AN Improved UTD Based Model For The Multiple Building Diffraction Of Plane Waves In Urban Environments By Using Higher Order Diffraction Coeficients

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ABSTRACT

This paper describes an improved model for multiple building diffraction modeling based on the uniform theory of diffraction (UTD). A well-known problem in conventional uniform theory of diffraction (CUTD) is multiple-edge transition zone diffraction. Here, higher order diffracted fields are used in order to improve the result; hence, we use higher order diffraction coefficients to improve a hybrid physical optics (PO)-CUTD model, the results show that the new model corrects errors of the PO-CUTD model. Therefore, the proposed model can find application in the development of theoretical models to predict more realistic path loss in urban environments when multiple-building diffraction is considered.

KEYWORDS

Higher order diffraction coefficient, Multiple-edge Diffraction, UTD

1. INTRODUCTION

Radio wave propagation models play an essential role in planning, analysis and optimization of wireless network. The most important propagation effect on wireless networks is attenuation of the signals due to urban structures. For this reason, finding a realistic model for predicting effects of buildings on radio wave propagation has become an important objective for designers of mobile and wireless networks. Moreover, in order to develop a propagation model for urban environments communication, it is necessary to calculate multiple building diffraction losses. [1-3]. For this purpose, the plane-wave incidence has been used by many researchers to predict the attenuation of the radio wave due to multiple diffraction process. A few models have been introduced, such as Bertoni and Wolfish model, in which the array of the buildings are assumed as a series of absorbing knife edges separated by a constant distance, and a direct numerical method is applied to evaluate the Kirchhoff–Huygens integral [4]. However, this method has some limitations; for instance, it is not valid when the base-station antenna is below the rooftop level. This case is important in path-loss prediction, especially when dealing with interference and multipath fade phenomena in the mobile communications. Furthermore, this proposed function is incorrect for very short range paths, when the settled field actually tends toward unity. It is also assumed that a great amount of buildings are involved, particularly for small values of α (angle of incidence), which may not be valid in practical cases. To overcome these limitations, Bonar and Saunders gave an explicit solution to the Walfish and Bertoni’s problem [5], where an attenuation function was derived in terms of a special function and they subsequently extended this solution to buildings of irregular heights and spacing [6]. Neve and Rowe were among the first groups of researchers to introduce the uniform theory of diffraction (UTD)-based model to predict the multiple knife-edge diffraction of plane waves [7]. In addition, Zhang modified the Neve and Rowe model, propounding a more appropriate base for theoretical analysis [8] and enhanced it by introducing wide-band characteristics of the environment [9]. Juan-Llacer and Cardona presented an explicit solution for multiple edge diffraction in mobile radio-wave propagation in terms of UTD-diffraction coefficients [10]. Kara and Yazgan improved the Zhang’s model by

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incorporating finite conducting right-angle wedges to the model building rows, instead of perfect conducting half-screens (knife-edges) [11]. Tzaras and Saunders proposed an enhanced heuristic UTD solution for multiple edge transition-zone diffraction by using slope diffraction terms [12]. Juan and Llacer introduced the UTD-PO (UTD-Physical Optics) method for evaluating the diffraction loss causes by an array of perfectly conducting wedges using plane-wave approximation [13]. Furthermore, Arablouei and Ghorbani launched a new UTD-based model for predicting the multiple diffraction loss due to buildings. They considered the plane-wave incidence and the buildings to be flat roofed and in parallel rows of dielectric blocks (building rows cross-sections are considered to be in rectangular shapes) [14]. Erricolo, Elia, and Uslenghi analyzed the radio-wave multiple diffraction by considering the building cross-sections to be rectangular and then their results were validated with measurements [15–16]. Finally, Rodriguez, Pardo, and Llacer introduced a new formulation expressed in terms of UTD coefficients for predicting the multiple diffraction formed by an array of finitely conducting buildings, by employing plane-wave approximation and rectangular building's cross-sections [17].

Torabi et al. have introduced a new model for calculating the reflection coefficient used in UTD method [18]. Tajvidy and Ghorbani proposed a new approach to calculate multiple building diffraction loss in microcell environments based on the spherical wave assumption. They showed that the plane wave approximation was not practical for microcell environments [19]. Torabi et al. have introduced a new diffraction coefficient for using in UTD [20]. They proved that there were some errors in the conventional diffraction coefficients and then correct them. In this paper, an improved model based on higher order diffraction coefficients is introduced for predicting multiple diffracted fields caused by an array of dielectric buildings. Authors in [17] used the concepts of PO to produce a solution in terms of UTD-diffraction coefficients in transition zone. Although, their results show that it is not necessary to introduce slope diffraction or higher order UTD diffraction coefficients to achieve a solution in a UTD context, there are some cases of diffraction that need some special care, as conventional UTD and PO are not accurate enough for multiple transition region diffraction in these cases. Detailed simulations are quoted in order to illustrate the new formulation and are compared with the PO-CUTD model.

2. MODEL CONFIGURATION

In Fig. 1 a mobile radio wave propagation path in a built-up area consisting of n buildings made of dielectric blocks with rectangular cross-sections is considered. This configuration can be seen as an array of dielectric joint wedges with interior right angles. Each building is assumed to have the same thickness, spacing, and constant average height relative to the base station antenna. The transmitter can have any arbitrary height (i.e., above, below, or at the same height as the building). As the transmitter is far away from the buildings, so that a plane wave with incident angle α impinges on them.

For the above-mentioned configuration, the diffracted fields were calculated at the observation point (we assumed that the observation point is located at the rooftop of the final building), using the proposed model and considering the effect of the higher order diffracted fields.

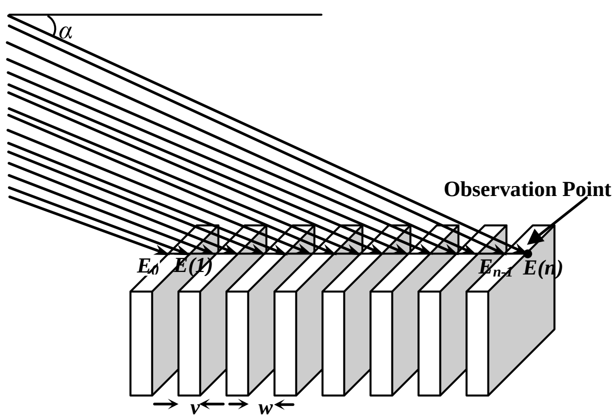

Figure 1: Radio wave propagation in presence of buildings.

3. BASIC THEORY

A. Diffraction Coefficient

The field diffracted by an edge is given by [21]:

$$\vec{E}^d(s) = \vec{E}^i(Q_d)\overline{D}\sqrt{\frac{\rho_e^i}{s(\rho_e^i + s)}}e^{-jks} . \quad (1)$$

$\vec{E}^i(Q_d)$ is the incident field at the diffraction point Q_d and $\overline{D}$ is the diffraction matrix, s is the distance between the observation and the diffraction points and finally the term ρ_e^i is the radius of curvature of the incident wave in the plane of incidence. When the incident wave front is spherical, it coincides with the distance between the diffraction point and the transmitter antenna (s'). In order to evaluate the diffraction coefficient for a finitely conducting wedge, we use Lubber’s heuristic coefficient:

$$D_{s,h}(\phi,\phi',L,n,\beta_0) = D_1 + D_2 + \Gamma_{s,h}(D_3 + D_4) \quad (2)$$

$$D_1 = \frac{-e^{-j\pi/4}}{2n\sqrt{2\pi k}}\cot\left(\frac{\pi + (\varphi - \varphi')}{2n}\right)F\left(kLa^+(\varphi - \varphi')\right) \quad (3)$$

$$D_2 = \frac{-e^{-j\pi/4}}{2n\sqrt{2\pi k}} \cot\left(\frac{\pi + (\varphi - \varphi')}{2n}\right) F\left(kLa^{-}(\varphi - \varphi')\right) \qquad (4)$$

$$D_3 = \frac{-e^{-j\pi/4}}{2n\sqrt{2\pi k}} \cot\left(\frac{\pi + (\varphi + \varphi')}{2n}\right) F\left(kLa^{+}(\varphi - \varphi')\right) \qquad (5)$$

$$D_4 = \frac{-e^{-j\pi/4}}{2n\sqrt{2\pi k}} \cot\left(\frac{\pi + (\varphi + \varphi')}{2n}\right) F\left(kLa^{-}(\varphi - \varphi')\right) \qquad (6)$$

F[x] is the Fresnel transition function, which is defined as [22]:

$$F(x) = 2j\sqrt{x}e^{jx} \int_{\sqrt{x}}^{\infty} \exp\left(-j\tau^2\right) d\tau \qquad (7)$$

The angles ϕ and ϕ' are shown in Fig.2; k is the wave number and n is given by:

$$n = \frac{2\pi - \beta}{\pi} \qquad (8)$$

β is the interior wedge angle.

In the above formulas, L is the distance parameter given by:

$$L = \frac{s\left(\rho_e^i + s\right)\rho_1^i \ \rho_2^i}{\rho_e^i\left(\rho_1^i + s\right)\left(\rho_2^i + s\right)} \qquad (9)$$

$\rho_{1,2}^i$ are radius of curvature of the incident wave front at the diffraction point. For a spherical incident wave front $\rho_e^i = \rho_1^i = \rho_2^i = s'$ and the distance parameter can be shown as follow:

$$L = \frac{ss'}{s + s'} \qquad (10)$$

The function $a^{\pm}\left(\delta^{\pm}\right)$ is given by:

$$a^{\pm}\left(\delta^{\pm}\right) = 2\cos^2\left(\frac{2n\pi N^{\pm} - \delta^{\pm}}{2}\right) \qquad (11)$$

$\delta^{\pm} = \phi \pm \phi'$ and $N^{\pm}$ are the integers which most nearly satisfy the equations:

$$2n\pi N^{+} - \delta^{\pm} = \pi \qquad (12)$$

$$2n\pi N^{-} - \delta^{\pm} = -\pi \qquad (13)$$

Note that $\Gamma_{s,h}$ is the Fresnel reflection coefficients of the wedge at the edge.

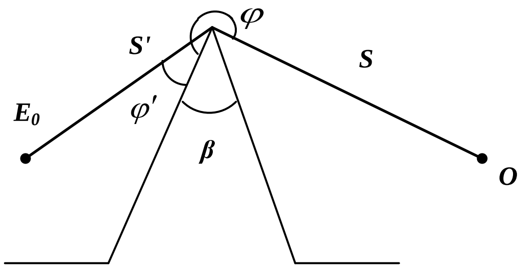

Figure 2: Diffraction by a straight wedge.

B. *Model's Equations*

According to the Heuristic solution [23], the field of the 6 times diffracted ray in Fig. 3 can be written as:

$$E_{UTD} = E_o e^{-jk(s_T - s_1)} \sqrt{\frac{1}{s_2 s_3 s_4 s_5 s_6 s_7}} \times \sum_{i,l,m,n+o=0}^{i+l+m+n+o=N_0} \frac{\partial^i D_1}{\partial \phi_1^i} \frac{1}{i!}\left(\frac{j}{ks_2}\right)^i \frac{\partial^{i+l} D_2}{\partial \phi_2'^i \partial \phi_2^l} \frac{1}{l!} \times \left(\frac{j}{ks_3}\right)^l \frac{\partial^{m+l} D_3}{\partial \phi_3'^l \partial \phi_3^m} \frac{1}{m!}\left(\frac{j}{ks_4}\right)^m \frac{\partial^{m+n} D_4}{\partial \phi_4'^m \partial \phi_4^n} \frac{1}{n!}\left(\frac{j}{ks_5}\right)^n \times \frac{\partial^{n+o} D_5}{\partial \phi_5'^n \partial \phi_5^o} \frac{1}{o!}\left(\frac{j}{ks_6}\right)^o \frac{\partial^o D_6}{\partial \phi_6^o} \qquad (14)$$

E_0 is the incident field at the diffraction point, $s_T = s_1 + \cdots + s_7$, and k is the wave number. Here, D_1 to D_6 are diffraction coefficients. In this formula the upper limit N_0 stands for a chosen order.

The field in (14) can be rewritten in a simpler form by defining $d_i\ (m;n)$ as [23]:

$$d_i(m,n) = \frac{1}{m!}\left(\frac{j}{ks_i}\right)^m \frac{\partial^{m+n} D_i}{\partial \phi_i'^m \partial \phi_i^n} \qquad (15)$$

Using (2), the following compact expression is obtained:

$$E_{UTD} = E_o e^{-jk(s_T - s_1)} \sqrt{\frac{1}{s_2 s_3 s_4 s_5 s_6 s_7}} \times \sum_{i+l+m+n+o=0}^{i+l+m+n+o=N_0} d_1(0,i) d_2(i,l) d_3(l,m) d_4(m,n) d_5(n,o) d_6(o,0) \qquad (16)$$

The expression in (1) can be generalized to more than six wedges. Using the above example, it should be obvious how to generalize the result to more than six edges[2].

For calculating the diffracted field in the transition zone, we explored and improved the approach presented in an earlier study [17]. The total field at the observation point (Fig. 1) was calculated using the summation of fields produced by a single and multiple diffraction process. In the proposed model, all beams except E_{n-1} are multiple diffraction terms, E_{n-1} is single diffraction from the last edge and, therefore, it is not combined with the multiple

[2] One explanation for long computation times when the number of edges turns out to be large is that the series in (16) will engage an enormous number of complex multiplications. The computation plan in [23] will reduce the number of complex multiplications significantly.

diffractions. To present an expression for the overall field calculation, the following argument can be used. If there was only one building between the transmitter and receiver, then the received field at the observation point can be given by

$$E(1)=E_0\left[e^{-jkv\cos\alpha}+\sqrt{\frac{1}{v}}D_x e^{-jkv}\right] \qquad (17)$$

The above formula is made out of two terms, the first term accounts the contribution of the direct field, for $\alpha<0$ (17) must be used without this term, and the second term is to calculate the diffracted field. D_x is denoted as:

$$D_x=D\left(\phi=\frac{3\pi}{2},\phi'=\frac{\pi}{2}+\alpha,L=v\right) \qquad (18)$$

$D(\phi,\phi',L)$ is the diffraction coefficient for an imperfect conducting wedge given in [24]. Furthermore, if the number of buildings is more than one (i.e., when there are a row of buildings $n\geq 2$), then the total received field ($E(n)$) can be summarized as follows:

$$E(n)=\frac{1}{2n-1}\left\{\sum_{m=0}^{n-2}E_m\left[e^{-jk[(n-m)(v+w)-w]\cos\alpha}+\frac{\left(\frac{1}{2}\right)^{n-m}}{\sqrt{v^{n-m}w^{n-(m+1)}}}D_{a,n,m}e^{-jk[(n-m)(v+w)-w]}\right]+E_{n-1}\left[e^{-jkv\cos\alpha}+\sqrt{\frac{1}{v}}D_x e^{-jkv}\right]+\sum_{p=1}^{n-1}E(p)\left[e^{-jk(n-p)(v+w)\cos\alpha}+\frac{\left(\frac{1}{2}\right)^{n-p}}{\sqrt{(vw)^{n-p}}}D_{b,n,p}e^{-jk(n-p)(v+w)}\right]\right\} \qquad (19)$$

$D_{a,n,m}$ and $D_{b,n,p}$ are the higher order diffraction coefficients (their values depend on the number of buildings). The factor of *1/2* in (19) is used to handle the special case of grazing incidence [24]. With reference to Fig.4, if we assume that only E_0 and $E(1)$ are the incident waves at the first and the second wedges, respectively, then the coefficients $D_{a,7,0}$ and $D_{a,7,1}$ are given by the following expressions:

$$D_{a,7,0}=\sum_{A=0}^{N_0}d_1(0,i)d_2(i,l)d_3(l,m)d_4(m,n)d_5(n,o)\ d_6(o,p)d_7(p,q)d_8(q,r)\cdot d_9(r,s)d_{10}(s,t)d_{11}(t,u)d_{12}(u,v)d_{13}(v,0) \qquad (20)$$

$$D_{b,7,1}=\sum_{B=0}^{N_0}d_1(0,i)d_2(i,l)d_3(l,m)d_4(m,n)d_5(n,o)d_6(o,p)d_7(p,q)d_8(q,0)\ d_9(r,s)\cdot d_{10}(s,t)d_{11}(t,u)d_{12}(u,0)$$

(21)

$$A=i+l+m+n+o+p+q+r+s+t+u+v$$

(22)

$$B=i+l+m+n+o+p+q+r+s+t+u$$

(23)

The upper limit N_0 is chosen properly (in this paper, $N_0=4$). E_m is the field impinges on the first corner of the roofs (left-placed wedge forming the rectangular building cross-section) as indicated in Fig.1. Hence, for $m\geq 1$, E_m can be defined as given in (24). In this formula $E(m)$ is the diffracted field from the last building's edge and can be considered as a single diffraction field. Thus, E_m can be described as

$$E_m=\frac{1}{2m}\left\{\sum_{q=0}^{m-1}E_m\left[e^{-jk(m-q)(v+w)\cos\alpha}+\frac{\left(\frac{1}{2}\right)^{m-q}}{\sqrt{(vw)^{m-q}}}D_{c,m,q}e^{-jk(m-q)(v+w)}\right]+\sum_{r=1}^{m-1}E(r)\left[e^{-jk[(m-r)(v+w)+w]\cos\alpha}+\frac{\left(\frac{1}{2}\right)^{m-r}}{\sqrt{v^{m-r}w^{m-(r+1)}}}D_{d,m,r}e^{-jk[(m-r)(v+w)+w]}\right]+E(m)\left[e^{-jkw\cos\alpha}+\sqrt{\frac{1}{w}}D_z e^{-jkw}\right]\right\} \qquad (24)$$

D_z is defined as:

$$D_z=D\left(\phi=\frac{3\pi}{2},\phi'=\alpha,L=w\right) \qquad (25)$$

If the incident fields are E_0 and $E(1)$, then $D_{c,6,0}$ and $D_{d,6,1}$ for seven buildings are given by:

$$D_{c,6,0}=\sum_{B=0}^{N_0}d_1(0,i)d_2(i,l)d_3(l,m)d_4(m,n)d_5(n,o)d_6(o,p)d_7(p,q)\ d_8(q,r)\cdot d_9(r,s)d_{10}(s,t)d_{11}(t,u)d_{12}(u,0) \qquad (26)$$

$$D_{d,6,1}=\sum_{B-u=0}^{N_0}d_1(0,i)d_2(i,l)d_3(l,m)d_4(m,n)d_5(n,o)d_6(o,p)d_7(p,q)\ d_8(q,r)\cdot d_9(r,s)d_{10}(s,t)d_{11}(t,0) \qquad (27)$$

After estimating E_m from (24) and substituting it into (19), $E(n)$ is calculated.

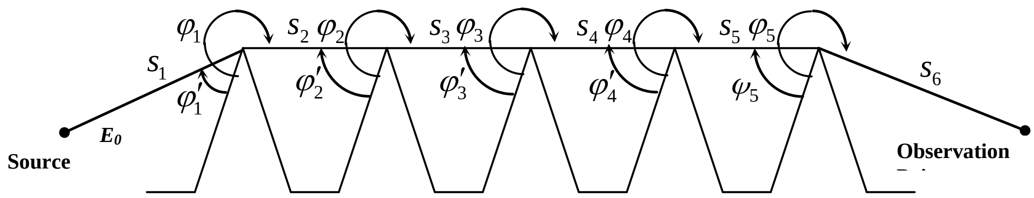

Figure 3: Ray geometry for diffraction by six straight wedges.

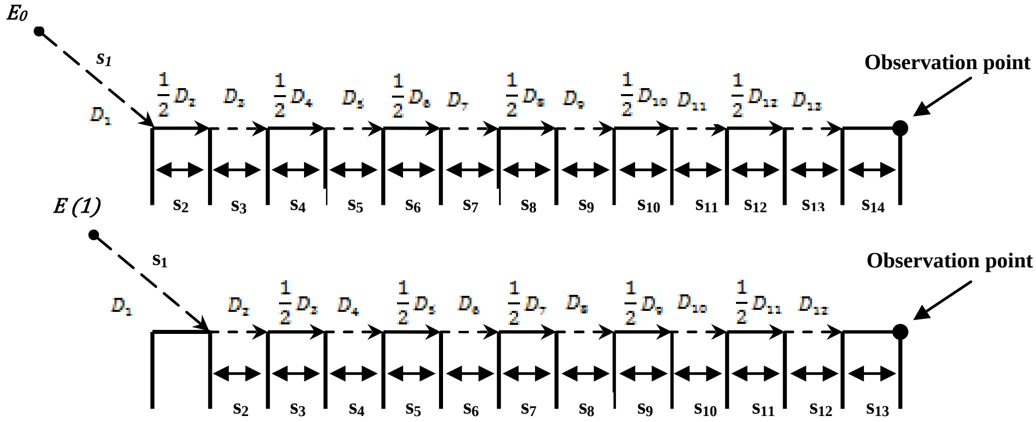

Figure 4: Diffraction mechanism by seven buildings when E_0 and $E(1)$ are considered.

4. Simulation

The normalized total electric field intensity at the observation point against number of existing buildings by using (19) is shown in Fig. 5, for $v = 28\lambda$, $w = 22\lambda$, $f = 922$ *MHz*, $\varepsilon_r = 5.5$, $\sigma = 0.023$ *S/m* and hard polarization. Applied permittivity and conductivity for the diffracting and reflecting building surfaces in the simulation are very close to buildings actual electrical properties.

In Fig. 6 the settled normalized diffracted field (for 10 buildings) is plotted against the angle of incidence of the fields α by using both the new model and the PO-CUTD model in [17]. Here $v = 28\lambda$, $w = 22\lambda$, $\varepsilon_r = 5.5$, $\sigma = 0.023$ *S/m*, $f = 922$ *MHz*, it is expected that $|E(n)/E_0|$ is settled in a fixed value of less than 1, however it is more than one at $\alpha > 5^o$ for the PO-CUTD model and it can be understood there is a critical angle between $5°$ and $6°$ in this model. Consequently we can observe from Fig. 6, more rational and explainable prediction is available by using this proposed model.

In order to review a special case, the normalized total electric field intensity at the observation point against number of existing buildings by using both the new model and the PO-CUTD model is shown in Fig. 7, for $v = 28\lambda$, $w = 50\lambda$, $f = 922$ *MHz*, $\varepsilon_r = 5.5$, $\sigma = 0.023$ *S/m*, $\alpha = 4°$ and hard polarization. Comparison of the two models shows that the PO-CUTD model predicts the amplification while the new model shows the attention. As we know when waves are sent through buildings, it is impossible to be amplified.

Comparing the two different sorts of the results shows that the new improved UTD-based model provides a prediction that is more acceptable and its trend is more accurate.

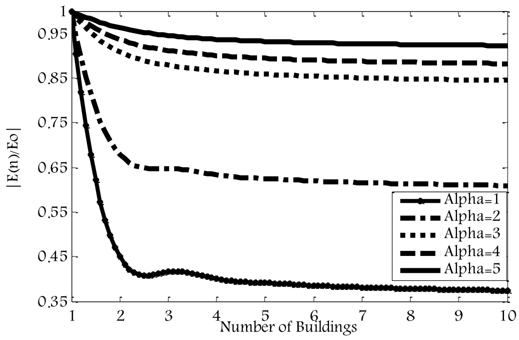

Figure 5: The normalized total electric field intensity at the observation point against number of existing buildings.

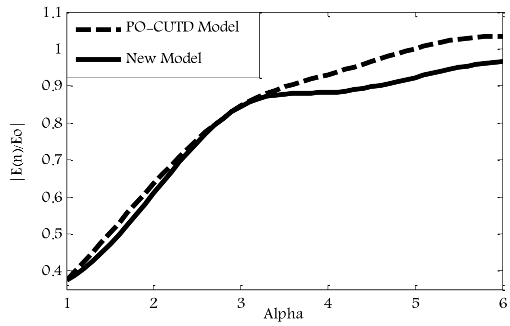

Figure 6: The normalized total electric field intensity at the observation point against the angle of incidence of the fields, for the new model and the PO-CUTD model.

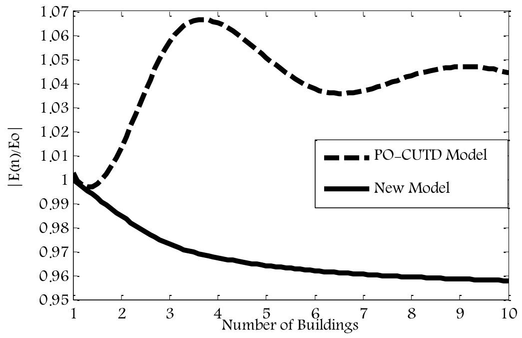

Figure 7: The normalized total electric field intensity at the observation point against number of existing buildings, for the new model and the PO-CUTD model.

5. CONCLUSION

In this paper, an improved model expressed in terms of higher order diffraction coefficients for prediction of the multiple diffraction produced by an array of flat roofed buildings considering the plane-wave incidence has been presented. The results showed that the proposed model can correct errors of the PO-CUTD model in [17]. Our results are the perfect examples for approving this subject. However, it is necessary that higher order diffraction coefficients are applied to improve results especially for $\alpha > 4°$.

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