B_1(\rho) & B_2(\rho)\end{bmatrix}\\
&C(\rho_i) = \begin{bmatrix} C_1(\rho) & C_2(\rho)\end{bmatrix}^T\\
&D(\rho_i) = \begin{bmatrix} D_{11}\,(\rho) & D_{12}\,(\rho)\\ D_{21}\,(\rho) & D_{22}\,(\rho)\end{bmatrix}
\end{aligned} \tag{6}$$

Where $S_0, S_1,\ldots,S_n$ are some properly chosen system matrices; A(.) , B(.) , C(.) , D(.) and E(.) are the fixed affine functions of some vector $\rho = (\rho_1,\ldots,\rho_n)$; The parameters $\rho_i$ are uncertain parameters. In this paper, uncertain parameters are given by the following vector:

$$\rho = \begin{bmatrix}\rho_1 & \rho_2\end{bmatrix} = \begin{bmatrix} R_s(t) & B_m(t)\end{bmatrix} \tag{7}$$

Now, according to the definition of (6), uncertain parameters of (7) and the local linearization approach (4) the matrix $A_i$, at the ith selected operating point, can be decomposed as:

$$A_i = A_{0i} + B_m(t)A_{B_m} + R_s(t)A_{R_S} \tag{8}$$

$$A_{0i} = \begin{bmatrix} 0 & 1 & 0 & 0\\ 0 & 0 & \frac{3}{2}\frac{\rho_0}{J_m}\left(L_d - L_q\right)x_4 & \frac{3}{2}\frac{\rho_0}{J_m}\varphi_f\\ 0 & \rho_0\frac{L_q}{L_d}x_4 & 0 & 0\\ 0 & -\rho_0\frac{\varphi_f}{L_q} & -\rho_0\frac{L_d}{L_q}x_2 & 0\end{bmatrix}$$

$$A_{B_m} = \begin{bmatrix} 0 & 0 & 0 & 0\\ 0 & -\frac{1}{J_m} & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{bmatrix}$$

$$A_{R_S} = \begin{bmatrix} 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & -\frac{1}{L_d} & 0\\ 0 & 0 & 0 & -\frac{1}{L_q}\end{bmatrix}$$

$$B_1 = \begin{bmatrix} 0\\ -\frac{1}{J_m}\\ 0\\ 0\end{bmatrix}$$

$$B_2 = \begin{bmatrix} 0 & 0\\ 0 & 0\\ \frac{1}{L_d} & 0\\ 0 & \frac{1}{L_q}\end{bmatrix}$$

The above model (8) is the quasi - linear affine model of the original system dynamics of IPMSM (1) in which $A_{Oi}$, $A_{Bm}$, $A_{Rs}$, $B_1$, $B_2$ are known real matrices with appropriate dimensions easily constructed from the systems parameters given in the nonlinear model (1). Knowing the fact that $x_2$ and $x_4$ are the only nonlinear terms that appear in $A_{Oi}$, this matrix is merely needed to be localized by using the T - S appeoach in (4). The overall fuzzy model is then rewritten as:

$$\dot{x}(t) = \sum_{i=1}^{r}\mu_i(v(t)\ \left[\ \left[A_i + \Delta A_i\right]x(t) + \right.$$

$$\left[B_{1i}+\Delta B_{1i}\right]w(t)+\left[B_{2i}+\Delta B_{2i}\right]u(t)\Big],\; x(0)=0$$

$$z(t)=\sum_{i=1}^{r}\mu_i(v(t)\;\Big[\left[C_{1i}+\Delta C_{1i}\right]x(t)+$$

$$\left[D_{11i}+\Delta D_{11i}\right]w(t)+\left[D_{12\,i}+\Delta D_{12\,i}\right]u(t)\Big]$$

$$y(t)=\sum_{i=1}^{r}\mu_i(v(t)\;\Big[\left[C_{2i}+\Delta C_{2i}\right]x(t)+$$

$$\left[D_{21i}+\Delta D_{21i}\right]w(t)+\left[D_{2\;\,i}+\Delta D_{2\;\,i}\right]u(t)\Big]$$

$$i=1,\ldots,r \tag{9}$$

where $v(t)=[v_1(t)\ldots v_p(t)]$ is the premise variable vector and $\mu_i(v(t)$ denotes the normalized time varying fuzzy weighting functions for each rule defined by

$$\mu_i(v(t)\;=\frac{\varpi_i(v(t)}{\sum_{i=1}^{r}\varpi_i(v(t)}$$

$$\varpi_i(v(t)\;=\prod_{k=1}^{p}M_k\,(v_k(t) \tag{10}$$

And it should be noted that

$$\varpi_i(v(t)\;\geq 0;\sum_{i=1}^{r}\varpi_i(v(t)\;>0; i=1,\ldots,r$$

$$\mu_i(v(t)\;\geq 0;\sum_{i=1}^{r}\mu_i(v(t)\;=1;\; i=1,\ldots,r$$

## 3- DESIGN OF ROBUST MIMED - SENSITIVITY GAIN - SCHEDULED $H_\infty$ CONTROLLER

### Gain- Scheduled $H_\infty$ Controller Design

Gain - Scheduling is a widely used technique for controlling the classes of nonlinear time varying systems [20, 21]. Rather than seeking a single robust controller for the entire operating range, Gain - Scheduling consists of designing controllers for each operating point and switches between controllers as the operating conditions vary. To get a feeling for the Gain - Scheduled methodology, the closed - loop control structure is depicted in Fig. 1.

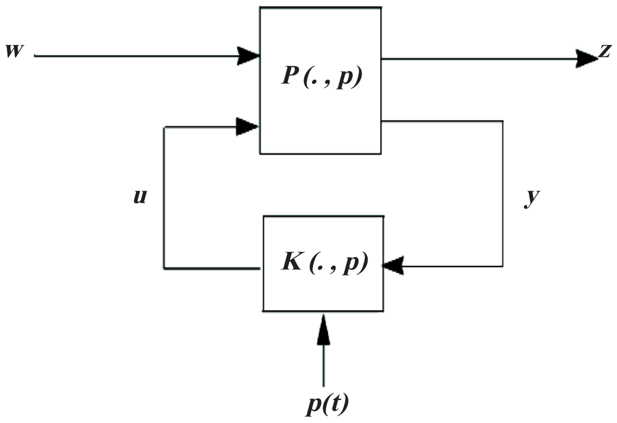



*Fig. 1. The control structure*

In this figure, P(. , ρ) is the parameter - dependent plant, K (. , ρ) is the Gain - Scheduled controller, the vector u represents the control signals, the vector y is the measured variables, w is the vector of exogenous signals and the vector z represents the main controlled variables. The plant P(. , ρ) is given in state - space form by

$$\dot{x}=A(\rho)x+B_1(\rho)w+B_2u$$
$$z=C_1(\rho)x+D_{11}(\rho)w+D_{12}u$$
$$y=C_2x+D_{21}w+D_{22}u$$
$$\rho(t)=(\rho_1(t),\ldots,\rho_n(t)\;\;;\underline{\rho}_i\leq\rho_i(t)\leq\overline{\rho}_i \tag{11}$$

where $\rho_i(t)$ are Time - Varying coefficients of the physical parameters that vary during operation, and A(.), $B_1$(.), $C_1$(.), $D_{11}$(.) are affine functions of $\rho(t)$. When these coefficients undergo large variations, it is often impossible to achieve high performance over the entire operating range with a single robust controller. Provided that the parameter values are measured in real time, it is then desirable to use the controllers that incorporate such measurements into the adjustment to the current operating conditions. Such controllers are said to be scheduled by the parameter measurements. This control strategy typically achieves a higher performance in the face of large variations in various operating conditions.

In this way, the main goal is to design a static output - feedback law u = K y such that:

- First, the closed - loop system becomes stable for all admissible parameter trajectories $\rho(t)$.
- And second, the worst - case closed - loop RMS gain from w to z does not exceed some level γ > 0 [22 - 24].

The LMI approach can be employed to generalize the plant P(. , ρ) so that for a given γ> 0, one can find necessary and sufficient conditions for the existence of internally stabilizing controllers K(. , ρ) such that: $\|F(p(.,\rho),K(.,\rho)\|_\infty < \gamma$ and the closed - loop system is asymptotically stable [20, 25]. The $H_\infty$ performance is directly optimized by solving the following LMI problem: Minimize over $R=R^T>0$ and $S=S^T>0$ such that

$$\begin{bmatrix} N_{12} & 0 \\ 0 & I \end{bmatrix}^T \begin{bmatrix} A_iR+RA_i^T & RC_{1i}^T & B_{1i} \\ C_{1i}R & -\gamma I & D_{1i} \\ B_{1i}^T & D_{1i}^T & -\gamma I \end{bmatrix} \begin{bmatrix} N_{12} & 0 \\ 0 & I \end{bmatrix} < 0$$

$$\begin{bmatrix} N_{21} & 0 \\ 0 & I \end{bmatrix}^T \begin{bmatrix} A_i^TS+SA_i & SB_{1i} & C_{1i}^T \\ B_{1i}^TS & -\gamma I & D_{1i}^T \\ C_{1i} & D_{1i} & -\gamma I \end{bmatrix} \begin{bmatrix} N_{21} & 0 \\ 0 & I \end{bmatrix} < 0$$

$$i = 1,\dots,N$$

$$\begin{bmatrix} R & I \\ I & S \end{bmatrix} \geq 0 \tag{12}$$

If the vector ρ(t) belongs to the specified region then the relationship between the numbers of fuzzy rules N and uncertain parameters n is given by $N=2^n$. In addition, $N_{12}$ and $N_{21}$ denote bases of the null spaces of $(B_2^T\ D_{12}^T)$ and $(C_2, D_{21})$, respectively. Theorem 1 derives $H_\infty$ controllers K(. , ρ) from any solution (R, S, γ) of (10) and as a result, achievable Gain - Scheduling controller K(. , ρ) is obtained as follows:

$$\dot{\xi} = A_k(\rho)\xi + B_k(\rho)y$$

$$\dot{u} = C_k(\rho)\xi + D_k(\rho)y \tag{13}$$

Loop - Shaping Methodology and the Mixed - Sensitivity Problem

Loop - Shaping is a design procedure to formulate the frequency - domain specifications as H∞ constraints problems [25 - 28]. To get a feeling for the Loop - Shaping methodology, consider the general pattern loop of Fig. 1. In this frequency domain method, the design specifications are reflected as gain constraints on the various closed - loop transfer functions. Where the main closed - loop transfer functions are sensitivity function and complementary sensitivity function, so gain constraints are shaping filters. The optimal $H_\infty$ control problem can be interpreted as minimizing the effect of the worst - case disturbance w on the output z. The closed - loop transfer function from w to z is given by $T_{zw}(S)$. Hence the optimal $H_\infty$ control seeks to minimize $\|F(p(.,\rho),K(.,\rho)\|_\infty$ over all stabilizing controllers K(. , ρ). Alternatively, we can specify some maximum value $\gamma$ for the closed - loop RMS gain so that $\|F(p(.,\rho),K(.,\rho)\|_\infty < \gamma$, Where $\gamma$ is guaranteed $H_\infty$ ratio between z and w. In this research, the closed - loop transfer function $T_{zw}(S)$ in frequency domain can be presented as follows:

$$T_{zw}(s) = F(p(.,\rho),K(.,\rho) = \begin{bmatrix} W_T(s)\,T(s) \\ W_S(s)\,S(s) \end{bmatrix} \tag{14}$$

Where S(s) is the sensitivity transfer matrix (the transfer function from r to e) and T(s) is the complementary sensitivity transfer matrix (the transfer function from r to y) that can be expressed as

$$S(s) = (I + G(s)K(s)^{-1}$$

$$T(s) = G(s)K(s)(I + G(s)K(s)^{-1} \tag{15}$$

The $W_S(s)$ and $W_T(s)$ are frequency dependent weighting functions (shaping filters), sensitivity weighting function and the complementary sensitivity weighting function respectively. As it is known, shaping T(s) is desirable for tracking problems, noise attenuation and robust stability with respect to multiplicative output uncertainties. On the other hand, S(s) relates to the error signals with references and disturbances so that the shaping of the sensitivity function will permit the performance (in terms of command tracking and disturbance attenuation) of the system to be controlled. Accordingly, in order to realize above requirement, following close - loop transfer functions can be normalized ass following forms:

$$\|W_S(s)\,S(s)\|_\infty < 1 \qquad \|W_T(s)\,T(s)\|_\infty < 1 \tag{16}$$

Whereas the rotor angular position and the electromagnetic torque are selected as the main objectives, as a result it can conclude that number of system outputs and tracking errors are 2. Therefore the size of weighting function matrices $W_S(s)$ and $W_T(s)$ are chosen 2×2 that can be expressed as follows

$$T(s) = T_{\text{yr}} = \begin{bmatrix} T_1 \\ T_2 \end{bmatrix} = \begin{bmatrix} T_{y_1 r_1} \\ T_{y_2 r_2} \end{bmatrix}$$

$$S(s) = T_{er} = \begin{bmatrix} S_1 \\ S_2 \end{bmatrix} = \begin{bmatrix} T_{e_1 r_1} \\ T_{e_2 r_2} \end{bmatrix} \quad (17)$$

Where $y_1$ and $y_2$ are angular position and electromagnetic torque; $r_1$ and $r_2$ are position command and torque reference input; $e_1$ and $e_2$ are the tracking errors. The sensitivity and the complementary sensitivity weighting functions $W_T(s)$, $W_S(s)$ are designed as two 2×2 square diagonal matrices. Where $W_{Tii}(s)$ and $W_{Sii}(s)$ are considered as the diagonal elements of $W_T(s)$, $W_S(s)$ respectively so that they can be rewritten as follows

$$W_T(s) = W_{Tii}(s)\, I_{2\times 2}$$

$$W_S(s) = W_{Sii}(s)\, I_{2\times 2} \quad (18)$$

The transfer functions $W_{Sii}(s)$ and $W_{Tii}(s)$ must be stable, minimum phase and additionally should be proper. Moreover, they must be low - pass and high - pass filters respectively. In order to determine the performance and robustness weights, practical formulas of $W_{Tii}(s)$ and $W_{Sii}(s)$ are suggested as follows

$$W_{Tii}(s) = \frac{d\,S + 1}{d\,S + 2}$$

$$W_{Sii}(s) = \frac{a\,S + W_c}{S + W_c\, b} \quad (19)$$

Where «a» is the high frequency disturbances gain; «b» is the gain for low frequency control signal; «d» is a constant; «Wc» is the crossover frequency. In other words «Wc» is the frequency which differentiates the high frequency disturbance signal and the low frequency control signal. «Wc» also indicates the minimum bandwidth of the transfer functions which are weighted. In order to design the optimal weighting functions, the Regular Weights Selection Method (RWSM) can be arranged as follows

a) Get the $i^{th}$ local linear subsystem Gi (. , ρ) (4)

b) Get new initial values a, b, d, $w_c$ and design $W_{Sii}(s)$, $W_{Tii}(s)$ and consequently $W_S(s)$ and $W_T(s)$ (18) - (19)

c) Build the plant P(. , ρ) by using merge (Gi (. , ρ) , $W_T(s)$ and $W_S(s)$

d) Design the local Gain - Scheduled controller K (. , ρ) based on LMI approach by using the parameter measurements ρ (t) (12)

e) Find the close - loop transfer functions: S(s): {$S_1$(s) and $S_2$(s)} and T (s): {$T_1$(s) and $T_2$(s)}

f) Check the following conditions:

$$\sigma_{\max}(S(jw)) < \gamma\, \sigma_{\min}(W_S^{-1}(jw))$$

$$\sigma_{\max}(T(jw)) < \gamma\, \sigma_{\min}(W_T^{-1}(jw))$$

$\gamma < 1$

Satisfactory steady state and transient responses of close - loop system for tracking and disturbance attenuation items;

→ K (. , ρ) is optimal and $W_T$(s) , $W_S$(s) are desirable

Consequently, the above method selects optimal weighting functions in order to formulate performance and robustness specifications of close - loop system. In the RWSM, the coefficients a, b, c, Wc should be decremented or incremented as far as whole of the clauses in each step can be valid and finally controller K(. , ρ) will be returned.

### C. PDC Concept and Proposed Control Loop Structure

In this section, the concept of parallel distributed compensation (PDC) is employed over the system (4) [18, 20]. According to PDC approach, the control law of the whole system is obtained through weighted sum of the local feedback control for various subsystems. That is:

$$IF\ v_1(t) is\ M_{i1}\ and ... and\ v_p(t) is\ M_{ip}\ THEN$$

$$u(t) = K_i(.,\rho)\, y(t)\ ;\ i = 1,...,r \quad (20)$$

Where Ki (. , ρ) are the local controller gains that are determined by LMI technique to achieve the design requirements. The control law of the whole system is created by PDC approach so that it can be expressed as

$$u(t) = \sum_{i=1}^{r} \mu_j\ K_j(.,\rho)\, y(t) \quad (21)$$

The main goal of this research is to design a suitable control which guarantees the robust performance in the presence of parameters variation and load torque disturbance. In this case, two control objectives are defined. First, rotor angular position and second, the electromagnetic torque of motor that both of them must track reference the trajectories $r_1$ and $r_2$, respectively. What's more, the load torque disturbance is an unknown exogenous input and magnitude of it may vary in a wide range. Consequently, in order to overcome this variation, electromagnetic torque of the motor should be adapted to the load torque disturbance and as a result, the second objective is a necessary point.

Moreover, to stay away from the reluctance effects and torque ripple, direct current must be tracked to zero. Consequently, in order to achieve these objectives, both of the tracking errors should be minimized. Mentioned goals are realized through construction of the objectives z in an appropriate control loop. In the proposed design, main objectives z can be defined as follows

$$\underline{z}_e = \begin{pmatrix} z_{e_1} \\ z_{e_2} \end{pmatrix} \qquad \underline{z}_y = \begin{pmatrix} z_{y_1} \\ z_{y_2} \end{pmatrix} \tag{22}$$

Under the above considerations, the structure of the Mixed - Sensitivity Gain - Scheduled control loop is the designed. According to this structure, the nonlinear dynamic is first approximated with some local linear models in four rules which are represented by T - S fuzzy approach. Both shaping filters $W_S(s)$ and $W_T(s)$ are then designed and augmented plant P (. , ρ) is built. The Gain - Scheduled controllers are employed for each linear sub plant based on LMI approach. Such the controllers are said to be scheduled by feedback of the Time - Varying parameter measurements *ρ (t)* of the system in real time. After that, the total linear system is obtained through using the weighted sum of the local linear system and is utilized rather than the original nonlinear system. Moreover, the control law of the whole system is designed by the PDC approach. Finally, by using the whole system and global controller, a tracking loop is applied in order to achieve desirable specifications such as tracking performance, bandwidth, disturbance rejection, and robustness for the close - loop system.

In the Gain - Scheduled control, in general, the plant is not exactly known and the parameters of the system vary at the time of switching. The main aim of this control approach is to change the controllers to improve the performance by using the Time - Varying parameter measurements in real time.Consequently; the mentioned control has a possibility to create shocking transients in controller output called 'bumps'. If current on - line controller is switched to a new controller, then different outputs at the switching instant are occurred called BAMP.

In our design, a bump less transfer block is used that allows the process to be changed between various controllers without upsetting the process. In other words, the controller is to be switched between updating parameters, whilst control signal hasn't been bumping the value suddenly.

## 4- SIMULATION RESULTS

The motor type used in this paper is 130 - 750MS - ZK - L2. The IPMSM is a three - phase, four - pole, 0.75 HP, with a 2000 rpm rated speed. The maximum voltage and the continuous rated armature current are set to 230 V and 12 A [1, 6, 7]. The parameters of the IPMSM are shown in Table I. According to (11), the $x_2$ and $x_4$ are nonlinear terms and referring to the IPMSM characteristic, it can be concluded that

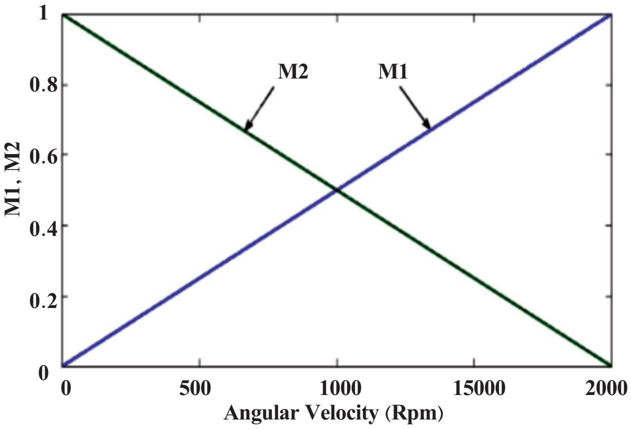
$$v_1(t) = x_2 \in [0 \quad 2000] \quad v_2(t) = x_4 \in [0 \quad 12]$$

Where $v_1$ and $v_2$ are fuzzy variables whose membership functions can be calculated as:

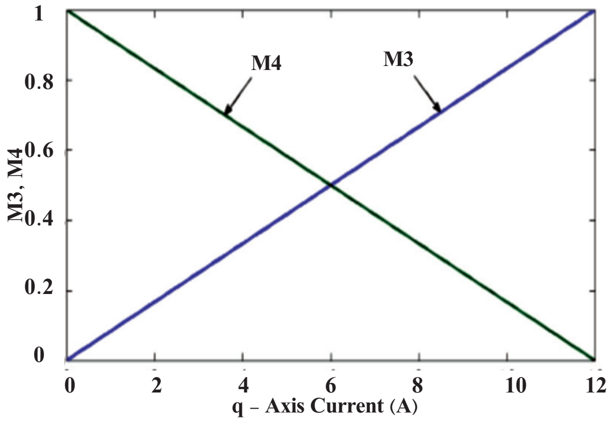
$$M_1 = \frac{v_1}{3000} \qquad M_2 = 1 - \frac{v_1}{3000}$$

$$M_3 = \frac{v_2}{6} \qquad M_4 = 1 - \frac{v_2}{6}$$

These membership functions are depicted in Fig.2.




(a)




(b)

*Fig. 2. (a) The membership functions for $M_1(v_1(t))$ and $M_2(v_1(t))$, (b) the membership functions for $M_3(v_2(t))$ and $M_4(v_2(t))$*

TABLE 1
PARAMETERS OF IPMSM

| Components | Rating values |
|---|---|
| Rs | 1.9 Ω |
| Bmo (without load) | 0.03 N m s/rad |
| Bmo (with load) | 0.0341 N m s/rad |
| Ld | 0.0151 H |
| Lq | 0.031 H |
| φf | 0.31 V s/rad |
| Jmo (without load) | 0.0005 kg.m2 |
| Jmo (with load) | 0.0227 kg.m2 |
| Po | 2 |

The T - S fuzzy model of the nonlinear system (1) is first modeled by using the fuzzy rules (4) with r=4. Referring to (8), all system matrices are constant except $A_{oi}$ that varies with respect to the rules as given below:

$$A_{0_1} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -1145 & 1860 \\ 0 & 49 & 0 & 0 \\ 0 & -20 & -82199 & 0 \end{bmatrix}$$

$$A_{0_2} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1860 \\ 0 & 0 & 0 & 0 \\ 0 & -20 & -82199 & 0 \end{bmatrix}$$

$$A_{0_3} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -1145 & 1860 \\ 0 & 49 & 0 & 0 \\ 0 & -20 & 0 & 0 \end{bmatrix}$$

$$A_{0_4} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1860 \\ 0 & 0 & 0 & 0 \\ 0 & -20 & 0 & 0 \end{bmatrix}$$

Referring to subsection 3.B, the weighting matrices $W_T(s)$ and $W_S(s)$ are designed as:

$$W_T(s) = W_{Tii}(s) I_{2\times2} = \frac{0.001S+1}{0.001S+2} I_{2\times2}$$

$$W_S(s) = W_{Sii}(s) I_{2\times2} = \frac{0.5S+30}{S+0.03} I_{2\times2}$$

The local controllers for each linear subsystem are calculated by using the proposed control loop, the above weighting matrices and (12). Finally, the overall fuzzy system and the control law of the whole system are obtained through a weighted average defuzzifer.

In the designed controller, the best performance of H∞ ($\gamma_{opt}$) is 0.92 that is guaranteed smaller than 1.

The complete proposed T - S fuzzy model accurately represents the nonlinear system in the region of [0, 12] A× [0, 2000] rpm on the $x_2$ - $x_4$ space for the various operating points.

The parameters $R_s(t)$ and $B_m(t)$ are supposed to differ from their nominal values $R_{so}$ and $B_{mo}$ and they are Time - Varying electrical parameters [2]. In addition, parameter trajectories of $R_s$ (t) and $B_m(t)$ in real time are shown in Fig.3.

In reality, the motor is used to convert the electrical energy into mechanical energy. Accordingly, an external load is added to the drive system. The external load is a 1 kg weight and is located at a certain location of the motor at 0.1s. As a result, this weight can provide the external load 1 N m that is introduced by a step input.

Fig. 4 demonstrates the disturbance rejection of angular position tracking for four different methods, when a step load torque is used. However, the IPMSM is first controlled to reach a fixed position, 90 degrees.

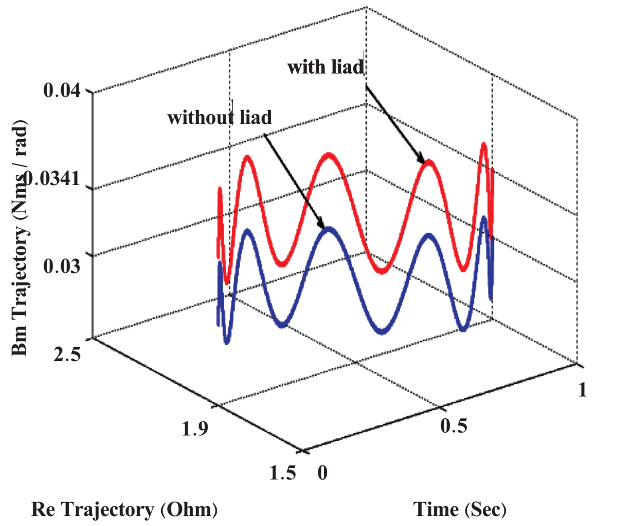



*Fig. 3. Trajectory of Time - Varying parameters in real time*

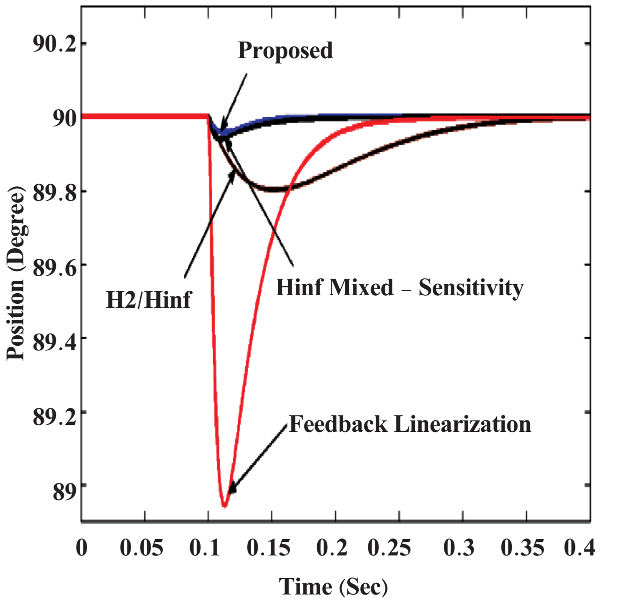



*Fig. 4. Disturbance rejection on angular position with step load torque (1 Nm)*

This figure shows the superiority of the proposed method in comparison with the $H_\infty$ mixed - sensitivity (see [7]), the $H_2/H_\infty$ controller (see [6]) and the feedback linearization technique (see [1]). As easily observed, the proposed method has the best disturbance attenuation. Table II also summarizes the disturbance rejection differences for four methods when the system is influenced by step load torque (1 Nm). According to this Table, the proposed method has the smallest undershoot value.

Fig. 5 shows the d - and q - axis current responses when external load 1 N m is used at 0.1 s.

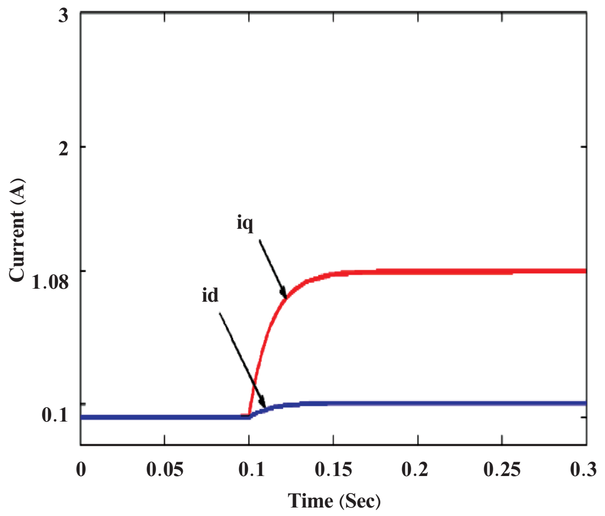



*Fig.5. d - q axis currents*

Fig. 6 shows the position responses of the certain step position (90 degrees) for the proposed method, $H_\infty$ mixed - sensitivity (see [7]), $H_2/H_\infty$ (see [6]) and feedback linearization (see [1]) controllers.

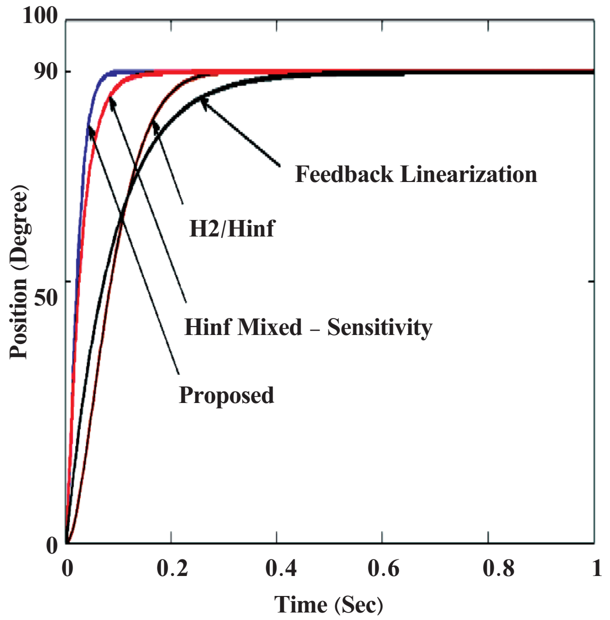



*Fig. 6. Comparison of transient responses of certain step position*

According to this figure, the proposed controller has the best values of settling time and rise time in comparison with the other methods, when parameters trajectories of $R_s(t)$ and $B_m(t)$ are changed in real time. It is clear that the system has good robustness when the parameters in the system dynamics are varied in a wide range.

Table II also summarizes the results of transient responses for these methods. Referring to this table, the proposed method has the smallest settling time value on position tracking responses.

**TABLE 2**
**COMPASISON OF DESIGN CHARACTERISTIC IN FOUR METHODS**

| Method | Settling time (Sec) | Under shoot value (Degree) |
|---|---|---|
| Proposed | 0.09 | 0.045 |
| H∞ mixed - sensitivity (see [7]) | 0.15 | 0.06 |
| H2/H∞(see [6]) | 0.28 | 0.2 |
| Feedback linearization (see [1]) | 0.4 | 1.1 |

Fig. 7 illustrates the electromagnetic torque tracking responses at different torque commands by using the proposed controller. According to this figure, the proposed system has satisfactory performance for various torque commands in order to overcome various load torque disturbances.

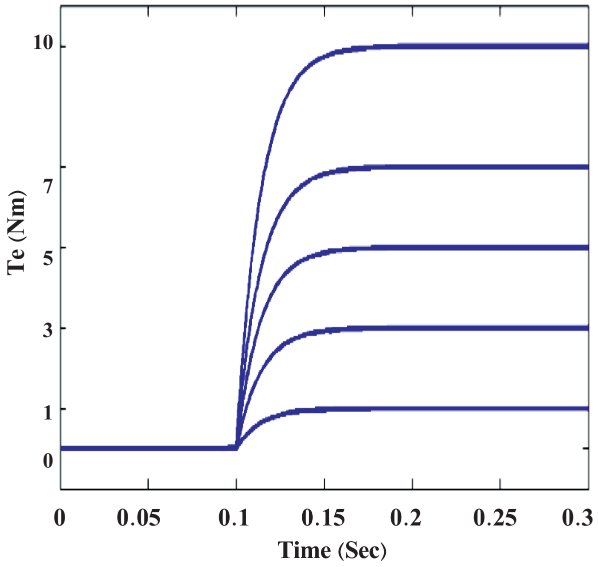



*Fig. 7. Electromagnetic torque tracking responses at different position reference*

Likewise, according to Figs. 6 and 7, the system has good robustness when the parameters in the system are varied over a wide range in real time.

## 5- CONCLUSION

In this paper, a robust Mixed - Sensitivity Gain - Scheduled $H_\infty$ controller has been designed for an MIMO nonlinear uncertain Time - Varying IPMSM system. First, a T - S fuzzy technique was employed to approximate the uncertain nonlinear parameter - dependent system. Then, both Loop - Shaping methodology and Mixed - Sensitivity problem were utilized to improve the frequency - domain specifications. After that, based on each linear model, a robust $H_\infty$ controller was designed by LMI technique in order to achieve the robustness against both system uncertainties and disturbance. The PDC was used to design the controller for the overall system and the total linear system was further obtained through using the weighted sum of the local linear systems. The simulation results on IPMSM showed that the robust control system has small position and current tracking errors and has desired the robustness against load torque disturbance and parameter variations. The proposed position and current control system had good transient, load disturbance rejection and tracking responses. The superiority of the proposed control scheme was approved through simulations in comparison with the feedback linearization, the H2/$H_\infty$ and the $H_\infty$ Mixed - Sensitivity controllers. Table II also summarizes the performance comparisons for four different methods. Referring to this table, the proposed method had the smallest settling time on position tracking response and the lowest amount of undershoot on the disturbance rejection response. The major achievements of this research are delivered as follows: (i) the performance requirements like good load disturbance rejection, tracking and fast transient responses in the proposed method were better than those of the other methods, (ii) the proposed method had satisfactory performance for various electromagnetic torque commands in order to overcome various load torque disturbances, (iii) The proposed controller had good robustness when parameter trajectories of $R_s(t)$ and $B_m(t)$ were changed in real time, and (iv) direct and quadrature currents of IPMSM didn't exceed the allowable limit.

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