Biomechanical Investigation of Empirical Optimal Trajectories Introduced for Snatch Weightlifting

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ABSTRACT

The optimal barbell trajectory for snatch weightlifting has been achieved empirically by several researchers. They have studied the differences between the elite weightlifters' movement patterns and suggested three optimal barbell trajectories (type A, B, and C). But they didn't agree for introducing the best trajectory. One of the reasons is this idea that the selected criterion by researchers might not be appropriate. Therefore we build a biomechanical model based on inverse dynamic approach to evaluate each trajectory while considering a specific mechanical criterion. We calculate the optimal motion of each trajectory that minimizes the actuating torques by using dynamic programming approach. We solve an example problem for a specific weightlifter that lifts a 100 (kg) barbell. According to our criterion, we recommend the pattern type C as the best trajectory. The most important result of this simulation is the cost assigned to each trajectory which gives us the ability to evaluate the trajectories clearly. This method is an appropriate tool for coaches to examine each trajectory for any specific weightlifter and make a good decision for selecting the best trajectory.

KEYWORDS

Sport Biomechanics, Simulation, Dynamic Programming, Optimization.

1. INTRODUCTION

Optimization of sport techniques is one of the main goals of sport biomechanics. The most effective technique for snatch weightlifting has been achieved empirically by several researchers in the form of biomechanical characteristics of optimal movement. Power produced by weightlifters, barbell trajectory, and velocity of barbell are some biomechanical characteristics which have been used to analyze the optimal performance. The barbell trajectory has been investigated over the years by several researchers [1]-[10].

Dividing the barbell trajectories to optimal and non-optimal is in agreement with the most of the above mentioned researchers. The researchers have studied the differences between the elite weightlifters' characteristics of motion and then categorized the optimal lifting motion patterns. After all, they introduced several optimal trajectories for snatch weightlifting. Vorobyev [10] suggested three barbell movement patterns (type A, B, and C) for snatch weightlifting (Figure 1) and Garhammer [4] showed that the pattern type A is the best trajectory according to his investigation. Baumann et al. [1] reported some results which showed the type B is the best, but Hiskia [7] who studied on a large number of weightlifters concluded that the type C is more common than the other types, and the Byrd's [2] suggestion was similar to pattern type B.

One of the reasons to this inconsistency for choosing the best type is this idea that the selected criterion by above researchers might not be appropriate. Their criterion was the weightlifters' success percentage to do the snatch. Considering none of the mechanical characteristics to introduce the best pattern is the disadvantage of this criterion. Also the researchers accept the dependency of the barbell trajectory to the specific personal parameters such as anthropometric and physical characteristics. Therefore we offer a mathematical approach to judge between the conflicts.

To do this, we build a biomechanical model for evaluating the cost of each barbell trajectory. This model is based on inverse dynamic approach to evaluate the motions while considering the specific mechanical criteria. The results of this model improve our knowledge about the limitation of each pattern and suggest us the

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best solution to the above mentioned problem. Since the increasing coaches' tendency to biomechanical analysis of weightlifting, biomechanical evaluation of optimal motion patterns help them to categorize the weightlifters' performance.

On the other hand the previous optimal patterns have not the ability to improve the performance of elite weightlifters themselves, because you cannot compare a good pattern with itself to reach to a better performance. Therefore, according to above mentioned deficiencies, the necessity for developing a pure mechanical model which considers the specific characteristics of each individual will be obvious. This model leads us to the best ideal technique by using the mechanical principles.

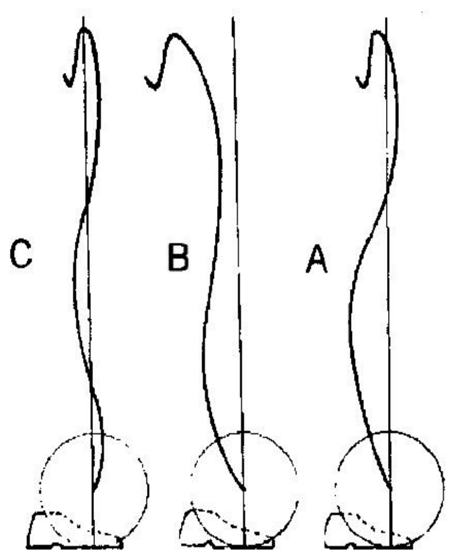

Figure 1: Three types of barbell trajectory in snatch weightlifting

We use an open kinematic chain as a model. This gives us the ability of using the mathematical approach for evaluating this technique. This model has five links in sagittal plane and has been used for modeling the lifting tasks by several researchers [11]-[13]. To evaluate each trajectory, we make the distal end of model (i.e. the position of barbell) move on the way of the trajectory. Because of the redundancy of the model there are many joint configurations that satisfy the desired motion. By using dynamic programming approach we choose the best answer and calculate its cost according to our selective criteria. Comparing the costs of trajectories which have been introduced by other researchers, we would be able to introduce the best of them. This mechanical criteria could be something like time, actuating joint torques [14], or energy consumption [15]. In recent years, some researchers have used actuating joint torque to introduce optimal patterns for lifting tasks. We choose the same criterion because of the reasons stated before [11] in which the relation between torques and injury is of more importance.

Let us summarize the problem as finding the minimum cost of each optimal trajectory by using the dynamic programming approach. The best trajectory is the one with minimum cost.

2. METHOD

To formulate the above mentioned problem, a set of motion equations and a criterion equation should be solved together. This situation forms a problem in optimal control domain. There are two different methods to solve this problem. The first is the indirect mathematical approach which gives us a unique solution [14] and the second is the direct search approach. This method searches between all solutions of motion equations to reach a solution which fulfills the criterion equation. We choose the latter because of its easier use and faster response. But the direct search without any specific search patterns is not suitable in this special problem. There are many algorithms to conduct the search approach like Genetic algorithm [16] and dynamic programming. The former is suitable when we try to find a new optimal trajectory and the latter is more suitable when we try to evaluate the previous trajectories.

By comparing the results of this model and other researchers' we will be able to examine the validity of our approach. But we should expect some differences because of two main reasons; first the intrinsic simplicity of our model and second, the deviation of each weightlifter from his/her perfect optimal trajectory.

A. Modeling

To build a biomechanical model of a weightlifter we should translate the physical property of human into the mathematical one. For this purpose, we can use the anthropometric models developed by several researchers. One of the comprehensive models has been introduced by Chaffin & Anderson [17]. By using this model, we have a multi-segment model that contains information about mass, center of gravity, length and moments of inertia of each segment which represents the whole body. In this model, the body segments convert into solid links and the body joints convert into simple revolute joints. We simplify this model to a two-dimensional sagittal plane model which can be used for modeling the weightlifting or other general lifting activities. This is a common assumption that has been used by several researchers [11], [13], and [18]. Now we should make a decision about the number of links we like to use and hence the number of degrees of freedom (DOF) which is the main factor that affects the complexity of model, and therefore it has a direct effect on time and cost of computing and solving the problem. The best model is the one that minimizes the complexity and simultaneously offers a good approximation of the whole motion. Several researchers used five DOF model to analyze lifting tasks [11], [13], and [18].

B. Equations of Motion

We use the five-link planar model in sagittal plane, which enables us to extract its motion equations. In Figure 2 the schematic diagram of this model at initial time can be observed. This model is made by five links by which shin, thigh, trunk, upper arm and forearm are represented, respectively named L1 to L5. Also, five body joints: ankle, knee, hip, shoulder and elbow are represented O1 to O5 respectively.

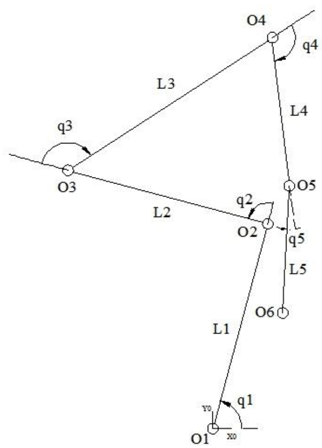

Figure 2: Biomechanical model of a weightlifter at initial position

The model motion can be described by the five relative joint coordinates which are defined by:

$$q_i = (\mathbf{X}_i, \mathbf{X}_{i-1})_{\mathbf{Z}_0} \qquad i = 1, \dots, 5 \quad (\mathbf{Z}_0 = \mathbf{X}_0 \times \mathbf{Y}_0) \tag{1}$$

Let us add the following complementary notations:

$$\mathbf{q} = (q_1, \dots, q_5)^{\mathrm{T}} \text{ , vector of joint coordinates} \tag{2}$$

$$\dot{\mathbf{q}} = (\dot{q}_1, \dots, \dot{q}_5)^{\mathrm{T}} \text{ , vector of joint velocities} \tag{3}$$

$$\ddot{\mathbf{q}} = (\ddot{q}_1, \dots, \ddot{q}_5)^{\mathrm{T}} \text{ , vector of joint accelerations} \tag{4}$$

Where $\dot{q}_i$ and $\ddot{q}_i$ are the first and the second time derivatives of q_i respectively. According to Figure 2, we define the dimensional characteristics of the model by:

$$\mathbf{O}_i\mathbf{O}_{i+1} = r_i\,\mathbf{X}_i\,, i = 1,\dots,5 \tag{5}$$

$$\mathbf{O}_i\mathbf{G}_i = a_i\,\mathbf{X}_i\,, i = 1,\dots,5 \tag{6}$$

Where r_i is the length, G_i is the center of gravity and a_i is the distance of G_i from proximal end of link L_i. Also m_i is the mass of link L_i and I_i^{zz} is the moment of inertia of link L_i with respect to the joint axis $(O_i\,\mathbf{Z_0})$.

Numerical values of these dimensional parameters are calculated based on body weight and height of a weightlifter, using the formula suggested by Chaffin & Anderson [17]. For obtaining the equations of motion the Lagrangian of the model is written as:

$$L(\mathbf{q}, \dot{\mathbf{q}}) = T(\mathbf{q}, \dot{\mathbf{q}}) - V(\mathbf{q}) \tag{7}$$

where V is the gravity potential and T is the kinetic energy defined by:

$$T(\mathbf{q}, \dot{\mathbf{q}}) = 1/2\,\dot{\mathbf{q}}^T \mathbf{M}(\mathbf{q})\,\dot{\mathbf{q}} \tag{8}$$

$\mathbf{M}$ is the mass matrix of the kinematics chain. Equations of motion may be derived by Lagrange's formula:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = Q_i^d + Q_i^a \quad , i = 1, \dots, 5 \tag{9}$$

where Q_i^a represents the joint actuating torque exerted by L_{i-1} on L_i at O_i and Q_i^d is joint dissipative torque. We neglect the Q_i^d because it is very small by comparison with Q_i^a.

C. Constraints

Initial and final conditions define the conditions of model at start position and at the end of second pulling phase (i.e. start of catching phase). Initial conditions are the angular position of each joint and the barbell velocity at the beginning of motion, i.e., "Lift-off" phase. Final conditions are the position and velocity of barbell at the end of second pulling phase (i.e. beginning of "catch" phase) of the snatch lift, and there is not any specific configuration at this position.

In order to respect joint stops, to prevent counter-flexion and to moderate total joint coordinate variations, we have to prescribe bounds on the joint coordinates, defined by the below constraints:

$$t \in [t^i, t^f], \quad i \le 5\,, \quad q_i^{\min} \le q_i(t) \le q_i^{\max} \tag{10}$$

Where t^i and t^f are the initial and final times and $q_i^{\min}$ and $q_i^{\max}$ are specified values [12]. In addition to state or kinematics constrains, we use control constraint terms for the inequalities defining limitation on torques acting on the mechanical system. Torques which are produced by actuators (i.e., muscles) have limited values [17]. Therefore we can write:

$$t \in [t^i, t^f], \quad |Q_i^a(t)| \le Q_i^{a,\max} \tag{11}$$

D. Criterion Function

We want to calculate the optimal motion of each trajectory by minimizing a performance criterion or dynamic cost. We minimize the actuating torques similar to several researchers who used this criterion to optimize human movement [11], [12], and [15]. Therefore the criterion function is:

$$J = \sum_{i=1}^{5} \int_{t_i}^{t_f} Q_i^{a^2}\, dt \qquad (12)$$

We extract the cost of each solution by using the function (12). But, before of this step we must solve the equations of motion (9) to compute the actuating torques. This is performed by using inverse dynamics approach. We employ the joint coordinates during each step, and use the numerical differentiating to calculate the joint velocities and accelerations. Once we have these data, it could be possible to calculate the actuating torques.

E. Optimization Algorithm

To evaluate barbell trajectories, we assign a cost to each one which is calculated as the sum of the actuating torques during motion. But because of redundancy in degrees of freedom, we encounter many solutions, i.e. many joint patterns combine together to make the same trajectory. If we choose the best solution for each trajectory, we will be able to compare the trajectories and introduce the best one. But the difficulty is how we could test all possible solutions. The dynamic programming approach satisfies our goal. We discrete the joint angles space and consider all possible solutions. To do this we divide our trajectory to small distances, start from the predefined initial point, go to second step, then third and so on. We know the position of each step according to given trajectories and assume the small amount tolerances. In each position we have many configurations of joint angles which position the end point (i.e. barbell) in the desired location. Therefore we have $n(i)$ choice in i th step. From the initial point we can go to $n(i)$ points in the second step and from each of these $n(i)$ points we can go to $n(i+1)$ points in third step (Figure 3) and continue this rule until reach the final state which is not unique in our problem.

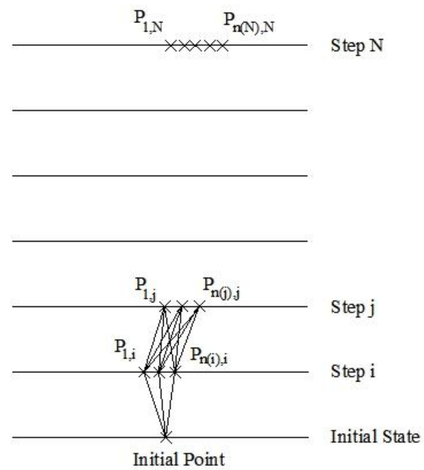

Figure 3: The motion steps, admissible points and possible paths in algorithm strategy

We calculate all costs from each point in i th step to each point in $(i+1)$ th step. We have many numbers as the cost related to each group of two adjacent points. We save them in a table and start the main algorithm, i.e. dynamic programming. We start from the initial point and calculate the cost of all the point in next step. The cost of each point in second step is defined and because there is only one way from initial point to each point in second step, this unique cost is minimum. Then, we calculate the minimum cost of each point in the third step. From each point we consider the corresponding cost to the previous points in second step and from the latter point to the first initial point. We check all possible motion from each point in third step to initial point and find the minimum of them as the minimum cost relating to each point in third step. We call this cost as the $\mathrm{Costmin}(P_{k,i})$ where $P_{k,i}$ is the point in i th step (here is third) and k refers to the order number of this point between $n(i)$ points in this i th step. We continue this procedure to reach the points in final step. The mathematical notation of the above mentioned procedure could be shown as:

$$\begin{aligned}&\mathrm{Costmin}(P_{k,i}) = \mathrm{Cost}(\mathrm{initial}, P_{k,i})\\ &\qquad ,k = 1,\ldots,n(i) \quad , \quad i = 1\end{aligned} \qquad (13)$$

$$\begin{aligned}&\mathrm{Costmin}(P_{k,j}) = \min[(\mathrm{Cost}(P_{l,i}, P_{k,j}) +\\ &\qquad \mathrm{Costmin}(P_{l,i})]\\ &\qquad k = 1,\ldots,n(j) \quad , \quad j > 1 \quad ,\\ &\qquad l = 1,\ldots,n(i) \quad , \quad i = j-1\end{aligned} \qquad (14)$$

Where $P_{k,i}$ is k th point in i th step and the total number of acceptable point in this step is $n(i)$.

After we calculate the minimum cost corresponding to each point, it is possible to examine all the points in final step and select the point with minimum cost among all the points in final step. This is the minimum cost from initial point to final state and it is the answer of desired cost related to the specified trajectory.

Now we are able to compare as many trajectories as introduced before. The comparison result shows us the best trajectory according to our desired criterion.

F. Snatch Weightlifting Technique

We solve a problem for a weightlifter with 80 (kg) mass and 1.80 (m) height who lifts a 100 (kg) barbell by snatch technique. Other dimensional parameters are calculated based on this information [17]. We select the summation of actuating torques of all joints as the optimization criterion and solve our problem between initial point and final state which represent the start of snatch and the start of catch phase respectively. Considering the snatch description as "vertically accelerating the barbell to a sufficient height, enabling the lifter to rapidly move beneath the bar and support it in an overhead full squat position" [19], the start of catch phase are selected in a manner that the barbell has a good condition to continue its motion and the weightlifter would be able to move under the bar quickly.

To validate the optimization algorithm, we test the program with two optimal trajectories which were obtained by optimizing the snatch weightlifting before [14], [16]. The optimal motions obtained with this algorithm are approximately similar to previous optimized motions. The minor differences are due to the allowable tolerances set for making the desired trajectory and some differences in constraints. This result assures us of proper working of the optimization algorithm. Figure 4 shows the optimal motions obtained by genetic algorithm and our method for a same trajectory.

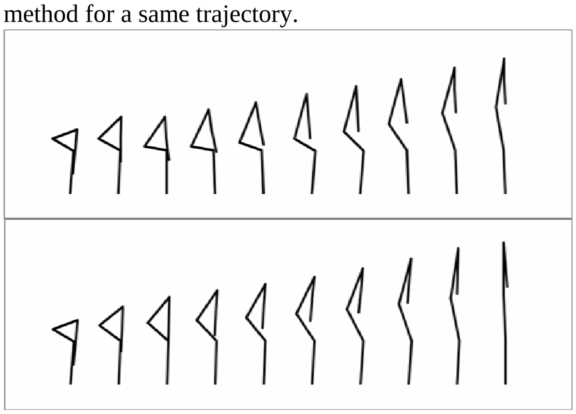

Figure 4: The best possible motion for a same trajectory obtained by our algorithm (up) and genetic algorithm (down)

3. Results

The most important question we try to answer is which trajectory is the best to recommend to the weightlifter. As we show in this section we find that according to our definition of "best" we recommend the pattern type C which is in agreement with Hiskia's suggestion [7] and in addition, it is in agreement with optimal trajectory obtained mathematically by Nejadian et al. [14].

Firstly, in Figure 5, we show the trajectories simulated in our approach. These trajectories can be compared with experimental suggestions introduced before by researchers. There are some dissimilarities between Figure 1 and Figure 5 because of insufficient precision of the trajectories shown in Figure 1 and some tolerances set to the introduced trajectory in our algorithm.

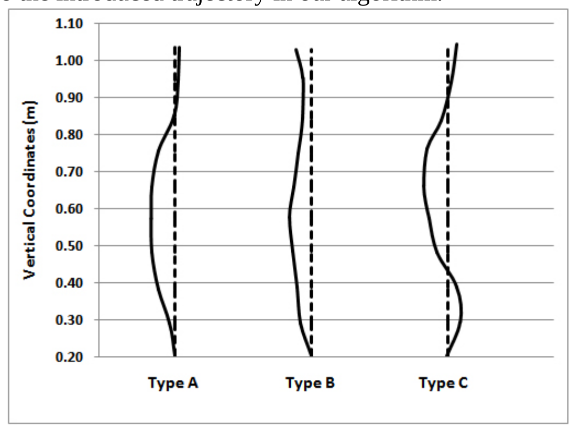

Figure 5: The simulated trajectories according to previous optimal patterns

We run our algorithm for three suggested trajectories. Figure 6 shows the best possible motion of model in each trajectory according to criterion function. The best motion is achieved by using dynamic programming in the described manner. As mentioned, because of the redundancy of model there are many ways to complete the motion. Comparing the trajectories we calculated the minimum possible cost relating to the best possible motion of each trajectory.

To assure these trajectories whether could be performed by weightlifters or not, we calculated the actuating torques of each joint. These actuating torques should be in their acceptable ranges. The actuating torques of hip joint shown in Figure 7 is in the defined acceptable range.

Finally the most important result of this simulation is the cost assigned to each trajectory. This information gives us the ability to compare the trajectories introduced by other researchers. In Figure 8 we show the cost of each trajectory as a percentage of the maximum torque cost. Since our problem has been solved for minimizing the actuating torques, we depict the costs related to this criterion and we rank the trajectories by using it.

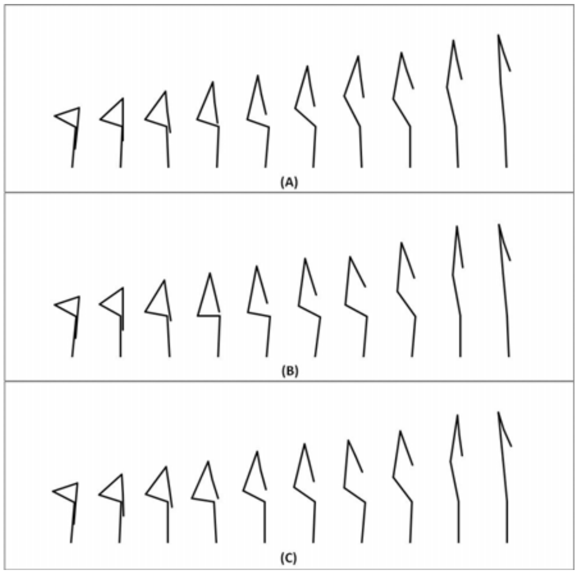

Figure 6: The best possible motion according to each trajectory; (A): trajectory type A, (B): trajectory type B, and (C): trajectory type C

4. DISCUSSION

In the procedure of modeling, optimizing, and evaluating the empirical optimal trajectories we introduce a mechanical base to judge about previous findings and give a measurement tool to evaluate and compare them. Now, each trajectory can be evaluated clearly and without any ambiguity if we agree in definition of "the best" trajectory. Unfortunately, we cannot see this clarity when we read the discussion of each researcher about the explanation of why his result is better than the others. Now we are able to discuss about the selected criterion for defining "the best". Some researchers may believe that the actuating torque is not suitable and it is better to select the power expenditure as the criterion for defining "the best". We choose the former because of reasons stated before which emphasis on the relation of this criterion with the injury that may be occurred during this technique.

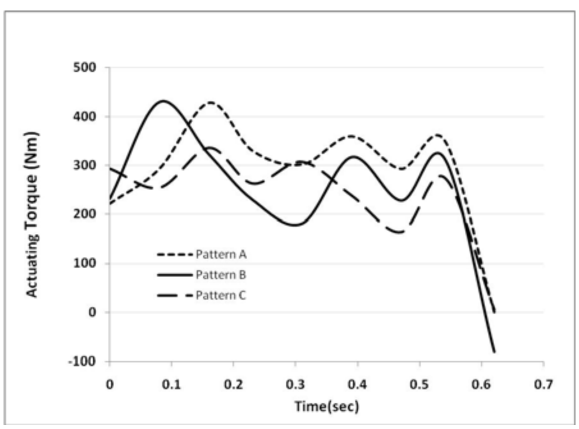

Figure 7: The hip actuated torques for optimal motion of each trajectory

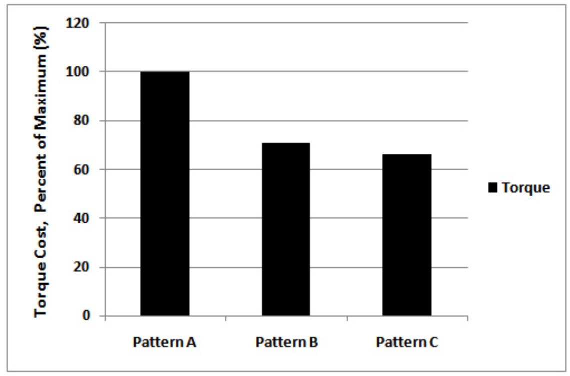

Figure 8: The cost of each trajectory based on actuating torques

Another point is the possible differences of trajectories simulated in our approach (Figure 5) with the exact trajectories introduced by researchers (Figure 1). These differences will be refined if we use the exact data. We believe that our approach gives us a powerful tool that can be used to examine any suggested trajectory and there is no limitation to restrict our investigation to three types stated before.

An interesting point in the best motion sequences (Figure 6) is the existence of double knee bend technique that we observe in previous findings and recommendation of best techniques. This technique is more obvious in the trajectories type B and C than the trajectory type A. Perhaps the motion sequences used by weightlifters whose trajectories are similar to type A, are not the best ones that the weightlifters can use regarding to mechanical cost. If this is true we can say that the real score of these weightlifters is even much worse than the value obtained by our simulation and this makes our recommendation of trajectory type C more reasonable.

In Figure 7 we see the range of actuated torque in hip joint and we can compare it in three recommended trajectories. As shown in this Figure the trajectory type C is more suitable than the others. If we note to this fact that the hip torque has the main role in snatch motion it can satisfy us to recommend the trajectory type C based on the hip joint criterion and it is in agreement with the results obtained when the criterion is the sum of all actuating torques (Figure 8).

Figure 8 shows the ranking of trajectories while considering their costs as the sum of the actuating torques during snatch motion from initial to catch phase. The trajectory type A has the highest cost of actuating torques and the trajectory type C has the lower cost regarding to this criterion. We should notice this fact that our model has been optimized by considering the actuating torques and therefore we select the best trajectory regarding to this criterion. Hence our answer for the best trajectory is type C. If someone wants to change the criterion of selecting the best, he/she should rerun the program considering his/her criterion for optimization.

An Important point is that these results have been obtained for a specific weightlifter, with some specific characteristics like mass and height, who lifts a specific weight. If these characteristics change according to another weightlifter, we will able to extract the results tailored to him/her. This is the big advantage of the modeling approach that we can repeat the evaluation of problem with any input data.

The method used in this research is an appropriate tool for coaches and gives them the ability to examine each suggested trajectory for any specific weightlifter and make a good decision for selecting the best trajectory. Finally we introduce a new method which can be refined by complicated models and forms a very useful evaluating tool.

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