Small Scale Effect on the Buckling Analysis of a Double-Walled Carbon Nanotube under External Radial Pressure Using Energy Method

Z. S. Mousavi [i], A. Ghorbanpour Arani [ii*] and M. Mohammadimehr [iii]

Received 21 April 2008; received in revised 5July 2009; accepted 4 Setember 2010

ABSTRACT

In this paper, using energy method, small scale effects on the buckling analysis of a double-walled carbon nanotube (DWCNT) under external radial pressure is studied. The constitutive equations derived for a DWCNT using the nonlocal theory of elasticity which Eringen are presented for the first time. By minimizing the second variation of the total energy for a DWCNT, hence, the value of the nonlocal critical buckling load is obtained. It is seen from the results that the nonlocal critical buckling load increases with increasing the circumferential wave number. Moreover, it is seen that the nonlocal critical buckling load is lower than that of the local one. It is shown from the results that the ratio of the critical buckling load decreases with increasing the length of nanotubes while it increases with decreasing the radius of the outer tube.

KEYWORDS

Nonlocal shell model; Buckling; DWCNT; External radial pressure; Energy method.

1. INTRODUCTION

Carbon nanotubes (CNTs) due to its wonderful properties have attracted attention, to many researchers in different fields such as chemistry, physics and engineering9. . Recently, one of the important topics is the buckling behavior of CNTs in the mechanical engineering. Ru [2] studied an elastic double-shell model for buckling of a double-walled carbon nanotube (DWCNT) embedded in an elastic medium based on the Winkler model. He obtained an explicit formula for the critical axial strain and concluded that the critical axial strain of the embedded DWCNT was lower than that of an embedded single-walled carbon nanotube (SWCNT). Wang et al. [3] investigated the axial compressed buckling of multi-walled carbon nanotube (MWCNT) subjected to combined internal or external pressure and axial compression stress. In their studies, MWCNT according to its radius to thickness ratio are thin, thick or solid. Their results show that the increase of the critical axial stress due to an internal radial pressure appears to be in qualitative agreement with some known results for filled SWCNTs obtained by molecular dynamics simulations. Guo et al. [4] studied about the bending buckling of SWCNTs using atomic scale finite element. Their studies show that increasing the bending angle enough, the kinks appear and the morphology of SWCNT is change abruptly. Lee and Chang [5] investigated temperature buckling of two type of SWCNT, zigzag and armchair under uniform temperature rise and presented a close form solution for the various length to diameter ratio. They modeled carbon nanotube as a Timoshenko beam to consider the effects of transverse shear deformation and rotary inertia. Ghorbanpour Arani et al. [6] investigated the torsional and the axially-compressed bucklings of an individual, embedded, multi-walled carbon nanotube (MWCNT) subjected to internal and external pressures. They considered the effects of the small length scale and the surrounding elastic medium. Their results showed that the internal pressure increased

i Z. S. Mousavi is with the Department of Mechanical Engineering, Faculty of Engineering, University of Kashan, Kashan, I.R. Iran (e-mail: z.mousavi@me.iut.ac.ir).

ii * Corresponding Author, A. Ghorbanpour is with the Department of Mechanical Engineering, Faculty of Engineering, University of Kashan, Kashan, I.R. Iran (e-mail: aghorban@kashanu.ac.ir ; a_ghorbanpour@yahoo.com)

iii M. Mohammadimehr is with the Department of Mechanical Engineering, Faculty of Engineering, University of Kashan, Kashan, I.R. Iran (e-mail: mmohammadimehr@gmail.com)

the critical load while the external pressure tended to decrease it. Ranjbartoreh et al. [7] derived expressions for the critical axial forces and pressures of DWCNTs and calculated the critical axial forces and pressures for different axial half-sine and circumferential sine wave numbers. They compared their results with those of the SWCNTs.

In this paper, small scale effects on the buckling analysis of a DWCNT under external radial pressure are investigated. The constitutive equations derived for a DWCNT using the nonlocal theory of elasticity which Eringen presented it for the first time. By minimizing the second variation of the total energy for a DWCNT, hence, the value of the nonlocal critical buckling load is obtained. The effects of the small scale and the van der Waals forces between the inner and the outer tube on the critical buckling load are taken into account.

2. NONLOCAL CONTINUUM SHELL MODEL

According to the Eringen nonlocal elasticity model [8], the stress state at a reference point in the body is regarded to be dependent not only on the strain state at this point but also on the strain states at all of the points throughout the body. The constitutive equations of the nonlocal elasticity can be written as [8]

$$(1-e_0^2a^2\nabla^2)\sigma = C_0 : \varepsilon \tag{1}$$

Symbol ':'denotes the inner product of tensor, C_0 is the tensile stress tensor of the isotropic elasticity, σ and ε represent the nonlocal stress and strain tensors, respectively. e_0 denotes a material constant $(e_0 = 0.82)$, and a is the internal characteristic length (e.g., length of C-C bond) $(a = 0.142\,nm)$.

Consider a thin cylindrical shell model with the radius R and the thickness h. Moreover, the x-direction is considered to be parallel to the axis of the cylinder, the θ-direction is tangent to a circular arc, and the z-direction is normal to the medium surface.

The following assumptions are used for the cylindrical shell model

- The cylindrical shell model is considered as a thin.
- Lateral deflections are negligible compared to thickness of the shell.
- During bending, lines normal to the middle surface of shell remain straight and normal.
- The material for this model is homogenous, isotropic and elastic solid.
- $\bar{\sigma}_{x,\theta}$ and $\bar{\sigma}_{\theta,\,x}$ are equal to zero.

Using Eq. (1) and applying the above assumptions, the constitutive equations for the plane stress state can be written as

$$\begin{aligned}
&\bar{\sigma}_x-(e_0a)^2\bar{\sigma}_{x,xx} = E/(1-\nu^2)(\bar{\varepsilon}_x+\upsilon\bar{\varepsilon}_\theta)\\
&\bar{\sigma}_\theta-(e_0a)^2\bar{\sigma}_{\theta,\theta\theta}/R^2 = E/(1-\nu^2)(\bar{\varepsilon}_\theta+\upsilon\bar{\varepsilon}_x)\\
&\bar{\sigma}_{x\theta}-(e_0a)^2(\bar{\sigma}_{x\theta,xx}+\bar{\sigma}_{x\theta,\theta\theta}/R^2) = E/(1+\nu)\bar{\varepsilon}_{x\theta}
\end{aligned} \tag{2}$$

where, E and υ are Young's modulus and Poisson's ratio.

Symbol '¯' denotes the stress and the strain at each point of the body and Symbol ',' means derivation.
Equation (2) can be rewritten as:

$$\begin{aligned}
\bar{\sigma}_x &= E/(1-\nu^2)(\bar{\varepsilon}_x+\nu\bar{\varepsilon}_\theta+(e_0a)^2\bar{\varepsilon}_{x,xx}+\\
&\qquad \nu(e_0a)^2\bar{\varepsilon}_{\theta,xx})\\
\bar{\sigma}_\theta &= E/(1-\nu^2)(\bar{\varepsilon}_\theta+\nu\bar{\varepsilon}_x+(e_0a/R)^2\bar{\varepsilon}_{\theta,\theta\theta}+\\
&\qquad \nu(e_0a/R)^2\bar{\varepsilon}_{x,\theta})\\
\bar{\sigma}_{x\theta} &= E/(1+\nu)(\bar{\varepsilon}_{x\theta}+(e_0a)^2\bar{\varepsilon}_{x\theta,xx}+\\
&\qquad (e_0a/R)^2\bar{\varepsilon}_{x\theta,\theta\theta})
\end{aligned} \tag{3}$$

3. VAN DER WAALS FORCES

The van der Waals force is modeled by radial pressure. Thus, the pressure on the outer tube can be expressed by

$$P_1 = c\,(w_1 - w_2) \tag{4}$$

where, w_1 and w_2 are the radial displacement of the outer and the inner tubes and c is the Van der Waals interaction coefficient that can be estimated as:

$$c = \frac{320\,erg\;/\,cm^2}{0.16\,d^2}\;(d\;=0.142\;nm) \tag{5}$$

The van der Waals forces between two tubes are equal and opposite, thus the pressure on the inner tube can be written as

$$P_2 = \;-\frac{R_1}{R_2}P_1 \tag{6}$$

where, R_1 and R_2 are the radii of the outer and the inner tubes.

4. BUCKLING ANALYSIS

Figure 1 illustrates a DWCNT under external radial pressure. It is a DWCNT of the inner radius R_2, the outer radius R_1, length L, thickness h.

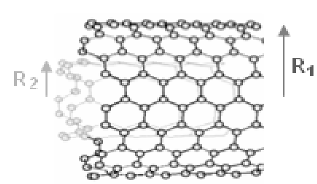

Figure 1: Model of a DWCNT

The total potential energy of DWCNT (V) is the sum of the strain energy U and the virtual work Ω of the applied pressure.

$$V = U + \Omega \quad (7)$$

The strain energy of the DWCNT can be expressed as

$$U = \frac{1}{2}\iiint(\bar{\sigma}_x\bar{\varepsilon}_x + \bar{\sigma}_\theta\bar{\varepsilon}_\theta + \bar{\sigma}_{x\theta}\bar{\varepsilon}_{x\theta})dV \quad (8)$$

Substitution of Eq. (3) into Eq. (8) yields

$$\begin{aligned} U = \frac{1}{2}\iiint\{ & E/(1-\upsilon^2)(\bar{\varepsilon}_x{}^2 + \upsilon\bar{\varepsilon}_x\bar{\varepsilon}_\theta + (e_0a)^2\bar{\varepsilon}_x\bar{\varepsilon}_{x,xx} \\ & + \upsilon(e_0a)^2\bar{\varepsilon}_x\bar{\varepsilon}_{\theta,xx} + \bar{\varepsilon}_\theta{}^2 + \upsilon\bar{\varepsilon}_\theta\bar{\varepsilon}_x + (e_0a/R)^2\bar{\varepsilon}_\theta\bar{\varepsilon}_{\theta,\theta\theta} + \\ & \upsilon(e_0a/R)^2\bar{\varepsilon}_\theta\bar{\varepsilon}_{x,\theta}) + E/(1+\upsilon)(\bar{\varepsilon}_{x\theta}{}^2 + (e_0a)^2 \\ & \bar{\varepsilon}_{x\theta}\bar{\varepsilon}_{x\theta,xx} + (e_0a/R)^2\bar{\varepsilon}_{x\theta}\bar{\varepsilon}_{x\theta,\theta\theta})\}dV \end{aligned} \quad (9)$$

The applied virtual work for the external radial pressure can be written as

$$\Omega = -R\iint Pwdxd\theta \quad (10)$$

where, P is the resultant radial outward pressure and w is the radial deflection in z direction due to the buckling. Equation (10) for the inner and the outer tube is obtained

$$\begin{aligned} \Omega_{inner} &= R_1\iint c(w_{outer} - w_{inner})w_{inner}dxd\theta \\ \Omega_{outer} &= R_1\iint\{P - c(w_{outer} - w_{inner})\}w_{outer}dxd\theta \end{aligned} \quad (11)$$

The relationships between the normal and the shear strains and displacements at the middle point can be written as

$$\begin{aligned} \varepsilon_x &= u_{,x} + \frac{1}{2}\beta_x{}^2 \\ \varepsilon_\theta &= \frac{v_{,\theta} + w}{R} + \frac{1}{2}\beta_\theta{}^2 \\ 2\varepsilon_{x\theta} &= \frac{u_{,\theta}}{R} + v_{,x} + \beta_x\beta_\theta \end{aligned} \quad (12)$$

where, $\beta_x = -w_{,x}$, $\beta_\theta = -\frac{w_{,\theta}}{R}$.

The normal and the shear strains at each point of the cylindrical shell can be defined as

$$\begin{aligned} \bar{\varepsilon}_x &= \varepsilon_x + z\,\kappa_x \\ \bar{\varepsilon}_\theta &= \varepsilon_\theta + z\,\kappa_\theta \\ \bar{\varepsilon}_{x\theta} &= \varepsilon_{x\theta} + z\,\kappa_{x\theta} \end{aligned} \quad (13)$$

where, $\kappa_x = -w_{,xx}$, $\kappa_\theta = -\frac{w_{,\theta\theta}}{R^2}$, $\kappa_{x\theta} = -\frac{w_{,x\theta}}{R}$.

Using Eq. (12) and substituting Eq. (13) into Eq. (9), the strain energy in terms of displacements is obtained. For obtaining the critical buckling load, the second variation of the total potential energy can be expressed as

$$\delta^2V = \delta^2U + \delta^2\Omega \quad (14)$$

where, $\Pi = \delta^2V$.

To investigate the possible existence of adjacent-equilibrium configurations, the small increments to the displacement variables are given and the two adjacent configurations represented by the displacements before and after the increment are examined, consequently, we have

$$\begin{cases} u \to u_0 + u_1 \\ v \to v_0 + v_1 \\ w \to w_0 + w_1 \end{cases} \quad (15)$$

If F is the function of displacements (u , v and w), for calculating the second variation of F (δ^2F), virtual displacement from u_0, v_0 and w_0 to u_0+u_1, v_0+v_1 and w_0+w_1 is considered. The second variation of F can be written as

$$\begin{aligned} \delta^2F = & F_{,uu}(u_0,v_0,w_0)u_1^2 + F_{,vv}(u_0,v_0,w_0)v_1^2 \\ & + F_{,ww}(u_0,v_0,w_0)w_1^2 + 2F_{,uv}(u_0,v_0,w_0)u_1v_1 \\ & + 2F_{,uw}(u_0,v_0,w_0)u_1w_1 + 2F_{,wv}(u_0,v_0,w_0)w_1v_1 \end{aligned} \quad (16)$$

In this study, the simply supported boundary conditions at two ends of tubes are considered as:

$$w_1(x=0) = w_{1,xx}(x=0) = w_1(x=L) = w_{1,xx}(x=L) = 0 \quad (17)$$

The buckling modes assumed to be of the following forms

$$\begin{cases} w_{1-outer\ tube} = A_{mn}\sin n\theta.\sin\frac{m\pi}{L}x \\ w_{1-inner\ tube} = B_{mn}\sin n\theta.\sin\frac{m\pi}{L}x \end{cases} \quad (18)$$

where, A_{mn}, B_{mn} are two real constants, m is the half axial wave number in the longitudinal direction, and n denotes the circumferential wave number.

For minimizing the second variation of the total potential energy ($\Pi = \delta^2V$) by Rayleigh-Ritz method, the following equations are obtained

$$\begin{cases} \partial\Pi/\partial A_{mn} = 0 \\ \partial\Pi/\partial B_{mn} = 0 \end{cases} \Rightarrow \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}\begin{Bmatrix} A_{mn} \\ B_{mn} \end{Bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (19a)$$

In order to obtain a non-trivial solution, it is necessary to set the determinant of the coefficient matrix in Eq. (19a) to zero.

$$M_{11}\,M_{22} - M_{12}\,M_{21} = 0 \quad (19b)$$

Solving Eq. (19b) yields the buckling load of the DWCNT for different wave numbers m and n that all processes to obtain the critical buckling load are shown in appendix A.

5. Discussions

The shell dimensions and its mechanical properties are considered as follows [7, 8]

$$\begin{aligned} & R_1 = 6.34\,nm,\ R_2 = 6\,nm,\ L = 30\,nm, h = 0.1\,nm \\ & e_0 = 0.82, a = 0.142\,nm \end{aligned} \quad (20)$$

Figure 2 shows the nonlocal critical buckling load under external radial pressure versus the half axial wave number for the circumferential wave number of 2, 6 and 16. By Maple software, the nonlocal critical buckling load (P_{cr}) is occurred at *m=1, n=16.* The value of the nonlocal critical buckling pressure for DWCNTs at *m=1, n=16* is 177.624 Mpa. It is seen that the nonlocal critical buckling load increases with increasing the circumferential wave number.

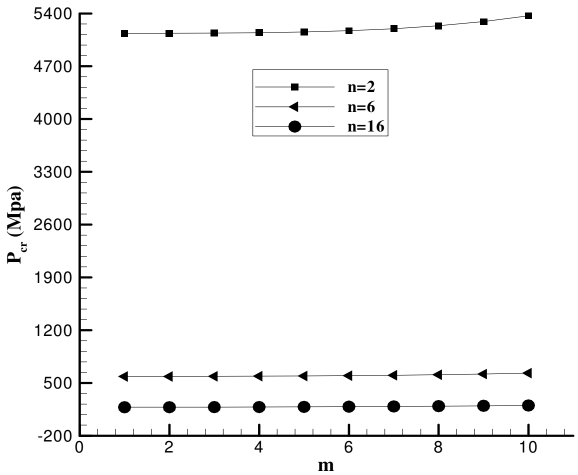

Figure 2: Applied radial pressure for various buckling mode using the nonlocal theory of elasticity ($e_0a = 0.11644\,nm$)

By Maple software, the local critical buckling load (P_{cr}) is occurred at *m=1, n=15.* The value of the local critical buckling pressure for DWCNTs at *m=1, n=15* is 184.726 Mpa. Figure 3 illustrates the local critical buckling load under external radial pressure versus the half axial wave number for the circumferential wave number of 2, 6 and 15. It is seen from the results that the local critical buckling load increases with increasing the circumferential wave number.

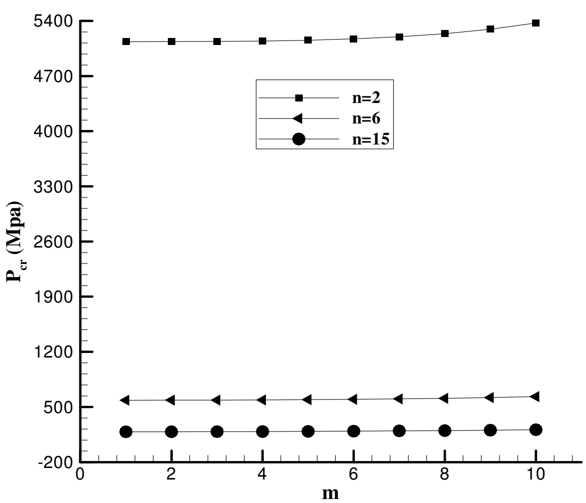

Figure 3: Applied radial pressure for various buckling mode using the local (classical) theory of elasticity ($e_0a = 0$)

To illustrate the influence of the small scale on the critical buckling load of a DWCNT in accordance with Eq. (19b), ψ , i.e., the ratio of the critical buckling load is defined as

$$\psi = \frac{P_{cr}(e_0a = 0)}{P_{cr}(e_0a = 0.11644\,nm)} \tag{21}$$

Figure 4 shows the ratio of the critical buckling load versus the half axial wave number for the circumferential wave number of 2, 6 and 10. It is concluded from this figure that the ratio of the critical buckling load increases with increasing m . Moreover, it is seen in accordance with Eq. (21) and this figure that the nonlocal critical buckling load is lower than the local critical buckling load.

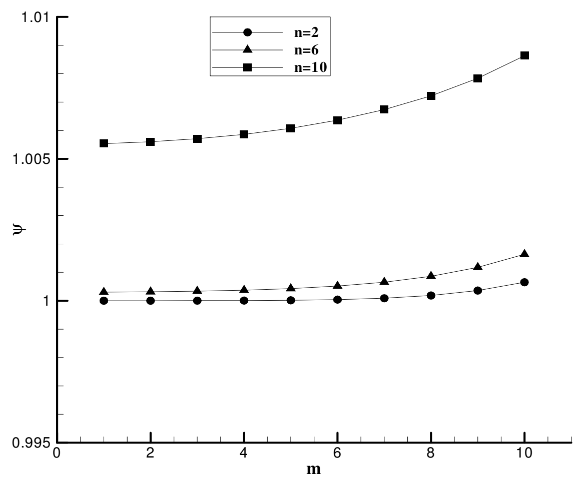

Figure 4: Influence of the small scale on the critical buckling load

Figure 5 demonstrates the effect of R_1 (the radius of the outer tube) on the critical buckling load that the obtained results for the values of the half axial and the circumferential wave numbers are equal to *6* and *10*, respectively. It is shown that the ratio of the critical buckling load decreases with increasing the length of nanotubes. Moreover, the ratio of the critical buckles load increases with decreasing the radius of the outer tube.

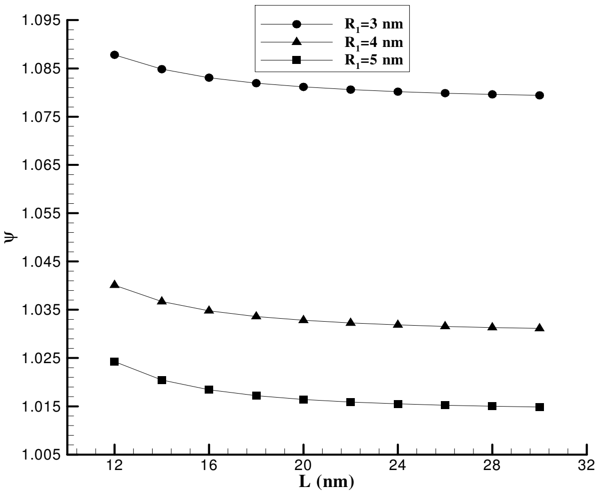

Figure 5: Effect of the small scale on the critical buckling load for different radius of the outer tube

6. CONCLUSION

Small scale effects on the critical buckling load of DWCNTs under radial pressure using energy method based on nonlocal elasticity was investigated in this paper. The van der Waals force was modeled by a radial pressured, which is appropriate by the difference between the radial displacement of the inner and the outer tubes. By minimizing the second variation of the total energy of DWCNT, the critical buckling load was obtained.

By Maple software, the nonlocal and the local critical buckling loads are occurred at ($m=1$, $n=16$) and ($m=1$, $n=15$), respectively that their values of the nonlocal and the local critical buckling pressures are equal to 177.624 Mpa and 184.726 Mpa. It was seen from the results that the nonlocal critical buckling load increases with increasing the circumferential wave number. Moreover, it was seen in accordance with Eq. (21) and Figure 4 that the nonlocal critical buckling load is lower than the local critical buckling load. Thus the effects of small scale must be considered in the analysis and design of the carbon Nanotube. Moreover, it was shown that the ratio of the critical buckling load decreases with increasing the length of nanotubes while it increases with decreasing the radius of the outer tube.

7. APPENDIX A

The critical applied radial pressure using the nonlocal theory of elasticity can be written as the following equation:

$P_{cr}=A/B.$

Where,

$A=A_1+A_2,$

$A_2=A_3+A_4+A_5,$

$A_3=A_6+A_7+A_8,$

$A_8=A_9+A_{10}.$

$A_1 = -144\, h\, E\, L^{12} (R_2R_1)^4 \{2c\, (\nu^2-1)\, (R_1)^3 + 2\, c\, (\nu^2-1)\, R_1 (R_2)^2 + E\, h\}$

$A_4 = -E\, \pi^4 (R_2R_1)^4 h^3 m^4 (L-m\, \pi\, e_0a)\, (L+m\, \pi\, e_0a)\, \{12\, E\, L^6\, h\, (R_1)^2 + 12\, E\, L^6\, h\, (R_2)^2 + E\, h^3\, m^4 (R_1R_2)^2 \pi^4 L^2 - E\, (e_0a)^2 m^6 h^3 (R_1R_2)^2 \pi^6 + 48\, c\, (\nu^2-1)\, L^6 R_1 (R_1R_2)^2 c\}$

$A_5 = -E\, L^{12} h^3 n^4 \{E\, h^3 (e_0a)^4 n^8 - E\, h^3 n^6 (R_2)^2 (e_0a)^2 - E\, h^3 n^6 (e_0a)^2 (R_1)^2 + E\, h^3 (R_1R_2)^2 n^4 - 12\, E\, h\, (e_0a)^2 n^2 (R_1)^4 - 12\, E\, h\, (R_2)^4 n^2 (e_0a)^2 + 12\, E\, h\, (R_1)^2 (R_1R_2)^2 + 12\, E\, h\, (R_2)^2 (R_1R_2)^2 - 24\, n^2 R_1\, c\, \nu^2 (R_2)^6 (e_0a)^2 + 24\, n^2 (e_0a)^2 (R_1)^7 c + 24\, n^2 (R_1)\, c\, (R_2)^6 (e_0a)^2 - 24\, n^2 (e_0a)^2 (R_1)^7 c\, \nu^2 + 24\, c\, (\nu^2-1)\, (R_1)^5 (R_1R_2)^2 + 24\, c\, (\nu^2-1)\, (R_2)^3 (R_1R_2)\},$

$A_6 = E^2 L^6 \pi^2 h^6 m^2 n^6 (e_0a)^2 (R_1)^2 \{-L^4 n^4 (e_0a)^2 - \pi^4 m^4 (e_0a)^2 (R_1)^4 + L^4 n^2 \nu\, (R_1)^2 + L^4 n^2 (R_1)^2 - L^2 \pi^2 m^2 n^2 (e_0a)^2 (R_1)^2 + L^2 \pi^2 m^2 (R_1)^4\},$

$A_7 = E^2 L^6 \pi^2 h^6 m^2 n^6 (e_0a)^2 (R_2)^2 \{-L^4 n^4 (e_0a)^2 - \pi^4 m^4 (e_0a)^2 (R_2)^4 + L^4 n^2 (R_2)^2 + L^4 n^2 \nu\, (R_2)^2 - L^2 \pi^2 m^2 n^2 (e_0a)^2 (R_2)^2 + L^2 \pi^2 m^2 (R_2)^4\},$

$A_9 = E\, L^2 \pi^2 (R_1R_2)^2 (e_0a)^2 h^3 m^2 n^2 \{-24\, L^6 \pi^2 (R_1)^3 (R_2)^4 m^2 c + 2E\, L^4 \pi^4 (R_1R_2)^2 h^3 m^4 n^2 \nu + 2EL^4 \pi^4 m^4 n^2 h^3 (R_2)^4 + 24\, L^8 (R_1)(R_2)^4 n^2 c\, \nu^2 - EL^4 \pi^4 (R_1)^2 n^4 m^4 h^3 (e_0a)^2 - EL^4 \pi^4 n^4 m^4 h^3 (e_0a)^2 (R_2)^2 + 2E\, L^2 \pi^6 (R_1)^4 (R_2)^2 h^3 m^6 + 24\, L^8 (R_1)^5 n^2 c\nu^2 - 24L^8 (R_1)(R_2)^4 n^2 c + 12EL^8 n^2 h\, (R_1)^2 - EL^2 \pi^6 (R_1)^4 n^2 m^6 h^3 (e_0a)^2 + EL^2 \pi^6 (R_1)^2 (R_2)^4 h^3 m^6 \nu + 2EL^2 \pi^6 (R_1)^2 (R_2)^4 h^3 m^6 - EL^2 \pi^6 (R_1)^2 n^2 m^6 h^3 (e_0a)^2 (R_2)^2 - EL^2 \pi^6 n^2 m^6 h^3 (e_0a)^2 (R_2)^4 - E\pi^8 m^8 h^3 (e_0a)^2 (R_2)^2 (R_1)^4 - 24L^6 \pi^2 (R_1)^5 (R_2)^2 m^2 c - 24\, L^8 (R_1)^5 n^2 c + 2E\, L^8 n^6 h^3 - E\, \pi^8 m^8 h^3 (e_0a)^2 (R_2)^4 (R_1)^2 + 24L^6 \pi^2 (R_1)^5 (R_2)^2 m^2 c\, \nu^2 + 24L^6 \pi^2 (R_1)^3 (R_2)^4 m^2 c\, \nu^2 + 2E\, L^6 \pi^2 (R_1)^2 h^3 m^2 n^4 + E\, L^6 \pi^2 (R_2)^2 h^3 m^2 n^4 \nu + 2EL^6 \pi^2 (R_2)^2 h^3 m^2 n^4 - E\, L^6 \pi^2 n^6 m^2 h^3 (e_0a)^2 + 2EL^4 \pi^4 m^4 n^2 h^3 (R_1)^4 + 2E\, L^4 \pi^4 (R_1)^2 (R_2)^2 m^4 n^2 h^3 + EL^2 \pi^6 (R_1)^4 (R_2)^2 h^3 m^6 \nu + 24\, EL^6 \pi^2 (R_1)^2 m^2 h\, (R_2)^2 + EL^6 \pi^2 (R_1)^2 h^3 m^2 n^4 \nu + 12EL^8 n^2 h\, (R_2)^2\}$

And,

$A_{10} = E\, L^6 \pi^2 (R_1R_2)^2 h^3 m^2 n^2 \{E\pi^4 (R_1)^2 (R_2)^4 h^3 m^4 \nu + E\, \pi^4 (R_1)^2 (R_2)^4 h^3 m^4 + E\, L^2 \pi^2 h^3 m^2 n^2 (R_1)^4 + 2\, E\, L^2 \pi^2 (R_1R_2)^2 h^3 m^2 n^2 \nu + E\, L^2 \pi^2 (R_1)^2 (R_2)^2 h^3 m^2 n^2 \nu^2 + EL^2 \pi^2 (R_1R_2)^2 h^3 m^2 n^2 + E\, L^2 \pi^2 h^3 m^2 n^2 (R_2)^4 + E\, \pi^4 (R_1)^4 (R2)^2 h^3 m^4 \nu + E\, \pi^4 (R_1)^4 (R_2)^2 h^3 m^4 + 24\, L^4 (R_1)^5 (R_2)^2 c\, \nu^2 + 24\, L^4 (R_1)^5 (R_2)^2 \nu^3 c - 24\, L^4 (R_1)^5 (R_2)^2 \nu\, c + 24\, L^4 (R_1)^3 (R_2)^4 c\, \nu^2 + 24\, L^4 (R_1)^3 (R_2)^4 \nu^3 c - 24\, L^4 (R_1)^3 (R_2)^4 \nu\, c + 24\, E\, L^4 (R_1R_2)^2 h\, \nu + EL^4 (R_1)^2 h^3 n^4 \nu + EL^4 (R_2)^2 h^3 n^4 \nu + 24\, E\, L^4 (R_1R_2)^2 h + E\, L^4 (R_1)^2 h^3 n^4 + E\, L^4 (R_2)^2 h^3 n^4 - 24\, L^4 (R_1)^5 (R_2)^2 c - 24\, L^4 (R_1)^3 (R_2)^4 c\}.$

Pcr (using classic elasticity) derived by substitute e_0a by zero. Thus we have:

$Pcr=A/B$

$A=A_1+A_2$

$A_2=A_3+A_4+A_5$

$A_1 = -144\, h\, E\, L^{12} (R_2R_1)^4 \{2c\, (\nu^2-1)\, (R_1)^3 + 2\, c\, (\nu^2-1)\, R_1 (R_2)^2 + E\, h\}$

$A_4 = -E\, \pi^4 (R_2R_1)^4 h^3 m^4 L^2 \{12\, E\, L^6\, h\, (R_1)^2 + 12\, E\, L^6\, h\, (R_2)^2 + E\, h^3 m^4 (R_1R_2)^2 \pi^4 L^2 + 48\, c\, (\nu^2-1)\, L^6 R_1 (R_1R_2)^2 c\}$

$A_5 = -\, E\, L^{12} h^3 n^4 \{E\, h^3 (R_1R_2)^2 n^4 + 12\, E\, h\, (R_1)^2 (R_1R_2)^2 + 12\, E\, h\, (R_2)^2 (R_1R_2)^2 + 24\, c\, (\nu^2-1)\, (R_1)^5 (R_1R_2)^2 + 24\, c\, (\nu^2-1)\, (R_2)^3 (R_1R_2)\},$

$A_3 = E\, L^6 \pi^2 (R_1R_2)^2 h^3 m^2 n^2 \{\, E\, \pi^4 (R_1)^2 (R_2)^4 h^3 m^4 \nu + E\, \pi^4 (R_1)^2 (R_2)^4 h^3 m^4 + E\, L^2 \pi^2 h^3 m^2 n^2 (R_1)^4 + 2\, E\, L^2 \pi^2 (R_1R_2)^2 h^3 m^2 n^2 \nu + EL^2 \pi^2 (R_1)^2 (R_2)^2 h^3 m^2 n^2 \nu^2 + E\, L^2 \pi^2 (R_1R_2)^2 h^3$

$m^2 n^2 + E L^2 \pi^2 h^3 m^2 n^2 (R_2)^4 + E\pi^4 (R_1)^4 (R2)^2 h^3 m^4 v + E \pi^4 (R_1)^4 (R_2)^2 h^3 m^4 + 24 L^4 (R_1)^5 (R_2)^2 c v^2 + 24 L^4 (R_1)^5 (R_2)^2 v^3 c - 24 L^4 (R_1)^5 (R_2)^2 v c + 24 L^4 (R_1)^3 (R_2)^4 c v^2 + 24L^4 (R_1)^3 (R_2)^4 v^3 c - 24 L^4 (R_1)^3 (R_2)^4 v c + 24 EL^4 (R_1R_2)^2 h v + E L^4 (R_1)^2 h^3 n^4 v + EL^4 (R_2)^2 h^3 n^4 v + 24EL^4 (R_1R_2)^2 h + EL^4 (R_1)^2 h^3 n^4 + E L^4 (R_2)^2 h^3 n^4 - 24 L^4 (R_1)^5 (R_2)^2 c - 24 L^4 (R_1)^3 (R_2)^4 c \}$.

8. ACKNOWLEDGMENT

The authors would like to thank the referees for their valuable comments, Mr. Hossein Atoof for his fruitful cooperation, and also, the Iranian Nanotechnology Development Committee for their financial support.

9. REFERENCE

[1] P. Ball, "Roll up for the revolution", Nature, vol. 414, pp. 142-144, 2001.

[2] C. Q. Ru, “Axially compressed buckling of a double walled carbon Nanotube embedded in an elastic medium", Journal of the Mechanics and Physics of Solids, vol. 49, pp. 1265-1279, 2001.

[3] C. Y. Wnag, C. Q. Ru ,A. Mioduchowski, “Axially compressed buckling of pressured multiwall carbon Nanotube”, International Journal of Solids and Structures, vol. 40, pp. 3893-3911, 2003.

[4] X. Guo, A. Y. T. Leung, X. Q. Jiang, Y. Huang, “”, Composite, vol. 39 (part B), pp. 202-208, 2008.

[5] H. L. Ball, W. J. Chang, "A closed-form solution for critical buckling temperature of a single-walled carbon Nanotube", Physica E, vol. 41, pp. 1492-1494, 2009.

[6] A. Ghorbanpour Arani, R. Rahmani, A. Arefmanesh, and S. Golabi, ”Buckling analysis of multi-walled carbon nanotubes under combined loading considering the effect of small length scale”, Journal of Mechanical Science and Technology, vol. 22, pp. 429-439, 2008.

[7] A.R. Ranjbartoreh, A. Ghorbanpour, and B. Soltani, ”Double-walled carbon nanotube with surrounding elastic medium under axial pressure”. Physica E, vol. 39, pp. 230-239, 2007.

[8] A. C. Eringen,”On differential equations of nonlocal elasticity and solutions of screw dislocation and surface waves”, Journal of Applied Physics, vol. 54, pp.4703-4710, 1983.