A Multi-Period 1-Center Location Problem in the Presence of a Probabilistic Line Barrier

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ABSTRACT

This paper investigates a multi-period rectilinear distance 1-center location problem considering a line-shaped barrier, in which the starting point of the barrier follows the uniform distribution function. In addition, the existing points are sensitive to demands and locations. The purpose of the presented model is to minimize the maximum barrier distance from the new facility to the existing facilities during the finite planning horizon. Additionally, a lower bound problem is generated. The presented model is mixed-integer nonlinear programming (MINLP); however, an optimum solution is reached.

KEYWORDS

Multi-period Center location problem, Probabilistic line barrier, Rectilinear distance

1. INTRODUCTION

The center location problem was first introduced in [49]; however, the continuous center problem was referred to [50]. Since we have a minimax objective function for the center model, it seems that it will be most applicable to emergency cases. Some potential applications of the center problem are: quick services (e.g., hospital emergency services, fire stations, and police stations), distribution (e.g., warehouses and garages), military purpose, government, and general (e.g., parks and hotels).

The solution space of the center problems are generally classified into *planar* and *network* space. One of the most common location problems is the restricted planar location problem that is classified into one of the categories: (1) *forbidden regions* (e.g., national parks or other protected areas), in which the opening a facility is prohibited but traveling through is permitted, (2) *congested regions* (e.g., big lakes or forest), in which opening a facility is prohibited but travelling through is possible with additional cost, and (3) *barrier regions* (e.g., military areas, mountain ranges, big rivers and the lake), in which both opening and travelling are forbidden. For the ease of better understanding, see Table 1.

TABLE 1
CATEGORIES OF RESTRICTED LOCATION PROBLEMS.

	Forbidden Regions	Congested Regions	Barrier Regions
Travelling	Allowable	Allowable with penalty	Unallowable
Establishing	Unallowable	Unallowable	Unallowable

There are too many environmental behaviors that influence the location of the facilities during the planning horizon. For instance, the demand or the position of the customers in different seasons may vary. So, *where* and *when* the facility must be opened is the concern of the decision makers. In a dynamic environment, considering time-dependent inputs impacts on strategic decision making processes. On the other side, opening and expansions the facility in long term are the key cases, in which decision makers must focus on. Therefore, many researchers are motivated to study dynamic location problems. Ref [54] classified the dynamic facility location problems into two principal classes: (1) location, in which a profitable site is selected for a defined time horizon and (2) location–relocation, in which a primary location, relocation or development of facilities is selected.

In this paper, the authors pay attention to the multi-period planar center location problem in the presence of a line barrier which randomly occurs on a given horizontal route. According to our knowledge, there is no work done in this field. This paper is presented as follows: Section 2

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is divided into two parts: (1) containing a review on the studies published about facility location problems with barriers and (2) including a brief survey on dynamic single facility location problems, separately. In Section 3, an MINLP model to solve the dynamic center problem in the presence of a probabilistic line barrier with the rectilinear distance metric is proposed. We present the lower bound problem for the presented model in Section 4. We illustrate an example in Section 5 to show the validity of the presented model in detail. Then we solve several sample problems in different size. Finally, Section 6 contains conclusion and further research directions.

2. Related Studies

A. *Location problems with a barrier*

So far most of the research has focused on the Weber problem and suggested different variations and modifications of this problem. Other objective functions, such as the minimax or maximin objective, have been rarely considered. Many studies to the center problem in the presence of barriers are based on rectilinear or block norm distances that allow for the problem decompositions and discretizations. A polynomial algorithm for restricted Euclidean center location problems is presented in [3]. Ref [10] and [11] considered a constrained rectilinear distance minimax location problem and presented a geometric solution approach. Nickel studied the restricted center location problems under polyhedral gauges in [44]. Ref [15] considered the rectilinear distances center facility location problem with polyhedral barriers and derived a finite dominating set result for the problem. Although similar ideas to a more general class of location problems were extended in [13] and [14]. After that, the same problem using the block norm distances in place of the rectilinear distances was studied in [16]. Ref [23] considered the minimax location problems in the presence of polyhedral barriers with the Euclidean distance. They proposed a solution approach based on propagation of circular wavefronts. Considering such barriers, the Euclidean multi-facility location–allocation problem and proposed two heuristics to solve the problem was presented in [5]. In the presence of the arbitrary shaped barriers, Ref [47] first considered a single finite-sized facility location problems with the Manhattan (i.e., rectilinear) distance metric. Based on the work of [47], a new facility location problem applying a contour line was presented in [33]. Ref [46] extended the work of [47] for finite facility location problem with only user–facility interactions. Ref [43] considered the rectilinear distance center problem in the presence of arbitrary shaped barriers.

Katz and Cooper first studied the planar Euclidean Weber problem and one circular barrier in [32]. They suggested a heuristic algorithm based on the sequential unconstrained minimization technique (SUMT) for solving this problem. In the same problem, Ref [37] divided the feasible region into some convex regions, in which the number of these convex regions is bounded by $O(N^2)$ where N is the number of existing facilities. Found a way of overcoming, the Weiszfeld technique and genetic algorithms (GAs) were applied in [6]. Aneja and Parlar studied the Euclidean Weber problem in the presence of convex or non-convex polyhedral barrier regions in [1]. Using simulated annealing (SA), they determined some candidate points, and then constructed a visibility graph to evaluate the shortest path between any candidate point and existing facility location. Ref [35] developed a reduction result for the same problem, in which the non-convex barrier location problem reduced to a set of convex location problems. Then, an exact and a heuristic algorithm were presented to solve such a planar location problem with barriers. Then, Ref [36] considered the Weber location problems in the plane in the presence of line barriers with a finite number of passages. She proved that the time complexity of the problem exponentially grows by increasing the number of passages. The big square small square (BSSS) method, a branch-and-bound based technique in [29], to approximate the global optima of the Euclidean Weber problem with the convex polyhedral forbidden regions was developed in [40]. In the presence of convex polyhedral barriers, Ref [7] addressed the Euclidean Weber problem and presented the FORBID heuristic method to decompose the feasible region. Larson and Sadiq worked on the discretization results for the rectilinear Weber problem with arbitrary shaped barriers in [39]. Generalized results of [39], considering both arbitrary shaped barriers and convex forbidden regions were provided in [4]. Ref [26] developed a similar discretization for a general class of distance functions. Formulations of a mathematical programming model, where facilities were finite-sized shape or point and barriers were rectangular, were proposed in [52]. For line barriers considering various distance functions, Ref [38] proposed an algorithm for multi-criteria location problems. Ref [8] presented a solution approach for the rectilinear Weber problem with a probabilistic line barrier. Furthermore, they provided an extensive overview of facility location problems in the presence of barrier regions. Then the multi-facility location problem in the presence of a probabilistic line barrier in extended in the Ref [56].

B. *Dynamic location problems*

Here, it is given the research papers published in this field briefly. The dynamic Weber problem was formerly introduced by Wesolowsky in [53]. Ref [34] investigated the dynamic location problem in a finite planning horizon and proposed a mixed-integer linear programming model.

Min studied on the facility location problems with considering dynamic relocation and expansion of capacitated public facilities as the objective functions in [42]. To solve the problem, a fuzzy goal programming approach was presented. Also, this study was a case that implemented successfully on Columbus public library facilities. Frantzeskakis and Watson-Gandy formulated a dynamic facility location problem over a given planning horizon. The dynamic programming and branch-and-bound methods were the proposed approaches to solve such problems in [22]. Galvão and Santibañez-Gonzalez (1990) described a multi-period facility location problem and developed a heuristic method based on dynamic programming and a Lagrangean heuristic method in [24]–[25]. Drezner and Wesolowsky investigated multiple time break and linear change of weights over time in [19]. They considered location on multiple time breaks and devise two algorithms for a minimax location problem. Ref [48] presented a dynamic capacitated plant location problem as a combinatorial optimization problem and proposed a Lagrangian relaxation-based technique to solve the problem. Ref [31] improved an exact algorithm for the dynamic facility location problem by integrating mixed-integer with dynamic programming methods.

Ref [21] developed a multi-criteria dynamic location problem, in which the total number of facilities to be located is uncertain. Ref [41] presented a multi-criteria multi-period model as a physical programming model that determines the optimal relocation site. They described that physical programming allows the decision maker to express criteria preferences not in the traditional form of weights but in terms of ranges of different degrees of desirability. Ref [30] formulated a mixed-integer programming model for multi-period multi-commodity two-echelon capacitated facility location problems and proposed a Lagrangean relaxation and a heuristic method to solve the problem. Ref [9] considered multi-period multi-stage multi-commodity capacitated facility location problem, in which the facility relocation is allowed. To solve such a problem, they proposed an algorithm by integrating a branch-and-bound method and dynamic programming over the planning horizon. Antunes and Peeters studied a complex dynamic location problem based on a real case in Portugal and used SA to solve this problem in [2]. Puerto and Rodríguez-Chía studied on a generalized dynamic single facility location problem in one and two dimensional spaces in [45]. They proposed a solution method based on the Weiszfeld algorithm to solve the problem. In addition, they proved the global convergence of their method. Based on [51], which described a branch-and-bound algorithm that uses a dual ascent procedure for the dynamic simple plant location problem, and [20], which evaluated the performance of seven approximate methods for dynamic location problems and combined different methods to find more effective solutions, a dynamic location problem was considered in [17] and an efficient primal-dual heuristic method to obtain a good results was proposed. Ref [18] considered capacitated and uncapacitated multi-objective dynamic location problems with relocation more than once during the planning horizon. They proposed the memetic algorithm to solve such problems. Ref [21] investigated a time-dependent weights single facility location-relocation problem for the rectilinear, squared Euclidean or Euclidean distances where the time horizon can be finite or infinite. They also considered a location-dependent relocation cost to make the research closer to the real-world situations. An optimal solution was found by their proposed algorithm.

3. PROBLEM DEFINITION

A. Assumptions

- Existing facilities are located in the feasible region in each period.
- New and Existing facilities' shape is point.
- Facilities are located on the plane (i.e., $\Re^2$).
- Location of the existing facilities is deterministic and dynamic during the planning horizon (location sensitive).
- Weight associated with demand of the existing facilities is deterministic and dynamic during the planning horizon (demand sensitive).
- Length of the planning horizon is finite.
- Rectilinear distance is considered.
- Barrier's shape is line. It means that the width of barrier is negligible.
- Length of the barrier is constant and known.
- Location of the barrier is probabilistic and distributed uniformly with known parameters.

B. Preliminaries definitions

Let Ex= $\{X_{ih} \in \Re^n,: i = 1,\ldots,I,\ h=1,\ldots,H\}$ be a finite set of existing facilities over the planning horizon, where I is the number of existing facilities and H is the length of planning horizon. Let $\{B_1, \ldots, B_G\}$ be a finite set of barrier and $\mathcal{B} = \bigcup_{g=1}^{G} B_g$. The interior of barrier regions, called int($\mathcal{B}$), is prohibited for opening a new facility, as well as traveling through int($\mathcal{B}$). Thus, the feasible region, $\mathcal{F} = \Re^n \setminus \text{int}(\mathcal{B})$, for locating and traveling is given by [55]. By our definition of $\mathcal{B}$ and $\mathcal{F}$, there is an exception only in *line barriers* that have an empty interior, in which travelling is also forbidden at non-interior points of a barrier.

To clarify the concept of distance function d_p^B, called the *p*-norm barrier distance, consider two arbitrary points $X, Y \in \mathcal{F}$. Suppose $d_p^B(X,Y) = \inf\{l(P_{X-Y})\}$, P_{X-Y}

is feasible X,Y path, where $l(P_{X\text{-}Y})$ is the length of the feasible X-Y path. Let $d_p(X,Y)$ be the p-norm distance between $X, Y \in \mathcal{F}$. Two arbitrary points (i.e., $X, Y \in \mathcal{F}$) are called *p-visible* if $d_p^B(X,Y) = d_p(X,Y)$, that is the presence of barrier is not in effect on the visibility of two points X,Y. On the other hand, the p-norm distance between $X, Y \in \mathcal{F}$ is called *p-shadow,* if:

$d_p^B(X_1,X_2) > d_p(X_1,X_2)$, that is the barrier is in effect. For one feasible point, $\mathbf{X} \in \mathcal{F}$, the set of *visible* points can be defined as:

$$\text{visible}_d(X) = \left\{ Y \in \mathcal{F} : d_p^B(X,Y) = d_p(X,Y) \right\}$$

In other words, the set *visible* contains points from the feasible region, Y, which are *p-visible* with a given feasible point X. For a feasible point $\mathbf{X} \in \mathcal{F}$, the set of *shadow* points is defined as:

$\text{shadow}_d(X) = \left\{ Y \in F : d_p^B(X,Y) > d_p(X,Y) \right\}$, (i.e., if the distance between X and Y is *p-shadow*, then it becomes barrier distance, [55]). In this paper, a rectilinear distance metric (i.e., p=1) is considered.

According to [27] and [28], the taxonomy of the presented location problem can be mentioned as $1/\Re^2/\mathcal{B}$ =1 *probabilistic line*/d_1^B/max$_{dynamic}$, in which position 1 contains the number of new facility must be located, position 2 shows 2-dimensional continuous location problems or planar problems, position 3 indicated the special features of location problems where barrier restriction in considered, position 4 gives the information about the distance function. In this paper, a rectilinear barrier distance is given, in which position 5 denotes the objective function. Indices, parameters and decision variables are stated as follows:

Indices and parameters

i	Index of the existing facilities
h	Index of periods
I	Number of the existing facilities
H	Planning horizon
w_{ih}	Weight of the ith facility in period h
x_{ih}	x-coordinate of the ith facility in period h
y_{ih}	y-coordinate of the ith facility in period h
b	y-coordinate of the barrier
l	Length of the barrier
L_1	Lower bound of uniform distribution function
L_2	Upper bound of uniform distribution function

$$s_{ih} = \begin{cases} 1, & \text{if } y_{ih} > b \\ 0, & \text{otherwise} \end{cases}$$

Decision variables

x	x-coordinate of the new facility
y	y-coordinate of the new facility

$$t_{ij} = \begin{cases} 1, & \text{if } i-\text{th existing facility and the new facility in period } h \text{ are located on the opposite halfplanes} \\ 0, & \text{otherwise} \end{cases}$$

$$a_{ij} = \begin{cases} 1, & \text{if } |x - x_{ih}| < l \\ 0, & \text{otherwise} \end{cases}$$

$$p = \begin{cases} 1, & \text{if the new facility in period } h \text{ is located on the upper halfplane} \\ 0, & \text{otherwise} \end{cases}$$

C. Mathematical model

Considering I existing facilities over H periods, the multi-period rectilinear center problems with barriers can be formulated by:

$$\min_{x,y \in \mathcal{F}} \left\{ \max_{i,h} \left\{ w_{ih} \cdot d_1^B\left(X, X_{ih}\right) \right\} \right\}, \quad (1)$$

where $X=(x,y)$ is the coordinates of the new facility and $X_{ih}=(x_{ih},y_{ih})$ is the coordinates of the ith existing facility in period h. Regarding to the line barrier that has a fixed y-coordinate at b and a probabilistic x-coordinate, we introduce a starting point of the line barrier as X_s where X_s is a continuous random variable with known parameters. The probability density function of X_s is as follows:

$$f(X_s)=\begin{cases}\frac{1}{L_2-L_1} & L_1\le X_s\le L_2,\\ 0 & otherwise,\end{cases}$$

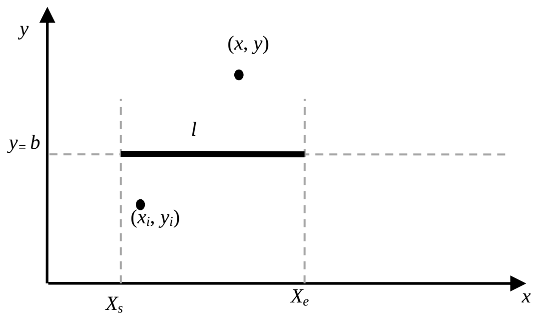

Figure 1: A probabilistic line barrier.

So, the difference between upper and lower limits of X_s is mentioned as: $r=L_2-L_1$. In addition, the ending point of the line barrier, called X_e, can be calculated as: $X_e=X_s+l$, (see Figure 2). Canbolat and Wesolowsky extensively discussed on the all possible cases of locating the existing facilities and the new facility related to the line barrier position on the plane and introduced the barrier conditions when it is in effect in [8].

They proved that the barrier conditions can be represented in the form of:

$$\max\{x-l,x_i-l\}\le X_s\le\min\{x,x_i\},\quad \forall i$$

Whereas calculating the exact distance between the optimal facility and the existing facilities, due to a probabilistic barrier, is impossible, the expected barrier distance must be considered. In addition, since the x-coordinate of the barrier location follows uniform distribution function, the y-coordinate of the distance function remains constant. So, expression (2) is enhanced.

$$\min_{x,y\in F}\left\{\max_{i,h}\left\{w_{ij}.E\left[d_1^B\left(\mathrm{x},x_{ih}\right)\right]+\left|y-y_{ih}\right|\right\}\right\}, \tag{2}$$

Equivalently, the expression (2) can be reformulated in the form of:

$$\min_{x,y\in F}\psi,$$

$$w_{ij}.\left(E\left[d_1^B\left(x-x_{ih}\right)\right]+\left|y-y_{ih}\right|\right)\le\psi \quad \forall i,h$$

The expected barrier distance was completely discussed in [8]. They proposed the expected barrier distance of x from x_i that can be generally written by:

$$E\left[d_1^B(x,x_i)\right]=\begin{cases}\frac{\left(l-|x-x_i|\right)^2}{2r}+|x-x_i|; & |x-x_i|<l\\ |x-x_i|; & |x-x_i|\ge l\end{cases},\quad \forall \mathrm{i}. \tag{3}$$

It means that when the absolute value of the distance difference of the new and the existing facility is less that the length of the line barrier, the expected barrier distance increases. However, a modification of Expression (3) in the dynamic environment is given by:

$$E\left[d_1^B(x,x_{ih})\right]=\begin{cases}\frac{\left(l-|x-x_{ih}|\right)^2}{2r}+|x-x_{ih}|; & |x-x_{ih}|<l\\ |x-x_{ih}|; & |x-x_{ih}|\ge l\end{cases},\quad \forall i,h. \tag{4}$$

To describe the above expression, the plane is divided into two half-planes. When the existing facility i in period h (i.e., X_{ih}) and the new facility (X) are located in the opposite half-planes, then the barrier affects on the expected distance (i.e., while X and X_{ih} locate in the same half-plane, the barrier will not affect on the distance).

We suppose that if the ith existing facility and the new facility locate in opposite halfplane in period h, t_{ih} is equal to 1; otherwise, it is zero. We can also state the binary variable t_{ih} in the form of:

$$t_{ih}=\begin{cases}1 & (y>b\,AND\,y_{ih}<b)\,OR\,(y<b\,AND\,y_{ih}>b)\\ 0 & (y>b\,AND\,y_{ih}>b)\,OR\,(y<b\,AND\,y_{ih}<b)\end{cases},\quad \forall i,h. \tag{5}$$

Then, we define a_{ih} as a binary variable that determines whether the barrier affects on the distance or not. We can pose a_{ih} as follows:

$$a_{ih}=\begin{cases}1, & if\ \left|x-x_{ih}\right|<l\\ 0, & if\ \left|x-x_{ih}\right|\ge l\end{cases}\quad \forall i,h \tag{6}$$

As the summary, the multi-period rectilinear center location problem in the presence of a probabilistic line barrier is presented as a mixed-integer nonlinear programming model by:

$$\min_{x,y\in F}\psi \tag{7}$$

Subject to:

$$w_{ih}\left(\frac{\left(l-|x-x_{ih}|\right)^2}{2r}a_{ih}t_{ih}+|x-x_{ih}|+\left|y-y_{ih}\right|\right)\le\psi \quad \forall i,h \tag{8}$$

$$2y.p-2b.p\ge y-b \tag{9}$$

$$\left|p-s_{ih}\right|=t_{ih},\ \forall i,h \tag{10}$$

$$2l.a_{ih}-2a_{ih}\left|x-x_{ih}\right|+\left|x-x_{ih}\right|\ge l,\ \forall i,h \tag{11}$$

$$t_{ih},a_{ih}\in\{0,1\},\ \forall i,h \tag{12}$$

$$p\in\{0,1\}, \tag{13}$$

$$x,y,\psi\ge 0 \tag{14}$$

It is clear that in none of the periods, the new facility location does not allow to locate in the route of barrier (i.e., $y_h\ne b$). The objective function (7) minimizes the maximum barrier distance in the conjunction of Constraint (8). Constraint (9) indicates that the new facility locates in the above halfplane or not. Constraint (10) guarantees that in each period when X_{ih} and X locate in the opposite halfplane, the barrier can be in effect. Constraint (11) depicts that in each period when the distance between X and X_{ih} is less that the length of barrier the barrier can affect on the distance. It is worthy to note that in period h when the two previous constraints are satisfied, the barrier affects on the distance between X and X_{ih}. Constraints (12) and (13) introduce the binary variables. Constraint (15) expresses the non-negative continuous variables.

The presented model is run by the Lingo 9.0 software, and then an optimal solution is obtained for a small-sized problem in a reasonable computational time. It is worth noting that obtaining an optimal solution for this type of complex, large-sized problem in a reasonable time by using traditional approaches is extremely difficult.

4. Lower bound

As location problems with barriers are generally difficult to solve, applying relaxation methods to reach a good lower bound are essential. Perhaps the simplest way to obtain lower bounds for location problems with barriers is to consider the corresponding unconstrained problems, simply discarding the barrier regions in $\mathcal{B}$. Ref [55] proved that the optimal solution of the unconstrained problem may not be feasible for the constrained problem with a barrier. While the barrier is static, the author introduced the restricted location problem involving forbidden regions $\mathcal{R}$, in which it is forbidden to place a new facility in the interior of the forbidden region; however, trespassing through is allowed. Based on [55], we present a lower bound problem for the multi-period center location problem with forbidden region to place and available to trespassing through the region which is a lower bound for the problem with forbidden region to place as well as to trespassing. Let z_{B}^{*} be the optimal objective value of a presented problem with barrier, z_{R}^{*} be the optimal objective value for such a problem with forbidden regions, and z_{nb}^{*} be the optimal objective value for such a problem without any restriction. To find the lower bound, this problem must be relaxed to the multi-period center location problem, while locating the facility on the route of the line barrier in not permitted. The desired problem with forbidden regions is presented below. This problem can be named as $1/\Re^2/\mathcal{R}=line/d_1/\max_{dynamic}$ according to the [27] and [28].

$$\min_{x,y\in\mathcal{F}_r,y\neq b}\psi' \qquad (15)$$

s.t.

$$w_{ih}\left(|x-x_{ih}|+|y-y_{ih}|\right)\leq\psi \quad \forall i,h\,, \qquad (16)$$

$$x_h, y_h, \psi'\geq 0$$

Regardless of $y\neq b$, the problem is equivalently transformed to the multi-period problem without any restrictions (i.e., $z_{\mathrm{nb}}^{*}\leq z_{\mathrm{R}}^{*}\leq z_{\mathrm{B}}^{*}$). So, we encounter with an unconstrained multi-period 1-center problem.

5. Numerical Examples

To better understand, we illustrate an example to seek the best new facility location among 10 existing facilities in the presence of a line barrier with the length of 20 on the plane during 2 periods in planning horizon, in which not only the demand of existing facilities are dynamic, also the location of existing facilities are dynamic during planning horizon. The data for the demands and coordinates of existing facilities for the typical example are given in Table 2, in which (x_{ih},y_{ih}); w_i denotes the ith existing facility location in period h and the demand of each facility point in period h. It is supposed that the x-coordinate of the line barrier follows the uniform density function with known parameters $U(0,60)$, and its y-coordinate is fixed at $b=40$. Fig. 2 depicts graphically the location of the existing facilities in two periods and the line barrier. Indeed, the location of the new facility in the presence of the line barrier is also depicted. To show the capability of the problem, the given multi-period center location problem, as a lower bound problem, is also solved and the related point is presented. The presented model is performed by the Lingo 9.0 software and an optimal solution is obtained. Moreover, the multi-period center location problem, as the lower bound problem, is also solved and the associated results are illustrated in Table 3. It is worth noting that optimal optimum solution is found.

To point out the efficiency of the proposed model, five sample problems in different size are solved. The data are generated via the uniform random generator function as the coordinates of the existing facilities location. Then, we transform the data to the interval of [20,60] by the uniform transformer function as, 20+(60-20)×α, where $\alpha\in[0,1]$ is the generated uniform random number. Via the same random generator function, the weights of all facilities are also generated in the range of [1,10]. Sample problems are then solved and the results are reported in Table 4. The results in Table 4 indicate the coordinates of the optimal center point during the number of period and the minimum objective value, respectively, for the problem in the presence of a probabilistic line barrier and without barrier.

Table 2

COORDINATES OF THE EXISTING FACILITIES.

	Existing facilities									
period	1	2	3	4	5	6	7	8	9	10
1	(41,46);3	(34,34);3	(23,23);2	(59,49);3	(34,52);2	(33,58);3	(52,35);4	(43,29);4	(41,25);3	(57,23);4
2	(28,44);2	(27,33);4	(49,34);3	(49,57);1	(35,26);1	(37,42);3	(56,22);3	(30,52);2	(53,46);1	(41,52);2

TABLE 3
RESULTS FOR THE EXAMPLE PROBLEM.

Multi-period Center location problem with barrier					Multi-period Center location problem				
*x**	*y**	Objective function	Solver type	Number of iterations	*x**	*y**	Objective function	Solver type	Number of iterations
47.000	38.414	101.6571	Global Opt	3654	42.058	33.343	101.1429	Global Opt	62

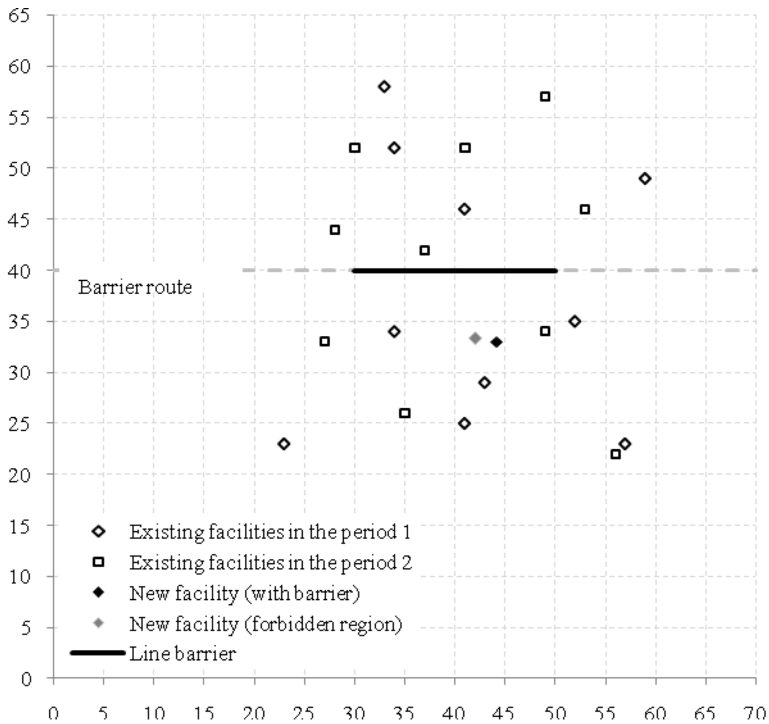

Figure 2: Typical example.

Additionally, the number of iterations (*Iter*) and the computational time is also provided for each sample problem for both, in the presence of barrier and forbidden region (stated as lower bound problem). The objective value obtained from the problem in the case of the presence of a probabilistic line barrier, is compared with the case of forbidden region. It is shown in Figure 3. It can be concluded that the differences are negligible. On the other side, for the computational time a comparison between the desired problem with barrier and with forbidden region is illustrated in Figure 4. It is clear that the larger size of problem in the presence of the probabilistic line barrier, the longer computational time. But in the case of forbidden region, there are not any meaningful differences between elapsed time of sample problems.

6. CONCLUSION

In this study, a general mixed-integer nonlinear programming (MINLP) model to find a point in the presence of a probabilistic line barrier has been presented. This model minimizes the maximum traveled weighted rectilinear barrier distance on the plane that has not been paid attention in the literature. The proposed model was solved by the Lingo 9.0 software and the global optimum has been obtained for each sample problem. Furthermore, a lower bound problem to find a good solution has been presented and a comparison with the original problem has been carried out. The results have been shown that for the large size sample problems which took a long computational time, the proposed lower bound problem could be applied with a negligible error on objective value and shorter computational time. As future research, considering other probability distribution can be extended.

Other objective function, such as maximin, can be regarded as another extension, in which the minimum distance is maximized. The application of this objective is in hazardous location problems. Using the Euclidian distance functions can be also another extension for this work. For the Euclidian distance, the visibility and the shadow conditions must strongly regard. Considering the mobility of the existing facilities is advised for the future topics.

TABLE 4
RESULTS FOR THE SAMPLE PROBLEMS.

Sample problem	Existing facility	Period		In the presence of Barrier region	In the presence of Forbidden region
10-2	10	2	*x**	39.403	39.402
			*y**	39.286	39.277
			*z**	228.830	228.75
			No. Iter.	235.000	132
			Time(sec.)	1	0
20-2	20	2	*x**	36.669	37.684
			*y**	40.001	41.211
			*z**	267.019	265.263
			No. Iter.	1815	96
			Time(sec.)	5	0
50-2	50	2	*x**	41	41
			*y**	40.733	41
			*z**	272.667	270
			No. Iter.	1333	107
			Time(sec.)	8	1
50-4	50	4	*x**	39.600	38.702
			*y**	39.979	40.877
			*z**	345.79	345.789
			No. Iter.	482	199
			Time(sec.)	5	1
100-4	100	4	*x**	36.958	37.431
			*y**	40.001	39.569
			*z**	360.365	360
			No. Iter.	750	69
			Time(sec.)	19	1

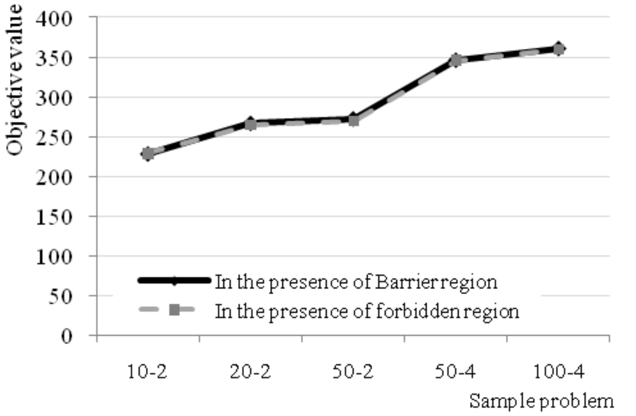

Figure 3: Comparison between objective values.

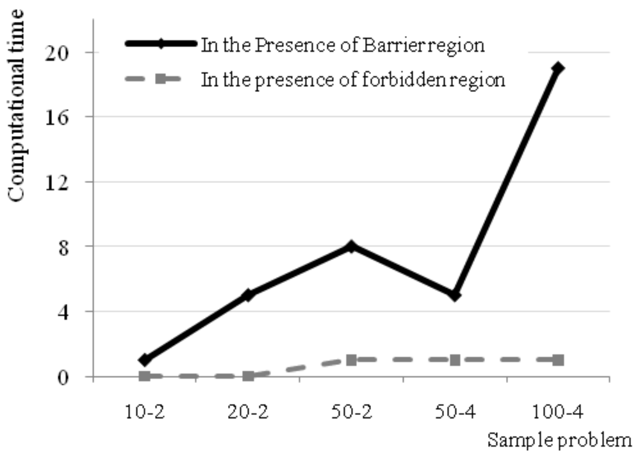

Figure 4: Comparison between computational times.

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